ОБЩИЕ ВОПРОСЫ АВТОМОБИЛЬНОГО ТРАНСПОРТА

УДК 539.5 = 111

SYNTHESIS OF VISCOELASTIC MATERIAL MODELS (SCHEMES)

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Abstract. *The principles of structural viscoelastic schemes construction for materials with linear viscoelastic properties in accordance with the given experimental data on creep tests are analyzed. It is shown that there can be only four types of materials with linear visco-elastic properties.*

Key words: *viscoelastic model, structural scheme, Hooke element, Newton element, Maxwell model, Kelvin model.*

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Аннотация. *Рассмотрены принципы построения структурных вязкоупругих схем для материалов с линейными вязкоупругими свойствами по заданным экспериментальным данным при испытаниях на ползучесть.*

Ключевые слова: *вязкоупругая модель, структурная схема, звено Гука, звено Ньютона, модель Максвелла, модель Кельвина.*

СИНТЕЗ В'ЯЗКОПРУЖНИХ МОДЕЛЕЙ (СХЕМ) МАТЕРІАЛІВ

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Анотація. *Розглянуто принципи побудови структурних в'язкопружних схем для матеріалів з лінійними в'язкопружними властивостями за заданими експериментальними даними при випробуваннях на повзучість.*

Ключові слова: *в'язкопружна модель, структурна схема, ланка Гука, ланка Ньютона, модель Максвелла, модель Кельвіна.*

Introduction

The stress-strain state modelling of the material with the viscoelastic output characteristics always reveals a question: which rheological model is the most relevant to describe the properties of this material based on the experiment data, for example, creep tests.

State of the art

This problem is not trivial and up to now researchers have to take quite controversial decisions that sometimes even contradict each other. For example, there are more than a dozen of rheological schemes with different structures offered in a specialized literature in the field of asphaltic concrete [1–5].

In many cases such a state makes it difficult to receive comparable results by different authors.

Goal and objective

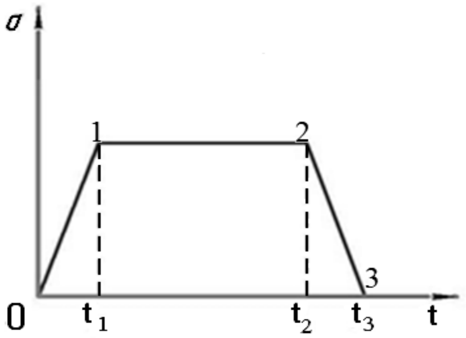
The main task solved in this paper is the creation of an integrated methodology describing the process of viscoelastic material models (schemes) creation based on the experimental data sets.

Expertise of conducting such researches shows that the creep diagrams are the most suitable to be used in order to achieve the stated goal [1, 4].

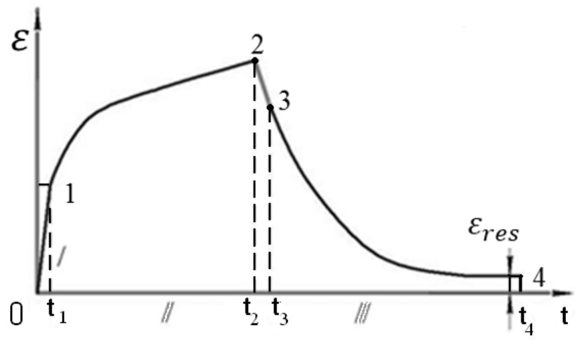
Creep diagram

Usually, there are three specimen loading modes that can be used to get the creep diagram [1–4]:
– axial compression;
– axial tension;
– shear (torsion).

A typical creep diagram is shown in Fig. 1 [1–4].



a



b

Fig. 1. A typical creep diagram: a – stress over time; b – deformation over time; (0–t_1), (t_2–t_3) – regions with mainly elastic deformations; (t_1–t_2) – creep region; (t_3–t_4) – region of smooth deformation decrease; ε_{res} – residual deformations

For the sake of convenience, for the further analysis it will be accepted that application and removal of the load happen instantaneously, which means that

$$t_1 = 0 \text{ and } t_2 = t_3, \quad (1)$$

which is acceptable when $t_2 >> t_1$.

Possible types of the diagrams

It is well known [5-8] that the total deformation of the material shown in Fig. 1b can be described with the following relation

$$\varepsilon_t = \varepsilon_b(t) + \varepsilon_v(t) + \varepsilon_{irr}(t), \quad (2)$$

where $\varepsilon_b(t)$ – reversible elastic deformations; $\varepsilon_v(t)$ – reversible viscoelastic deformations; $\varepsilon_{irr}(t)$, – irreversible (residual) deformations.

Depending on the presence or absence of one or another component, (2) can describe eight different material models (schemes), see Table 1.

Table 1 – Types of viscoelastic models

Type	$\varepsilon_b(t)$	$\varepsilon_v(t)$	$\varepsilon_{irr}(t)$
1	+	+	+
2	–	+	+
3	+	–	+
4	–	–	+
5	+	+	–
6	–	+	–
7	+	–	–
8	–	–	–

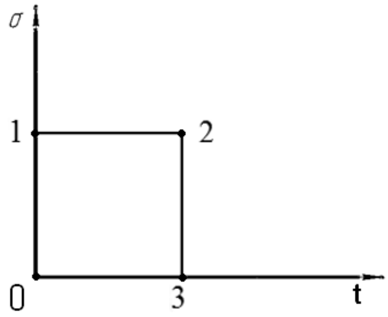
In the Table 1:
– «+» means presence of the corresponding component;
– «–« means absence of the corresponding component.

Absence of regions (0–t_1) and (t_2–t_3) in Fig. 1b, 2b is a sign of the reversible elastic component absence. Linear growth of $\varepsilon(t)$ when σ = =const and absence of the region (t_3–t_4) with the smooth decline of $\varepsilon(t)$ in the Fig. 1b, 2b is a sign of the reversible viscoelastic deformations component absence.

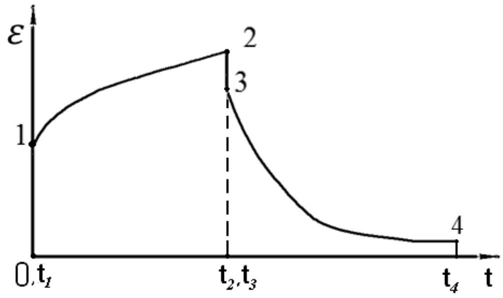
Synthesis of the simplest models

Taking into account (1), the graphs from Fig. 1 will transform to what is shown in Fig. 2.

Material model type 8 in the table 1 has no physical meaning, hence it is not considered below. Material model types 3, 4 and 7 can be obtained using only the simplest viscoelastic and Maxwell elements [9] (Fig. 3, 4). The output characteristics of the models (schemes) shown in Fig. 3, subjected to loading law shown in Fig. 2a, are presented in Fig. 4.

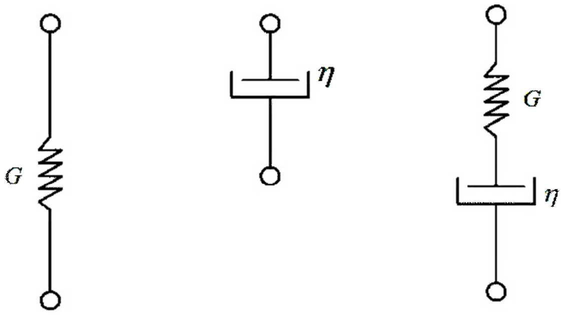


a



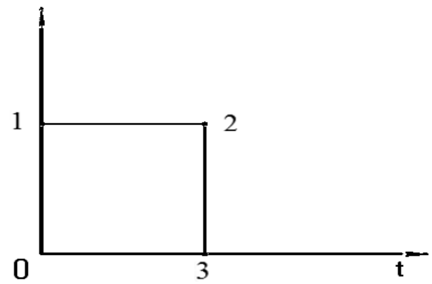
b

Fig. 2. Creep diagram with the «instantaneous» application and removal of the load: a – loading force; b – deformation

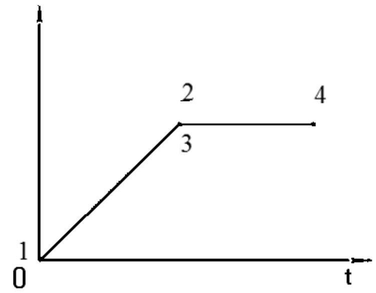


a b c

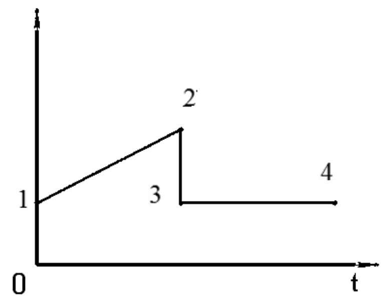
Fig. 3. The simplest elements and Maxwell element G, η – elastic and viscous characteristics of corresponding elements: a – Hooke element as an implementation of the model type 7; b – Newton element as an implementation of the model type 4; c – Maxwell element as an implementation of the model type 3



a



b



c

Fig. 4. The output characteristics of the simplest viscoelastic material models: a – for the model in Fig. 3a; b – for the model in Fig. 3b; c – for the model in Fig. 3c

Synthesis of models using generalized Maxwell and Kelvin models

The models 1, 2, 5 and 6 require reversible viscoelastic deformations so they can be described using the generalized Maxwell and Kelvin models shown in Fig. 5.

Why these models? According to [10], the linear structural model of any complexity can be reduced to a generalized Maxwell model. But there can be made one more conclusion based on the results of [10, 11]: both, complete and incomplete generalized Maxwell models can be transformed into the equivalent generalized Kelvin models and vice versa.

Another advantage of these models is their relatively simple mathematical formulation [11, 12].

The correspondence table between the transformations of these structures can be obtained from their classification [10] (Fig. 6, 7). In this way, all model types from Table 1 are defined.

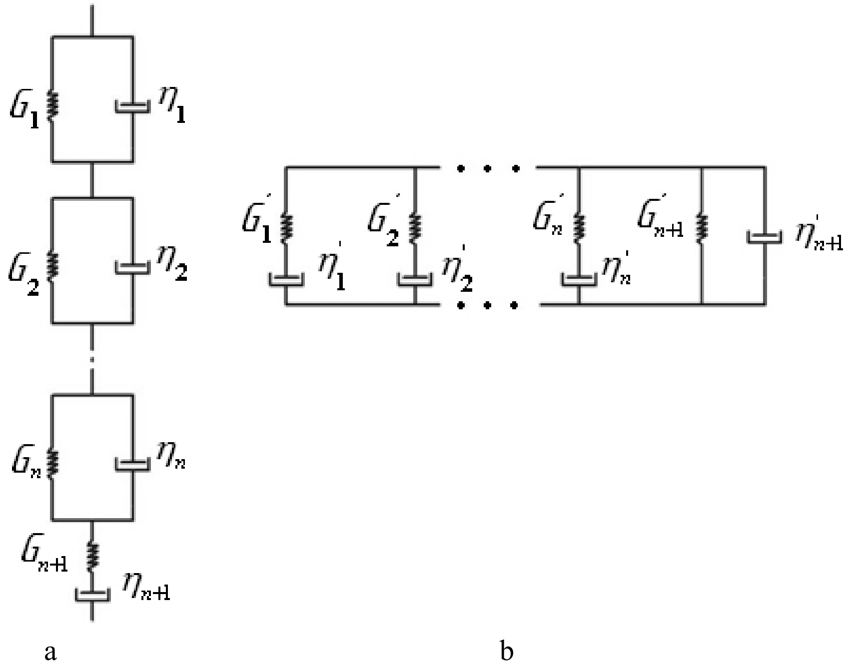


Fig. 5. Complete generalized models: a – Kelvin; b – Maxwell

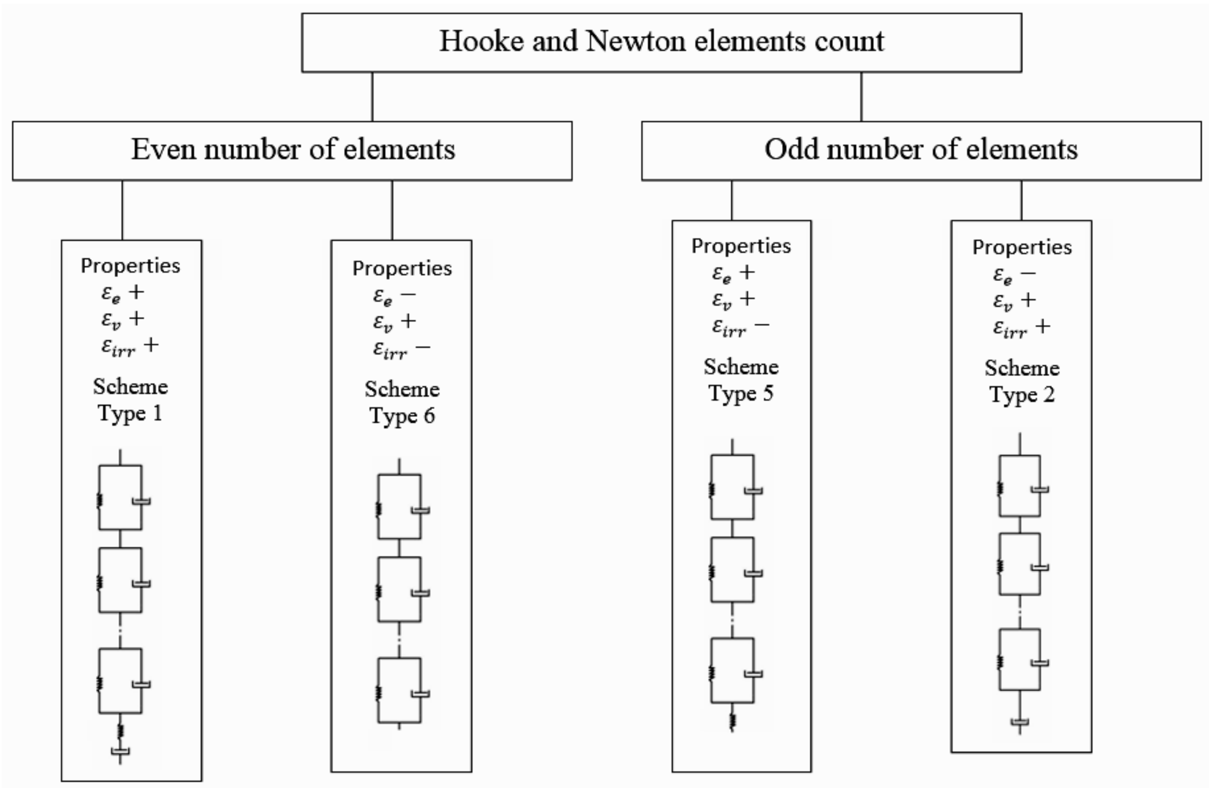


Fig. 6. Classification of the generalized Kelvin models

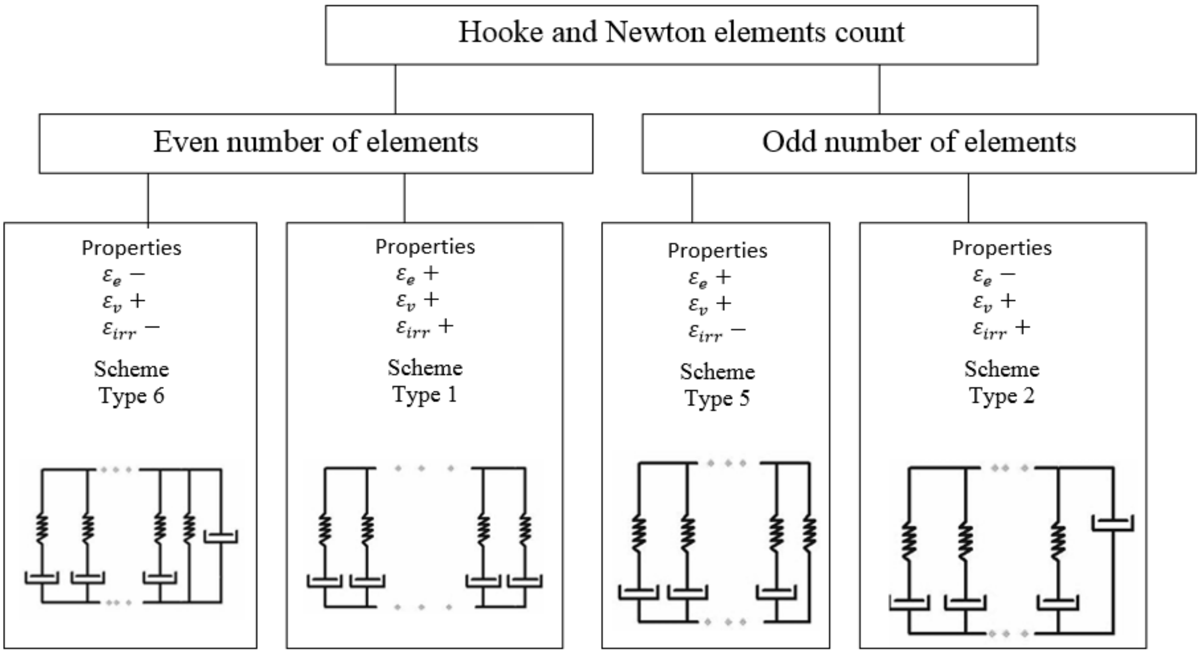
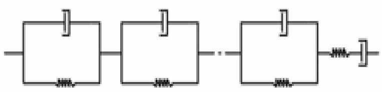
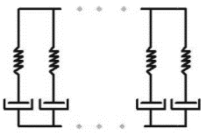
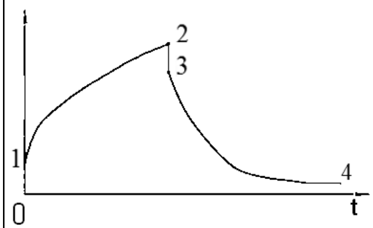
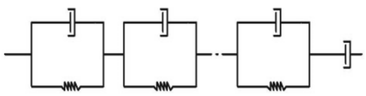
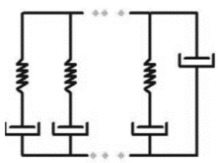
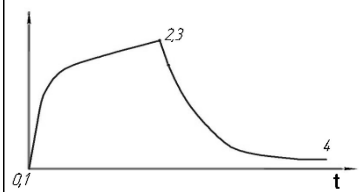
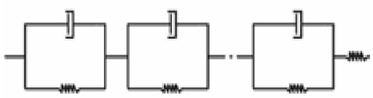
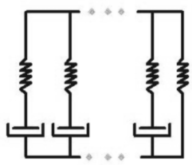
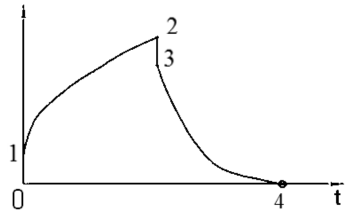
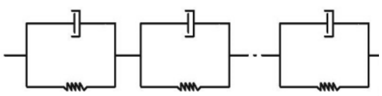
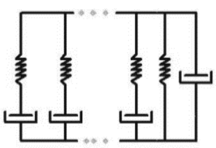
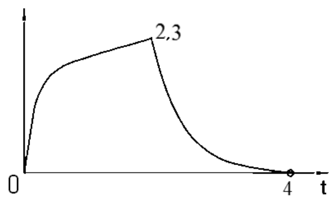


Fig. 7. Classification of the generalized Maxwell models

Table 2 Equivalent generalized Kelvin and Maxwell schemes

Scheme type	Generalized Kelvin scheme	Generalized Maxwell scheme	Output characteristics under load law shown in Fig. 2a
1			
2			
5			
6			

Conclusions

1. The methodology of viscoelastic models (schemes) synthesis based on the experimental characteristics of the material under the creep load is offered.
2. The generalized Kelvin and Maxwell models classification is offered.
3. There are seven types of rheological models offered depending on the experiment data.

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Статья поступила в редакцию 15 сентября 2014 г.