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THEORY OF MECHANISMS AND MACHINES

Lecture notes

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These lecture notes serve as a comprehensive guide to the Theory of Mechanisms and Machines, providing students and professionals in mechanical engineering with a solid foundation in the subject. The content covers key topics such as structure, kinematics, dynamics, force analysis, mechanical transmissions and friction. It delves into the principles governing the design and operation of mechanisms and machines, emphasizing their practical applications in various engineering fields. The lecture notes aim to foster a deep understanding of the fundamental concepts and mathematical methods essential for analyzing and synthesizing mechanical systems. This resource is invaluable for anyone seeking a detailed introduction to the Theory of Mechanisms and Machines, offering insights into the intricate world of mechanical motion and the pivotal role of machines in modern engineering.

The presented material allows you to learn the theoretical foundations of mechanics for the possibility of further study of the following special disciplines.

The lecture notes is intended for students of technical universities majoring in mechanical engineering.

CONTENT

THEORY OF MECHANISMS AND MACHINES (TMM)

Department of Machine Components and Theory of Mechanisms and Machines

Department of MC and TMM is located on the second floor – auditory 236

https://dl.khadi.kharkov.ua/course/view.php?id=2522

Lecture 1

INTRODUCTION TO TMM

A machine is a device that performs certain actions to convert energy or materials in order to replace or facilitate the physical labor of a person.

Machines are classified according to their functions into:

a) power machines; b) working machines.

Energy machines are machines designed to convert some form of energy into mechanical energy and vice versa.

Working machines are machines designed to transform materials.

Working machines are: transport and technological.

Transport machines are machines designed to change the location of material.

Technological machines are machines designed to change the shape, properties, composition, state of materials.

Machines are made up of mechanisms.

In engineering, a mechanism is a device that transforms input forces and motion into a desired set of output forces and motion.

Mechanisms, depending on the purpose, are:

1) Mechanisms of motors and converters (ICE, electric motors, steam engines, turbines; pumps, compressors, hydraulic drives).

2) Transmission mechanisms – transmit motion over a distance (gearboxes, vehicle transmission).

3) Actuators – directly affect the medium being processed (dozer blade, machine tool cutter, drill, milling cutter).

4) Control, control and regulation mechanisms (limiters, regulators, steering gear).

The theory of mechanisms and machines is one of the main engineering disciplines. It is devoted to the study of the most general issues of research and design of mechanisms and machines. Such issues include:

1) study of the structure of mechanisms;

2) determination of positions of mechanisms and points trajectories;

3) determination of speeds and accelerations of any points and links (units) of the mechanism;

4) research and design of various mechanisms (gear, cam and lever);

5) determination of various forces (external, reactions, friction, inertia) acting on the links of the mechanism;

6) the study of the energy balance of machines (Efficiency, etc.);

7) study of the true law of motion of machines under the action of given forces;

8) the study of controlling the speed of the machine;

9) the study of balancing inertial forces in machines, etc.

The theory of mechanisms and machines based on the methods of **mathematics, physics** and **especially theoretical mechanics** to solve its problems.

The theory of machines and mechanisms (TMM) is a science that studies the structure, kinematics and dynamics of mechanisms due to their analysis and synthesis.

Kinematics is a subfield of physics, developed in classical mechanics, that describes the motion of points, bodies (objects), and systems of bodies (groups of objects) without considering the forces that cause them to move.

Dynamics is a subfield of physics, developed in classical mechanics, that describes the motion of points, bodies (objects), and systems of bodies (groups of objects) with considering the forces that cause them to move

Mechanism analysis is the study of the structural, kinematic and dynamic properties of a mechanism.

Mechanism synthesis is the design (creation) of mechanisms with given structural, kinematic and dynamic properties.

Mechanisms are made up of links.

Links can consist of one or more parts.

A kinematic pair (KP) is kinematic pairs, or joints, are considered to provide ideal constraints between two links, such as the constraint of a single point for pure rotation, or the constraint of a line for pure sliding, as well as pure rolling without slipping and point contact with slipping. A mechanism is modelled as an assembly of rigid links and kinematic pairs.

A kinematic pair (KP) is a movable joint of two links.

Kinematic chain (KC) is a system of links interconnected by kinematic pairs.

Kinematic chains are a connected system of links that form kinematic pairs with each other.

Kinematic chains by the nature of the relative motion of the links are divided into **plane** and **spatial**.

A kinematic chain is called **flat** if the points of its links describe trajectories that lie in parallel planes.

A kinematic chain is called **spatial** if the points of its links describe non-planar trajectories or trajectories lying in intersecting planes.

Self-Study and Review

1. What is the fundamental purpose of studying the Theory of Mechanisms and Machines, and how does it relate to engineering and technology?
2. Can you explain the basic concepts and terminology used in the Theory of Mechanisms and Machines, and provide examples of how they are applied in practical engineering design?
3. How does the study of mechanisms and machines play a crucial role in various engineering disciplines, and what are some real-world applications that demonstrate its significance?

4. What are the primary challenges and objectives in the field of mechanisms and machines, and how does this knowledge contribute to improving the efficiency and functionality of mechanical systems?

5. What is the primary definition of a "mechanism" in the context of this theory?

6. In the introductory terminology, what distinguishes a "machine" from a "mechanism," and why is this differentiation important?

Lecture 2

STRUCTURAL ANALYSIS OF MECHANISMS

Structural analysis tasks:

– drawing of the kinematic scheme of the mechanism;

– enumeration of the mechanism’s links.

– calculating the degree of freedom of the mechanism.

– identifying redundant connections in the mechanism.

The mechanism consists of one fixed link (rack) and one or more movable links which moving relative to the rack.

The kinematic scheme is the most simplified drawing of the mechanism (Fig. 1–Fig. 6), taking into account only those dimensions of the links that affect the law of motion and the force interaction of the links.

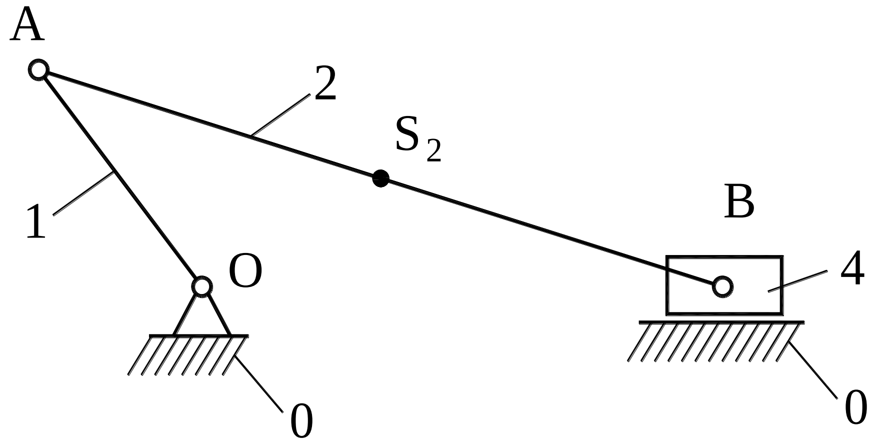


Fig. 1. Slider-crank (crank-and-slider) mechanism

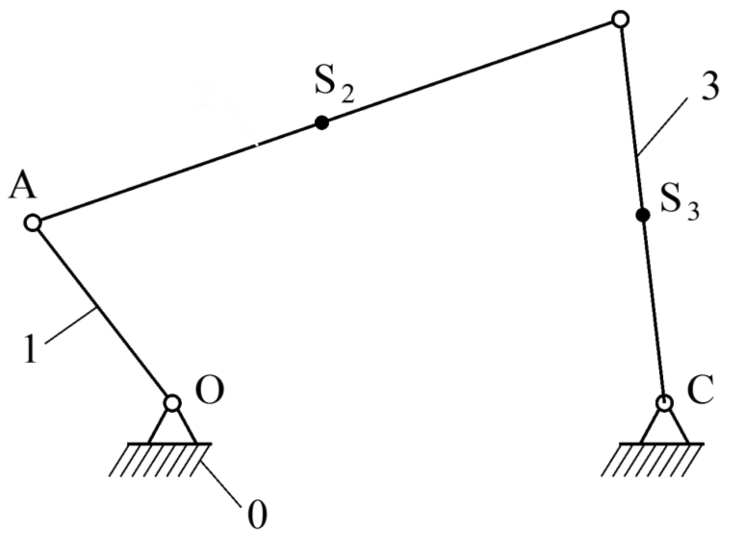


Fig. 2. Rocker-crank mechanism / drag-crank mechanism, (four-link mechanism)

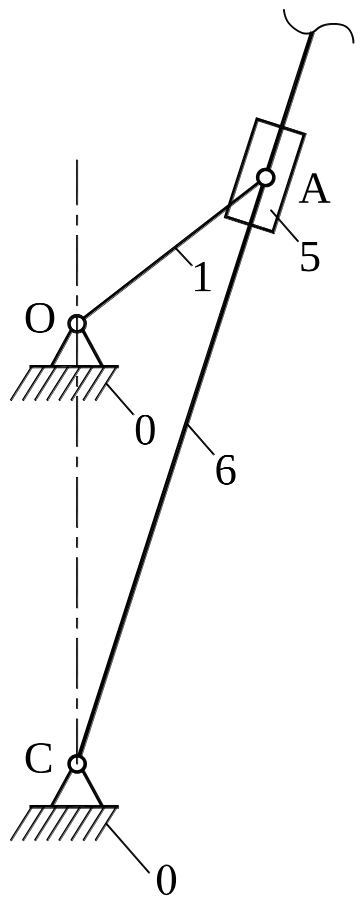


Fig. 3. A crank and slotted-lever mechanism

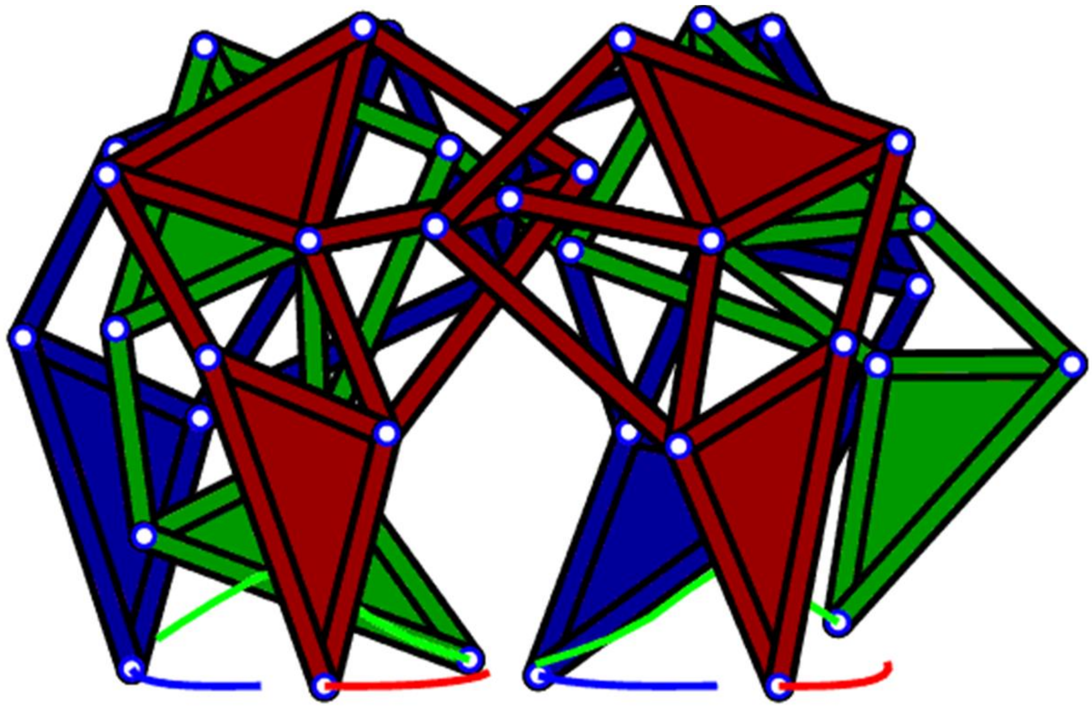


Fig. 4. From walking animation of a Strandbeest with six legs

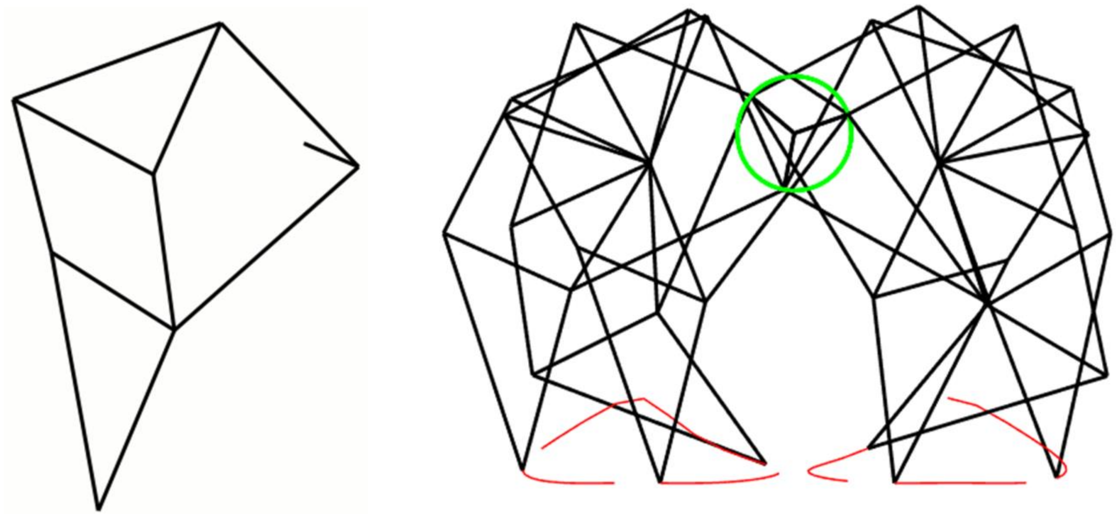


Fig. 5. Black and White Version

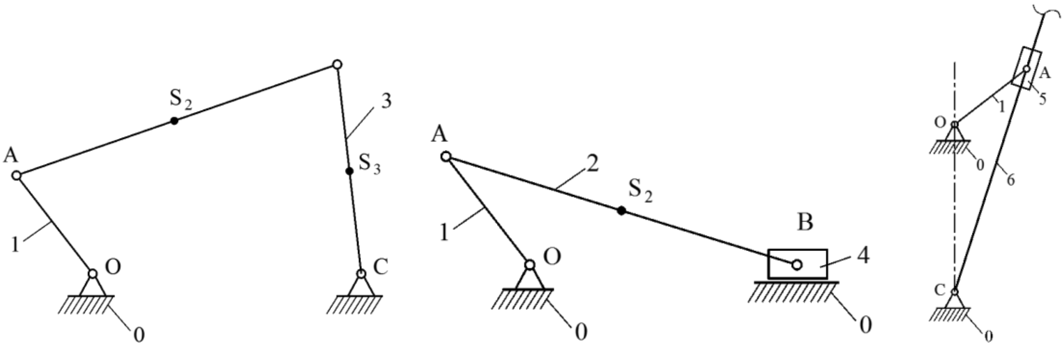


Fig. 6. Links of basic mechanisms

The links have their own names depending on the nature of the motion:

0 – **fixed link** (**rack**) – basic link that is not moving;

1 – **crank** – a link performing a full-turn rotary motion;

2 – **connecting rod** (**coupler**) – a link performing compound (often plane) motion;

3 – **rocker** (**drag**) – a link performing a reciprocating rotary motion or reciprocating swinging motion but not full-turn ;

4 – **slider** – a link performing a translational motion;

5 – **stone** is a slider performing a compound (complex) motion (often plane);

6 – **slotted-lever** – movable stone guide.

The input link is the link to which the motion is imparted.

The output link is the link that makes the required motion.

Intermediate links are the links connecting the input and output links.

The links in the mechanisms are interconnected using kinematic pairs.

A kinematic pair (Fig. 12) is a connection of two contacting links, allowing their relative motion.

A rigid body, freely moving in space, has “**six**” degrees of freedom (DOF). (Fig. 7, Fig. 8)

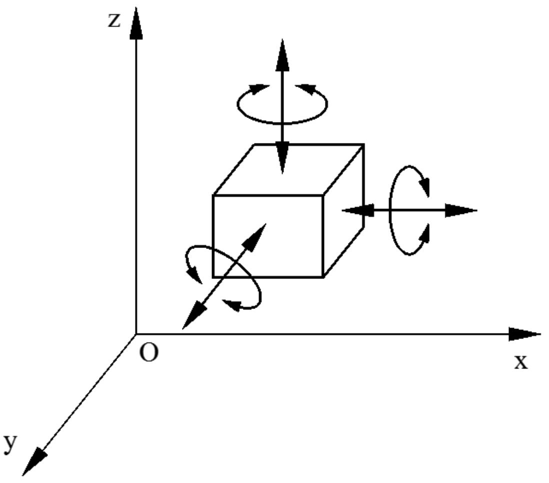


Fig. 7. Degrees of freedom

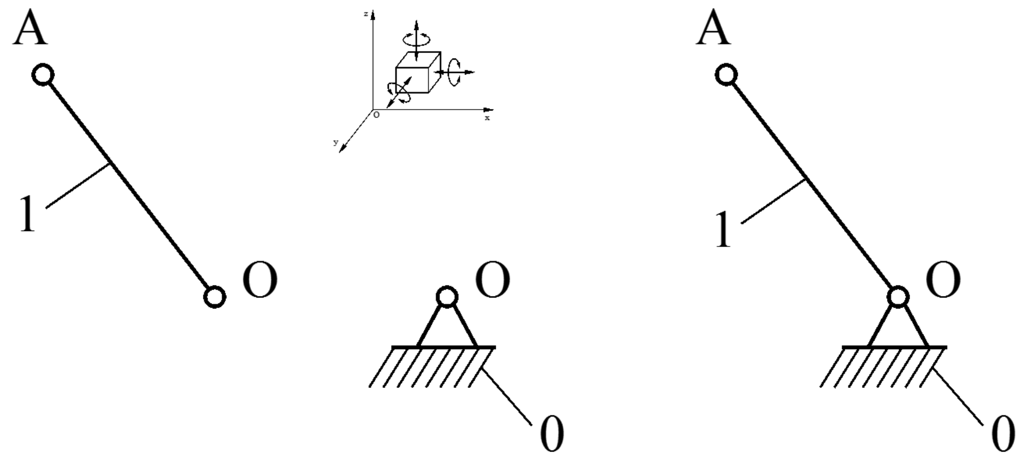


Fig. 8. Number of DOF and constraint conditions

H – number of degrees of freedom.

S – number of constraint conditions.

For KP, H cannot be equal to six, since then the body does not come into contact with another body. For the KP, H cannot be equal to zero, since then the bodies are rigidly connected to each other.

The conditions for the existence of a kinematic pair are: $5 \geq S \geq 5$ and $1 \leq H \leq 5$.

$$S + H = 6 \rightarrow S = 6 - H,\ H = 6 - S.$$

S defines the class of the kinematic pair.

Rotational kinematic pair of the fifth class (Fig. 9).

$$S = 5.\ H = 1.$$

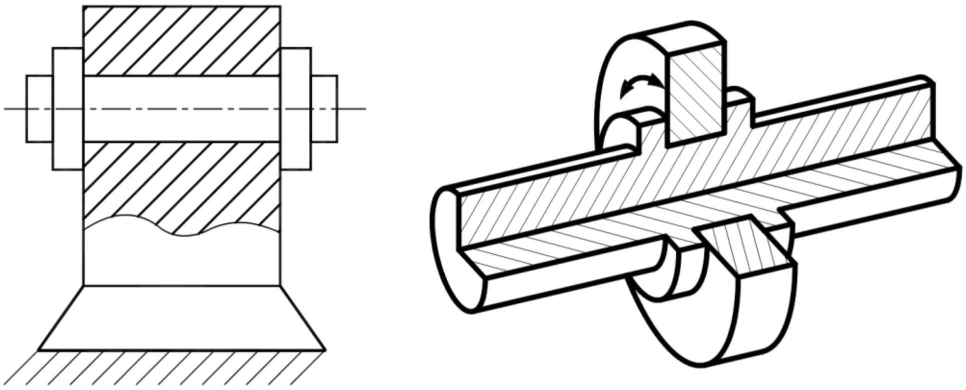


Fig. 9. Rotational kinematic pair

Translational kinematic pair of the fifth class (Fig. 10).

$$S = 5. \ H = 1.$$

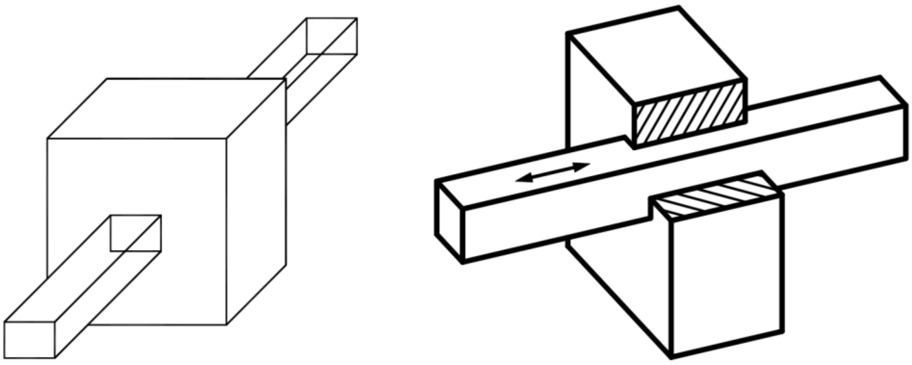


Fig. 10. Translational kinematic pair

Cylindrical kinematic pair of the fourth class (Fig. 11).

$$S = 4. \ H = 2..$$

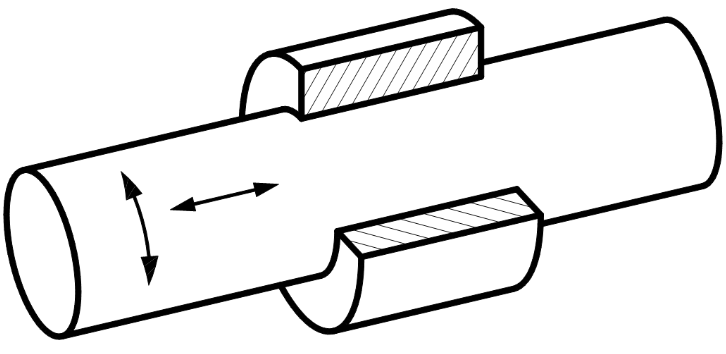


Fig. 11. Cylindrical kinematic pair

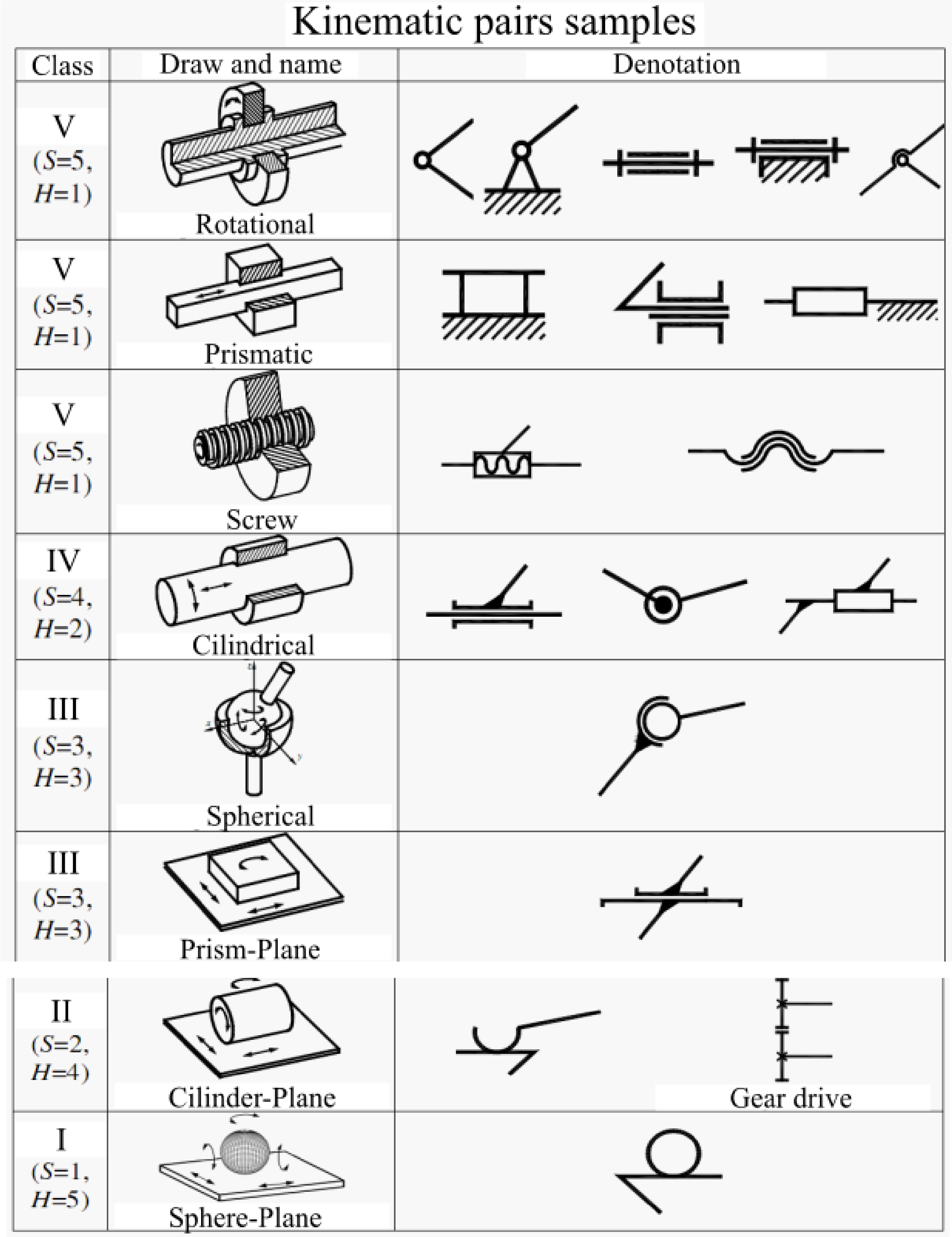


Fig. 12. Kinematic pairs samples

By the number of links, the number and class of kinematic pairs, the degree of mobility of the mechanism is determined.

Kinematic pairs are divided into:

1) Higher

2) Lower.

The Higher KPs are KPs in which the contact of the contacting surfaces is carried out along a line or at a point.

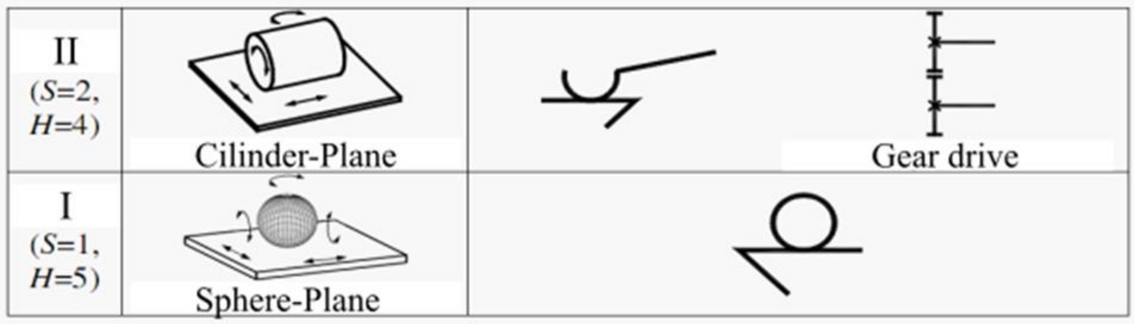


Fig. 13. Higher kinematic pairs

The Lower KPs are KPs in which the contact of the contacting surfaces is carried out on a surface or plane.

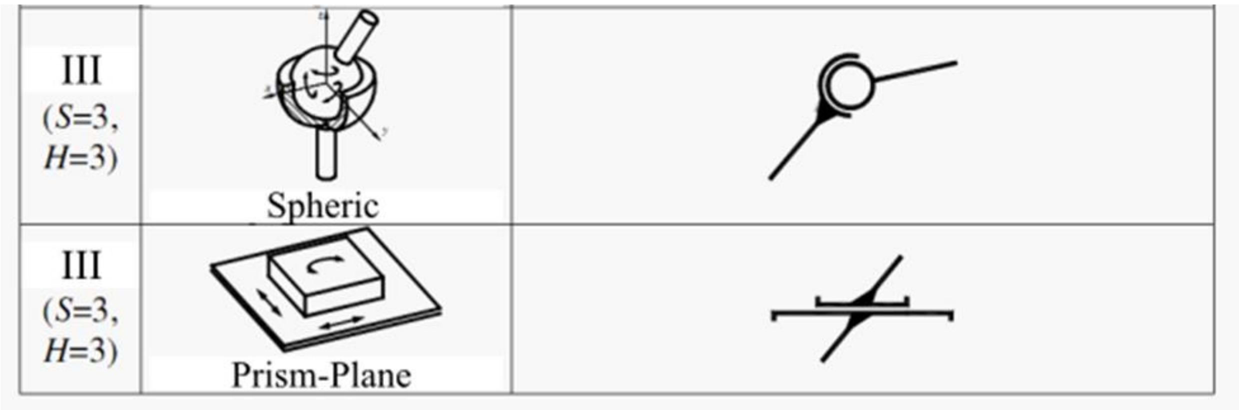


Fig. 14. Lower kinematic pairs

The constancy of the contact of the links is carried out due to the closure.

The closure can be:

– by force;

– by geometric.

The force closure can be carried out due to the forces of gravity, the forces of elasticity of the springs.

The geometric closure can be achieved through the shape of the links.

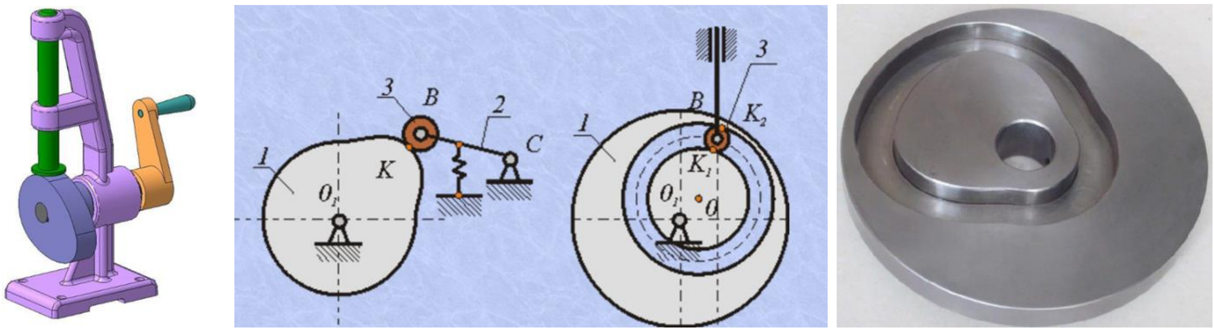


Fig. 15. Cams and cam mechanisms

Self-Study and Review

1 What are the key components of a mechanical system that are considered during structural analysis?

2 Could you explain the role of joints in the structural analysis of mechanisms?

3 How do you define the term "degree of freedom" in structural analysis, and why is it important?

4 How does the structural analysis of mechanisms and machines contribute to the design and optimization of mechanical systems?

5 Can you explain the difference between a binary, ternary, and quaternary linkage mechanism?

Lecture 3

THE NUMBER OF DEGREES OF FREEDOM (DOF) FOR PLANE MECHANISM. CHEBYSHEV'S FORMULA.

The degree of mobility of a mechanism / the number of degrees of freedom (DOF) is the number of independent motions that its links can perform in relation to the rack.

For plane mechanisms, the number of degrees of freedom of the mechanism is determined by the Chebyshev's formula

$$w = 3 \cdot n - 2 \cdot p_5 - p_4$$

$$w = 3 \cdot n - 2 \cdot p_L - p_H ,$$

где w – the number of DOF (the number of mobilities);

n – the number of moving links;

p_L – the number of lower kinematic pairs;

p_H – the number of higher kinematic pairs.

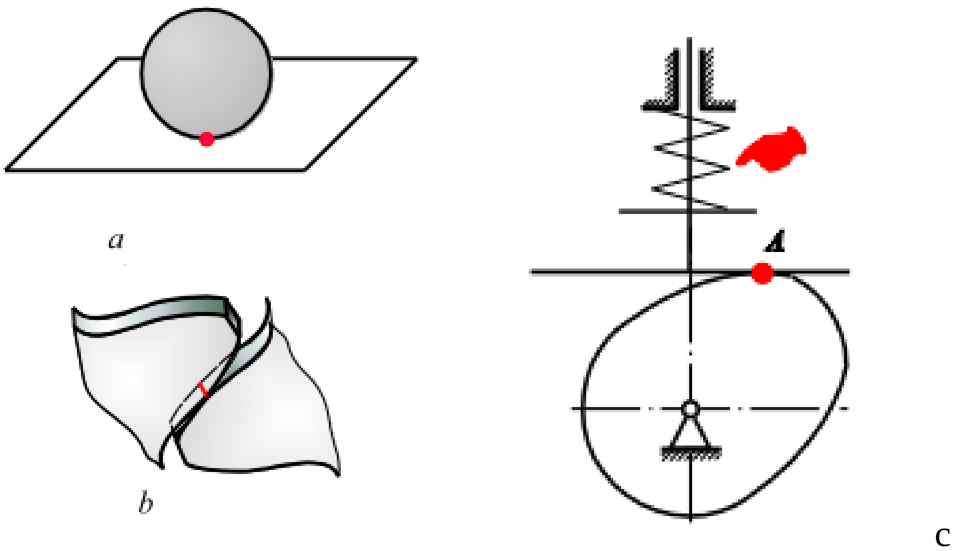


Fig. 16. Higher kinematic pairs:

a – Ball and plane – Point contact;

b – Gear mechanism – Line contact;

c – Cam mechanism – Line contact due to the cams thickness

Example of determining the number of degrees of freedom of the mechanism is shown bellow ().

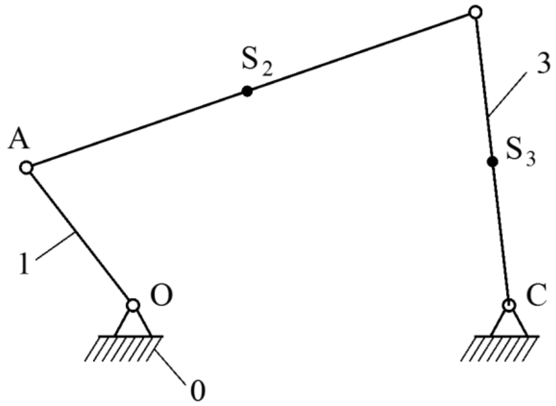


Fig. 17. Crank-and-rocker mechanism

$$w = 3 \cdot n - 2 \cdot p_5 - p_4 = 3 \cdot 3 - 2 \cdot 4 = 1.$$

$$n = 3;\ p_5 = 4;\ p_4 = 0.$$

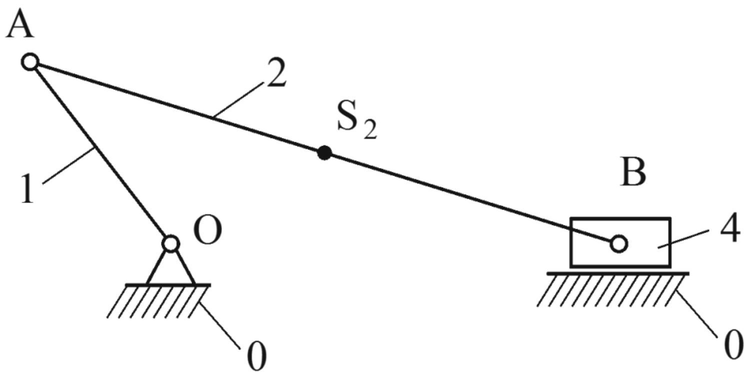


Fig. 18. Crank-and-slider mechanism

$$w = 3 \cdot n - 2 \cdot p_5 - p_4 = 3 \cdot 3 - 2 \cdot 4 = 1.$$

$$n = 3;\ p_5 = 4;\ p_4 = 0.$$

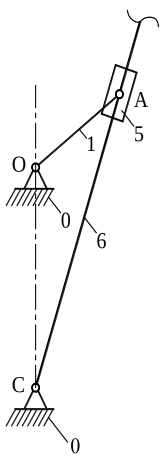


Fig. 19. Crank-and-guide mechanism

$$w = 3 \cdot n - 2 \cdot p_5 - p_4 = 3 \cdot 3 - 2 \cdot 4 = 1.$$

$$n = 3;\ p_5 = 4;\ p_4 = 0.$$

Please find the number of DOF for the presented mechanisms (Fig. 20, Fig. 21)

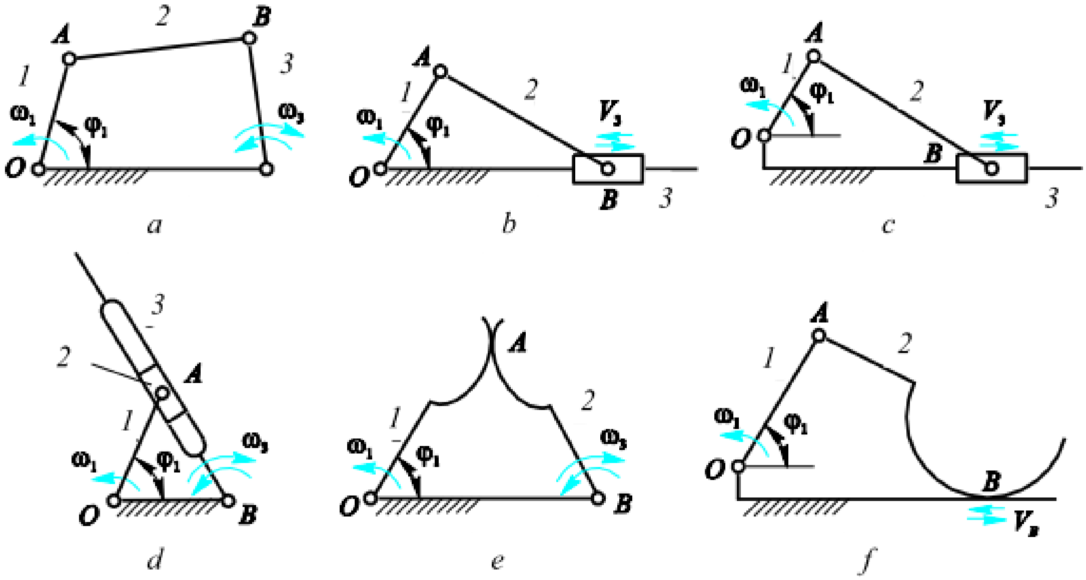


Fig. 20. Examples of mechanisms as closed kinematic chains:

a – crank-and-rocker mechanism; b and c – crank-and-slider mechanisms;

d – crank-and-guide mechanism; e and f – three-bar linkages with both lower and higher kinematic pairs

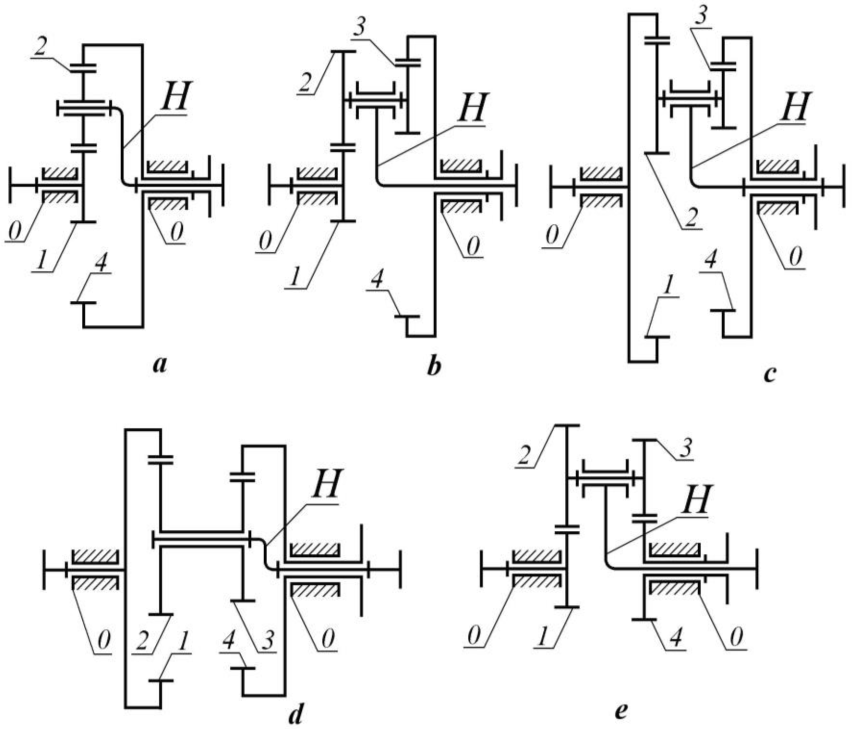


Fig. 21. Examples of satellite mechanisms:

a – single-stage James planetary gear, b and c – double-stage James planetary gears

d and e – David planetary gears with two internal gears (d) and two external gears (e)

Self-Study and Review

1. How is the number of degrees of freedom determined for a given mechanism?
2. Can you explain the relationship between the number of links and the degrees of freedom in a mechanism?
3. How are constraints and joints related to the calculation of degrees of freedom in a mechanism?
4. What role do kinematic pairs play in determining the degrees of freedom of a mechanical system?
5. How does overconstraint affect the degrees of freedom in a mechanism, and what are its implications?
6. Can you provide examples of mechanisms with different degrees of freedom and explain their practical applications?

Lecture 4

KINEMATICS OF LEVER MECHANISMS

Kinematics is a branch of theoretical mechanics that studies the movement of bodies without taking into account the masses and forces that caused this movement.

Basic parameters of movement:

– The Linear Position or Position of the body (point)

– The Linear Displacement

– The Angular Position (also known as orientation, or attitude) of the body.

– The Angular Displacement

– The Trajectory

– The Linear Velocity

– The Angular Velocity

– The Linear Acceleration

– The Angular Acceleration

Short definition:

Trajectory – a line (curve) along which a point moves (for example, the center of mass of a solid).

A trajectory or flight path is the path that an object with mass in motion follows through space as a function of time. In classical mechanics a complete trajectory is defined by position and momentum, simultaneously.

Types of motion

Simplest:

1) Translation

2) Rotation around a fixed axis

Complex:

3) Compound motion

A special case:

3.1) Plane motion ()

Describe the trajectories of link points (Fig. 22, Fig. 23)

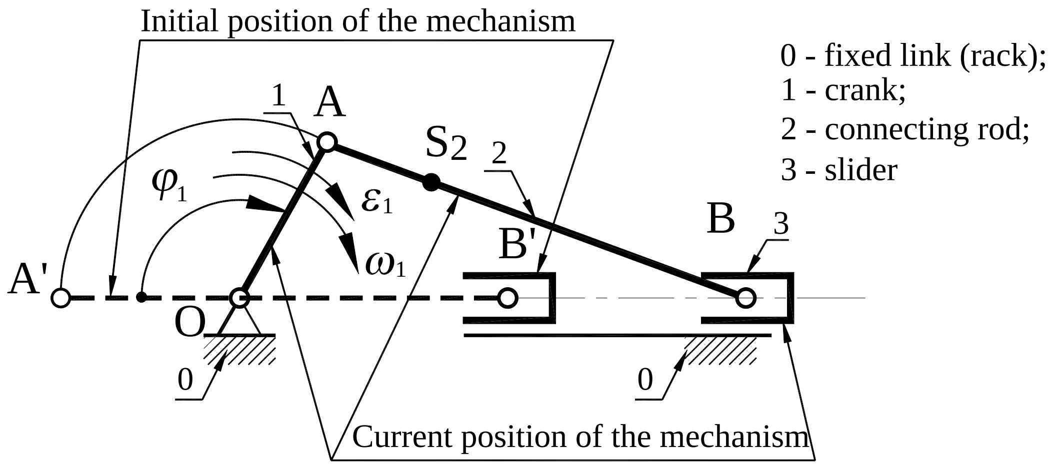


Fig. 22. Crank-and-slider mechanism

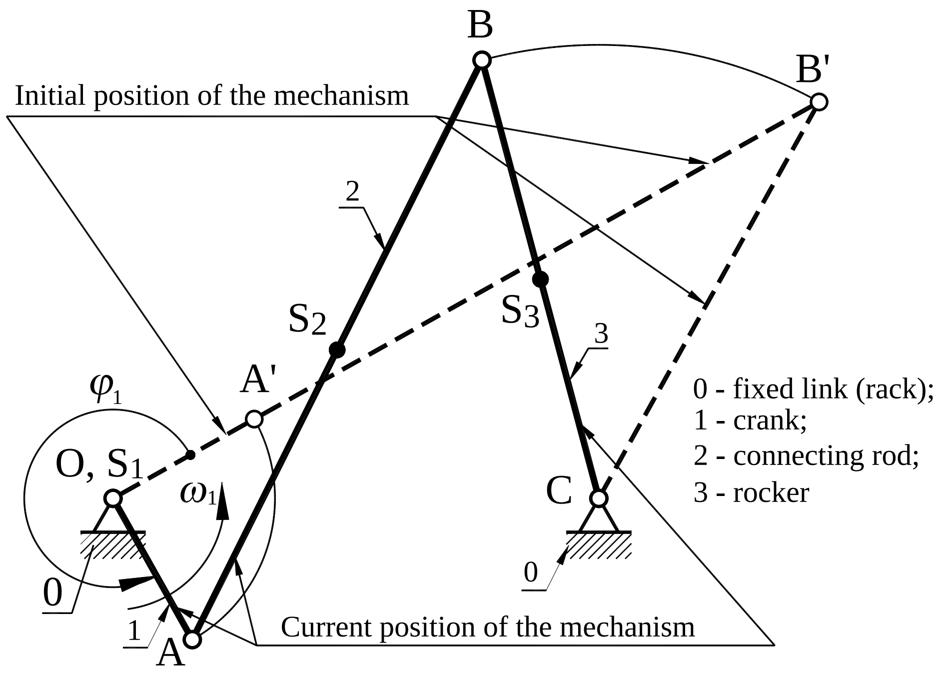


Fig. 23. Crank-and-rocker mechanism

The trajectory of movement of point O –

Link 1 – makes a …

Trajectory of movement of point A -

Link 3 – makes a …

Trajectory of movement of point B –

Link 2 – makes a …

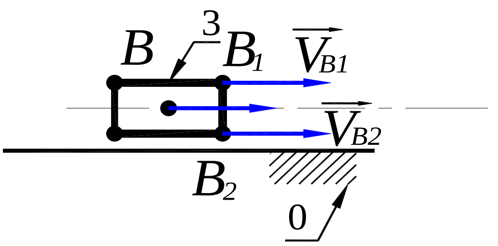
(Link 2 is producing planar motions).

Comparison of translational and rotational motion. Basic formulas

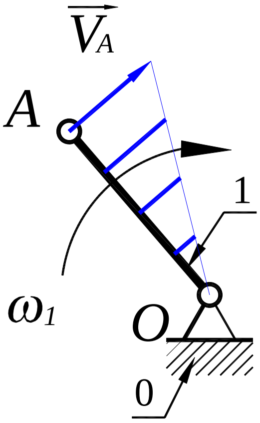
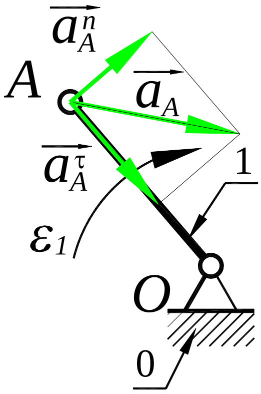
Translation	Rotation
1) Displacement S (X, Y), m	1) The Angular Displacement φ, rad (radians), $^\circ$ (degrees). $$180^\circ = \pi \text{ radians},$$ $$1 \text{ revolution} = 360^\circ = 2\pi \text{ radians}$$
2) Velocity V, m/s (km/h) $$V = \frac{dS}{dt}$$	2) Angular Velocity ω, rad/s (1/s, s^{-1}) $$\omega = \frac{d\varphi}{dt}$$ n – rotation frequency, r/s (rpm) $$\omega = 2\pi \cdot n_s = \frac{\pi \cdot n_m}{30}$$
3) Acceleration a, m/s^2 $$a = \frac{dV}{dt} = \frac{d^2 S}{dt^2}$$	3) Angular Acceleration ε, rad/s^2 $$\varepsilon = \frac{d\omega}{dt} = \frac{d^2\varphi}{dt^2} \quad (1/s^2,\ s^{-2})$$

Motion properties

I Translation

	1) $\overrightarrow{V_B} = \overrightarrow{V_{B1}} = \overrightarrow{V_{B2}}$ 2) $\overrightarrow{V_B} \parallel \overrightarrow{a_B}$ 3) $\overrightarrow{a_B} = \overrightarrow{a_{B1}} = \overrightarrow{a_{B2}}$

II Rotation around a fixed axis

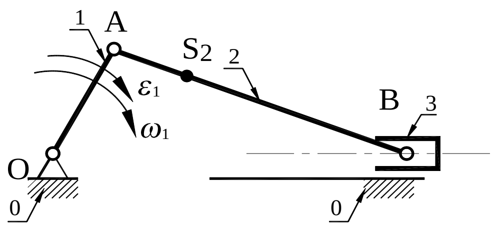
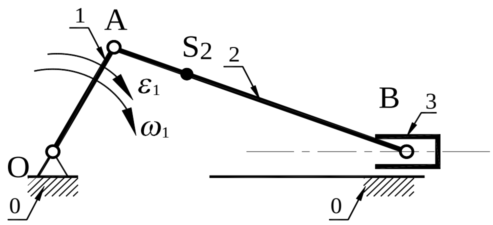
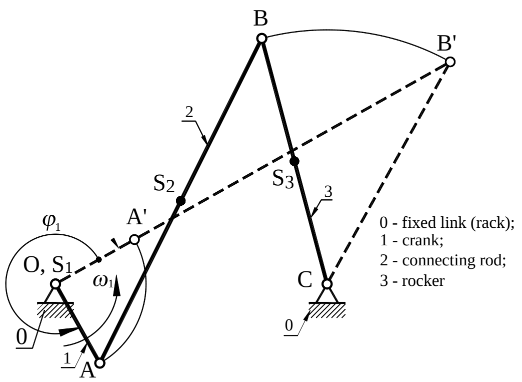
 	1) $\overrightarrow{V_0} = 0,\ V_0 = 0$ $a_0 = 0$ 2) $\overrightarrow{V_A} \perp OA$ $V_A = \omega_{OA} \cdot l_{OA} \quad (V_A = \omega_1 \cdot l_1)$ 3) $\overrightarrow{a_A} = \overrightarrow{a_A^n} + \overrightarrow{a_A^\tau},\ a_A = \sqrt{(a_A^n)^2 + (a_A^\tau)^2}$ $a_A^n = (\omega_{OA})^2 \cdot l_{OA};\ a_A^\tau = \varepsilon_{OA} \cdot l_{OA}$

Normal Acceleration is called centripetal.

The Normal Acceleration vector is always directed from point to center $\overrightarrow{a_A^n} \| OA$.

The Tangential Acceleration is directed towards the angular acceleration of the body (link) $\varepsilon_1 = \varepsilon_{OA},\ \overrightarrow{a_A^\tau} \perp OA$.

III 3.1. Plane motion

	1) $\overrightarrow{V_B} = \overrightarrow{V_A} + \overrightarrow{V_{B/A}}$ $(\overrightarrow{V_B} = \overrightarrow{V_{A/O}} + \overrightarrow{V_{B/A}})$ $\overrightarrow{V_A}$ – velocity of the p. A (linear, absolute); $\overrightarrow{V_B}$ – velocity of the p. B (linear, absolute); $\overrightarrow{V_{B/A}}$ – velocity of the point B around (relative) point A; velocity of relative motion (relative velocity); $V_{B/A} = \omega_2 \cdot l_{AB}$; $\overrightarrow{V_{B/A}} = -\overrightarrow{V_{A/B}}$ $\overrightarrow{V_{B/A}} \perp AB$
	2) $\overrightarrow{a_B} = \overrightarrow{a_A} + \overrightarrow{a_{B/A}}$ $\overrightarrow{a_A} = \overrightarrow{a_A^n} + \overrightarrow{a_A^\tau}$, $\overrightarrow{a_{B/A}} = \overrightarrow{a_{B/A}^n} + \overrightarrow{a_{B/A}^\tau}$ $\overrightarrow{a_B} = \overrightarrow{a_A^n} + \overrightarrow{a_A^\tau} + \overrightarrow{a_{B/A}^n} + \overrightarrow{a_{B/A}^\tau}$ $a_{B/A}^n = (\omega_{AB})^2 \cdot l_{AB}$; $a_{B/A}^\tau = \varepsilon_{AB} \cdot l_{AB}$ $a_{B/A}^n = (\omega_2)^2 \cdot l_{AB}$; $a_{B/A}^\tau = \varepsilon_2 \cdot l_{AB}$
	$\overrightarrow{a_B} = \overrightarrow{a_A} + \overrightarrow{a_{B/A}}$ $\overrightarrow{a_B} = \overrightarrow{a_B^n} + \overrightarrow{a_B^\tau}$ $\overrightarrow{a_B^n} + \overrightarrow{a_B^\tau} = \overrightarrow{a_A^n} + \overrightarrow{a_A^\tau} + \overrightarrow{a_{B/A}^n} + \overrightarrow{a_{B/A}^\tau}$ $a_B^n = (\omega_{OB})^2 \cdot l_{OB}$; $a_B^\tau = \varepsilon_{OB} \cdot l_{OB}$ $a_B^n = (\omega_3)^2 \cdot l_{OB}$; $a_B^\tau = \varepsilon_3 \cdot l_{OB}$

In plane motion for the velocity and the acceleration of point B, we can write a vector equation that is solved:

a) analytically (decomposed into 2 equations of projections on a plane);

b) graphic and analytically (by the method of plans, etc.);

c) numerically.

Self-Study and Review

1 How does the study of kinematics in linkage mechanisms contribute to the design and optimization of mechanical systems?

2 What are the key methods used to analyze the motion and displacement in linkage mechanisms?

3 How is angular velocity defined, and what are its units of measurement?

4 Can you explain the relationship between linear and angular velocity in rotational motion?

5 What are the key differences between translational and rotational motion?

6 What is translational motion, and how does it differ from other types of motion?

7 How is linear velocity defined, and what are its units of measurement?

Lecture 5

KINEMATICS OF LEVER MECHANISMS

III 3.2. Resultant motion (Compound motion / Complex motion) /general case/

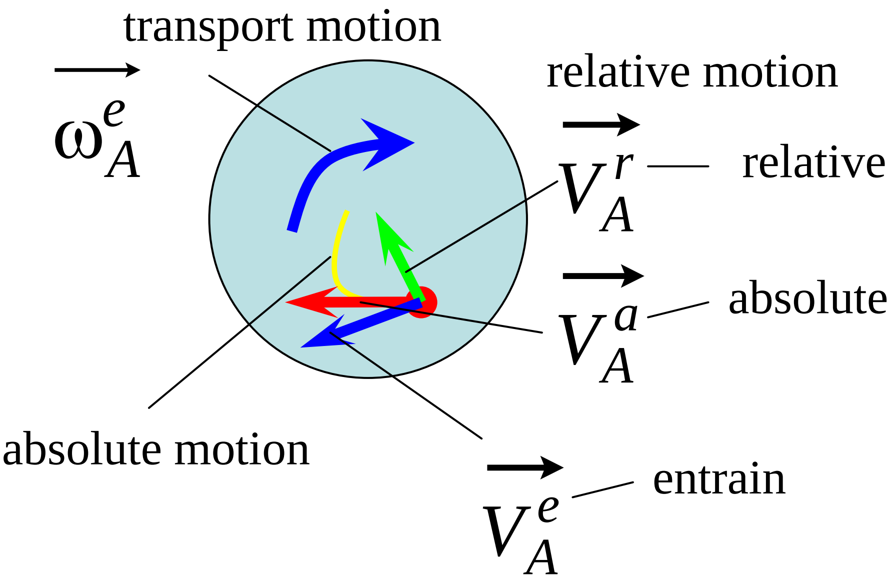


Fig. 24. Compound motion / Complex motion

Velocity (Composition of Velocities)

$$\overrightarrow{V_A^a} = \overrightarrow{V_A^e} + \overrightarrow{V_A^r}$$

$\overrightarrow{V_A^a}$ – the absolute velocity of the point A;

$\overrightarrow{V_A^e}$ – the transport (entrain, bulk) velocity of the point A;

$\overrightarrow{V_A^r}$ – the relative velocity of the point A;

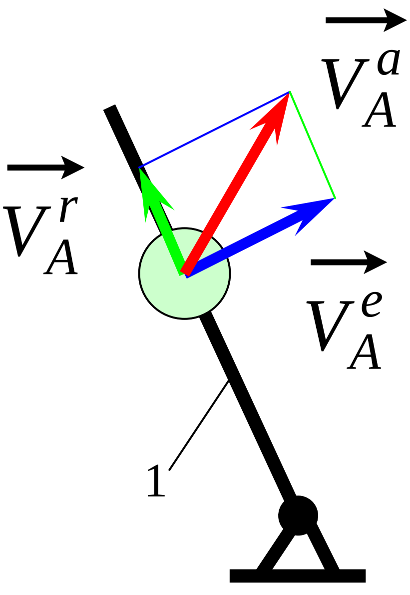


Fig. 25. Velocities for compound motion

Acceleration(Composition of Accelerations)

$$\overrightarrow{a_A^a} = \overrightarrow{a_A^e} + \overrightarrow{a_A^r} + \overrightarrow{a_A^c};$$

$$\overrightarrow{a_A^e} = \overrightarrow{a_{An}^e} + \overrightarrow{a_{A\tau}^e},$$

$\overrightarrow{a_A^e}$ – the transport total acceleration of the point *A*;

$\overrightarrow{a_{An}^e}$ – the transport normal acceleration of the point *A*;

$\overrightarrow{a_{A\tau}^e}$ – the transport tangential acceleration of the point *A*;

$\overrightarrow{a_A^r}$ – the relative acceleration of the point *A*;

$\overrightarrow{a_A^a}$ – the absolute acceleration of the point *A*;

$\overrightarrow{a_A^c}$ – the Coriolis acceleration of the point *A*

Coriolis Acceleration

$$\overrightarrow{a_A^c} = \overrightarrow{a_A^{cor}} = \overrightarrow{a_A^C};$$

$$\overrightarrow{a_A^c} = 2\overrightarrow{\omega^e} \times \overrightarrow{V_A^r};$$

$$a_A^c = 2 \cdot \omega^e \cdot V_A^r \cdot \sin\alpha;$$

$$\alpha = 90° \Rightarrow a_A^c = 2 \cdot \omega^e \cdot V_A^r.$$

Zhukovsky's Rule for determining Coriolis Acceleration direction $\overrightarrow{a_A^c}$:

For determining the direction Coriolis Acceleration vector $\overrightarrow{a_A^c}$ you need a vector of relative velocity $\overrightarrow{V_A^r}$ turn on 90 degrees in the direction of the angular velocity of the transport motion ω^e.

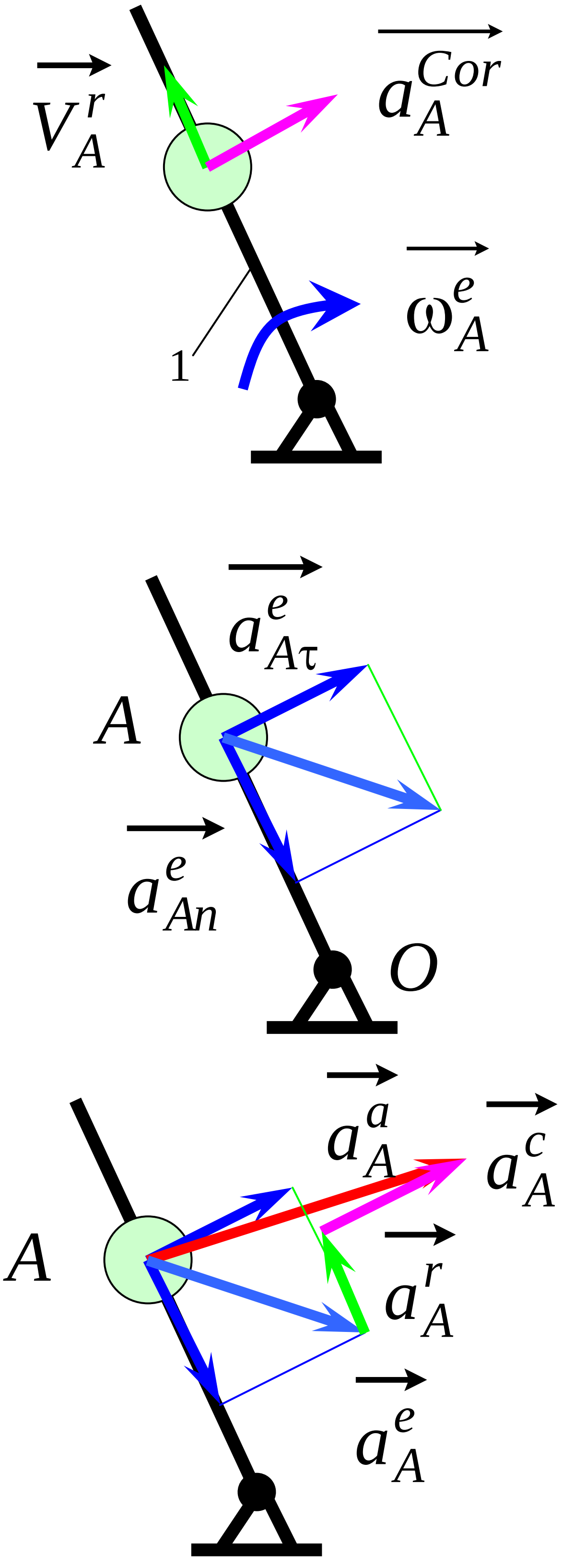


Fig. 26. Accelerations for compound motion

Self-Study and Review

1 How do you describe the superposition of translational and rotational motion in complex systems?

2 What are the key principles and equations used to analyze complex motion in mechanisms and machines?

3 How is velocity and acceleration analysis performed for complex motions?

Lecture 6

VELOCITIES PLAN AND ACCELERATIONS PLAN

Velocities plan is a vector diagram that shows the velocities vectors of all points of the mechanism to scale.

The accelerations plan is a vector diagram, on which the accelerations vectors of all points of the mechanism are shown to scale**.**

The scale is the ratio of the actual size to the line segment corresponding to it.

The Scale of Kinematic Schema

$$\mu_l = \frac{l_{OA}}{OA}\left(\frac{m}{mm}\right),$$

where l_{OA} is the length of the link OA,

OA is the line segment on kinematic schema, which corresponds to the link OA.

The drawing of the plan of velocities begins with the velocity of the point A:

$$V_A = \omega_1 \cdot l_{OA} \text{ (m/s)}.$$

Velocity vector of point A $\vec{V}_A$ (line segment $p_V a$ on diagram of velocities vectors) is perpendicular to the crank OA and directed towards the angular velocity of the crank ω_1. The length of the line segment $p_V a$ is recommended to be taken as 100 mm.

The Scale of Velocities

$$\mu_V = \frac{V_A}{p_V a}\left(\frac{\text{m/s}}{\text{mm}}\right).$$

Velocity vector of point B is determined by graphically solving the vector equation

$$\overrightarrow{V_B} = \overrightarrow{V_A} + \overrightarrow{V_{B/A}}$$

where $\overrightarrow{V_A}$ Velocity vector of point A (is already on the velocities plan. This is a line segment $p_V a$);

$\overrightarrow{V_{B/A}}$ the velocity of the relative rotational motion of point B around point A, direction $\overrightarrow{V_{B/A}}$ (line ab on the velocities plan) is perpendicular to the radius of rotation, i.e. to the link AB);

$\overrightarrow{V_B}$ – the velocity of point B in its rotational or translational motion relative to the rack, direction $\overrightarrow{V_B}$ (line $p_V b$ on the velocities plan) is perpendicular to the rocker BC or parallel to the movement of the slider (to the rack).

The order of drawing a velocities plan:

1) Choosing the pole of the velocities plan p_V;

2) Choosing the scale of the velocities plan μ_V;

3) Draw a line segment $p_V a$;

4) Draw a line ab through the point a ;

5) Draw a line $p_V b$ through the point p_V ;

6) There is a point a at the intersection of the lines ab and $p_V b$.

Using the drawn velocities plan, you can find the velocities of any point of each link of the mechanism by multiplying the corresponding segment from the velocities plan by the calculated velocities scale μ_V.

In addition, it is easy to determine the magnitude and direction of the angular velocities of the links having rotational motion (including for relative rotation). For example, the angular velocity of the connecting rod 2.

$$\omega_2 = \frac{V_{B/A}}{l_{AB}} = \frac{(ab)\mu_V}{l_{AB}} \left(\frac{\text{rad}}{\text{s}}\right),$$

and the direction of angular velocity ω_2 is determined by the transfer of the vector $\overrightarrow{V_{B/A}}$ from the plan of velocities to point B on the kinematic schema of the mechanism.

Similarity theorem for Vector diagram of velocities (plan of velocities)

The plan of velocities for points of any link is a figure similar to the figure formed by the same points on the link, and rotated relative to the links (kinematic schema) by 90 degrees.

Drawing the accelerations plan

It is first necessary to write down the system of vector equations.

The drawing of the accelerations plan should be started by determining the acceleration of point A:

$$\overrightarrow{a_A} = \overrightarrow{a_A^n} + \overrightarrow{a_A^\tau},$$

where $\overrightarrow{a_A}$ is the total acceleration of the point A,

$\overrightarrow{a_A^n}$ is the normal acceleration of the point A :

$$a_A^n = (\omega_1)^2 \cdot l_{OA},$$

Since the normal acceleration is directed along the radius to the center of rotation, then

$\overrightarrow{a_A^n}$ is parallel to OA and directed from point A to point O.

$\overrightarrow{a_A^\tau}$ is the tangential acceleration of the point A :

$$a_A^\tau = \varepsilon_1 \cdot l_{OA}.$$

The tangential acceleration is directed towards the angular acceleration of the body (link) $\varepsilon_1 = \varepsilon_{OA}$, $\overrightarrow{a_A^\tau} \perp OA$.

In the drawing, we select a point (pole) p_a and parallel to OA (in the direction from point A to point O) we draw line segment $p_a n_1$ (the length of the segment is recommended to be taken as 100 mm).

Accelerations Plan Scale:

$$\mu_a = \frac{a_A^n}{p_a n_1} \left(\frac{\text{m/s}^2}{\text{mm}}\right).$$

Next, we draw the line segment $n_1 a$ perpendicular to the segment $p_a n_1$. The length of the segment $p_a n_1$ is determined by the formula through the scale:

$$n_1 a = \frac{a_A^\tau}{\mu_a}.$$

The $p_a a$ line segment corresponds to the full acceleration of point A, we can measure its value on the plane of accelerations.

$$a_A = p_a a \cdot \mu_a$$

We draw a plan of accelerations for links 2 and 3. The vector equation for this links has the form:

$$\underline{\vec{a}_B} = \underline{\underline{\vec{a}_A}} + \underline{\underline{\vec{a}^n_{B/A}}} + \underline{\vec{a}^\tau_{B/A}}; \qquad \underline{\vec{a}_B} = \underline{\underline{\vec{a}^n_A}} + \underline{\underline{\vec{a}^\tau_A}} + \underline{\underline{\vec{a}^n_{B/A}}} + \underline{\vec{a}^\tau_{B/A}};$$

$$\parallel \mathrm{AB} \quad \perp \mathrm{AB} \qquad\qquad \parallel \mathrm{OA} \perp \mathrm{AB} \parallel \mathrm{AB} \perp \mathrm{AB}$$

$$(\overrightarrow{p_a b}) = (\overrightarrow{p_a a}) + (\overrightarrow{an_2}) + (\overrightarrow{n_2 b}); \quad (\overrightarrow{p_a b}) = (\overrightarrow{p_a n_1}) + (\overrightarrow{n_1 a}) + (\overrightarrow{an_2}) + (\overrightarrow{n_2 b});$$

where $\vec{a}_B$ the absolute acceleration of the point B;

$\vec{a}^n_{B/A}$ is the vector of normal acceleration of the relative rotational motion of point B around point A, the value of which is determined by the formula $a^n_{B/A} = \omega_2^2 \cdot l_{AB}$ (directed parallel to the link AB from point B to the center of relative rotation point A);

$\vec{a}^\tau_{B/A}$ is the vector of the tangential acceleration of the relative rotational motion of point B around point A, (the magnitude of this acceleration is unknown before the drawing of the plan of accelerations, but we knew the direction of this vector, it is perpendicular to link AB / line AB on kinematic schema);

In accordance with the vector equation from point ***a*** of the accelerations plan, we draw the line segment an_2

$$an_2 = a^n_{B/A} / \mu_a,$$

который изображает нормальное ускорение $\vec{a}^n_{B/A}$ в выбранном масштабе μ_a. The line segment an_2 is parallel to AB and is directed from point B to point A. Further, through point n_2 (the end of the segment an_2,) we draw a straight line $n_2 b$ perpendicular to AB, that is, in the direction of acceleration $\vec{a}^\tau_{B/A}$.

From the pole p_a in the direction of the vector $\vec{a}_B$ we draw a straight line $p_a b$ to the intersection with the straight line previously drawn from the point n_2. The point of intersection, i.e., point b determines the values of the line segments $p_a b$ and $n_2 b$.

Using the accelerations plan, you can find the angular acceleration of the rotating links. For example, the angular acceleration of the connecting rod 2 (rad/s^2)

$$\varepsilon_2 = a^\tau_{B/A} / l_{AB} = (n_2 b \cdot \mu_a) / l_{AB},$$

where l_{AB} is length of connecting rod (link 2), m;

n_2b is the segment of the plan of accelerations, displaying the vector $\vec{a}^{\tau}_{B/A}$, measured in mm.

To determine the direction ε_2, we transfer the vector to point B on the kinematic schema of the mechanism (similar to the velocity vector $\vec{V}_{B/A}$). The direction ε_2 is the same as the direction of the vector $\vec{a}^{\tau}_{B/A}$.

The accelerations of the centers of gravity of links 2 and 3 (for the crank-and-roke mechanism, that is, in the case when link 3 rotates) is determined in accordance with the similarity theorem for the accelerations plan. For example, for link 2: $as_2 = ab \cdot AS_2 / AB$, then $a_{S_2} = p_a s_2 \cdot \mu_a$.

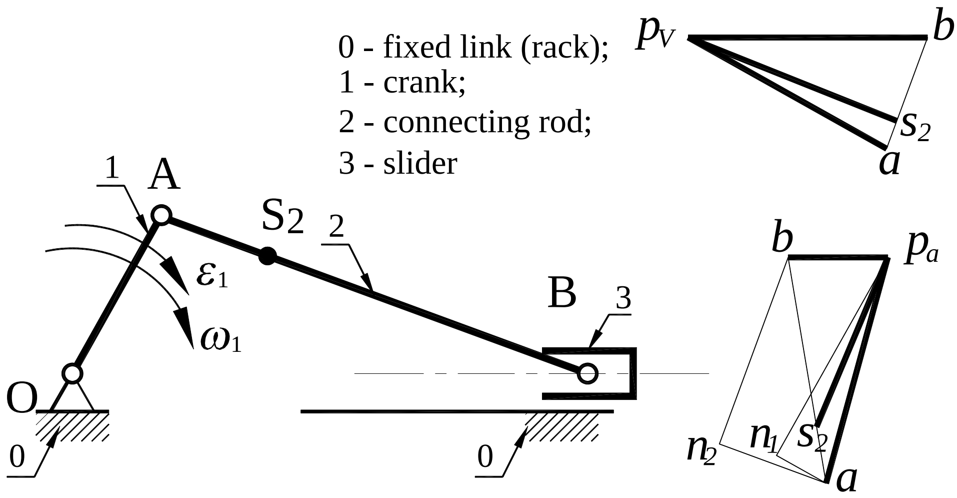


Fig. 27. Velocities and accelerations plans for slider-crank mechanism

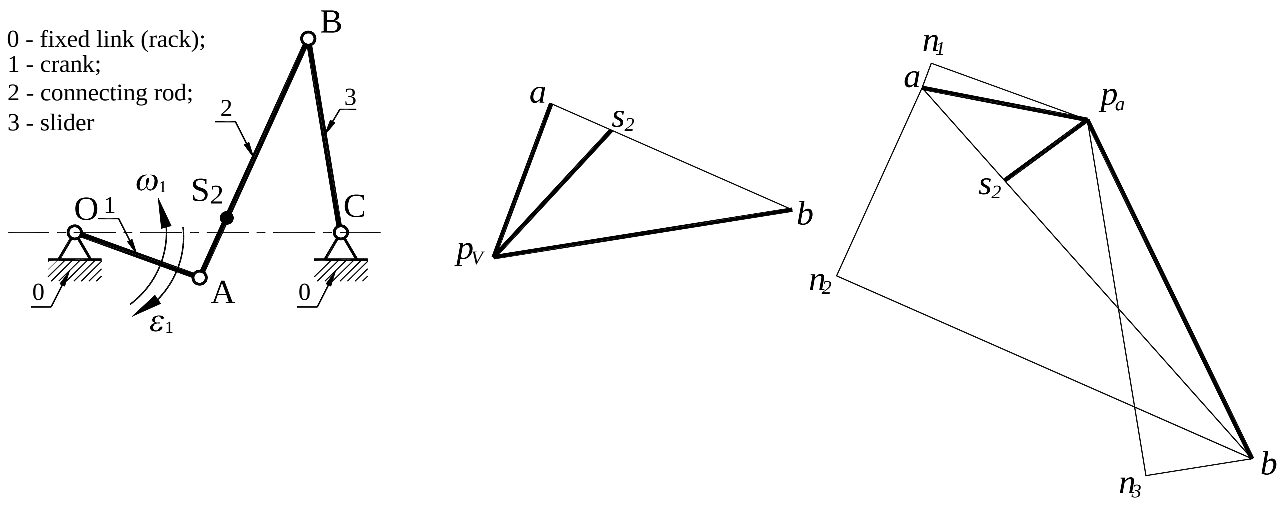


Fig. 28. Velocities and accelerations plans for rocker-crank mechanism

Self-Study and Review

1 Can you provide examples of how velocity diagrams are used in practical engineering applications?

2 How does the choice of kinematic pairs affect the construction of velocity diagrams for a mechanism?

3 What is a Coriolis component, and how is it represented in a velocity diagram?

4 How do you handle situations where a mechanism experiences change in direction in a velocity diagram?

5 What software tools or modern technologies are commonly used for automating the process of velocity diagram construction in engineering practice?

Lecture 7

REDUCING OF FORCES AND MASSES

Reducing of forces

Force is called reduced to the point of the mechanism, if it, being applied at a point and directed in a motion direction of this point on tangent to the motion path, delivers the same power, as all forces acting on the mechanism.

$$N_{RF} = N_{\Sigma}.$$

Power

$$P = \vec{F} \cdot \vec{V};$$

$$P = F \cdot V \cdot \cos\alpha;$$

$$P = M \cdot \omega, \quad P = T \cdot \omega$$

where P – power, W (sometimes mechanical power indicates as N);

F – force;

V – velocity;

α – the angle between the vectors of force and velocity;

M – moment of forces or more often T – torque;

ω – angular velocity.

Crank-and-slider mechanism

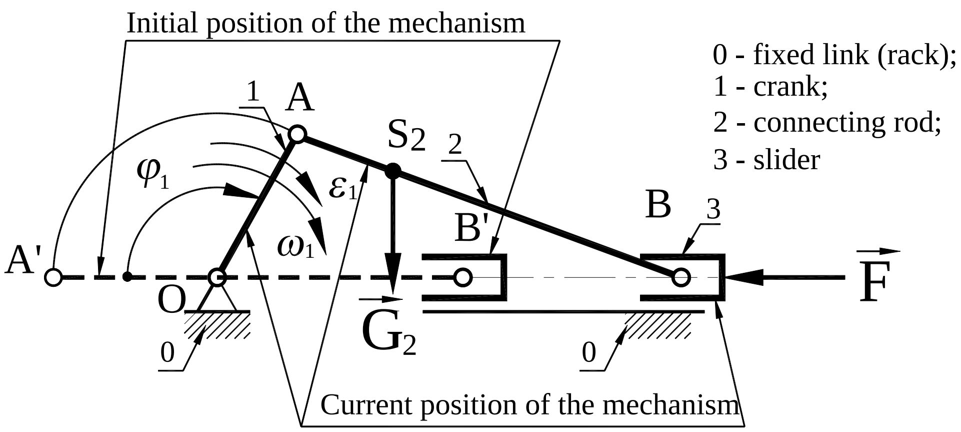


Fig. 29. Reducing of forces for slider-crank mechanism

$$N_{RF} = F_R \cdot V_A = M_R \cdot \omega_{OA}$$

$$N_{\Sigma} = \sum_i N_i = N_{G2} + N_F;$$

$$N_F = -F \cdot V_B;$$

$$N_{G2} = G_2 \cdot V_{s2} \cdot \cos\alpha$$

$$F_R \cdot V_A = G_2 \cdot V_{s2} \cdot \cos\alpha - F \cdot V_B$$

$$F_R = \frac{G_2 \cdot V_{s2} \cdot \cos\alpha - F \cdot V_B}{V_A}$$

Graphic method for definition of the Reduced Force

(Zhukovsky Lever)

To determine the reduced force, it is necessary:

1) draw a plan of velocities;

2) turn all external forces acting on the mechanism by 90 degrees (into the direction of the crank rotation) and apply them to the corresponding points on the plan of velocities

or apply all external forces acting on the mechanism to the plan of velocities rotated by 90 degrees (into the direction of the crank rotation);

3) find the total moment of these forces relative to the pole of the plan of velocities and divide it into a segment corresponding to the velocities of the reference point (***$p_V a$***).

$$F_R = \frac{G_2 \cdot \mu_V (p_V s_2) \cdot \cos\alpha - F \cdot \mu_V (p_V b)}{\mu_V (p_V a)} = \frac{G_2 \cdot h_2 - F \cdot (p_V b)}{(p_V a)}$$

$$F_R = \frac{1}{(p_V a)} \sum_i \pm F_i \cdot h_i$$

Reduced Moment or Moment Reduced Force

$$M_R = F_R \cdot l_{OA}.$$

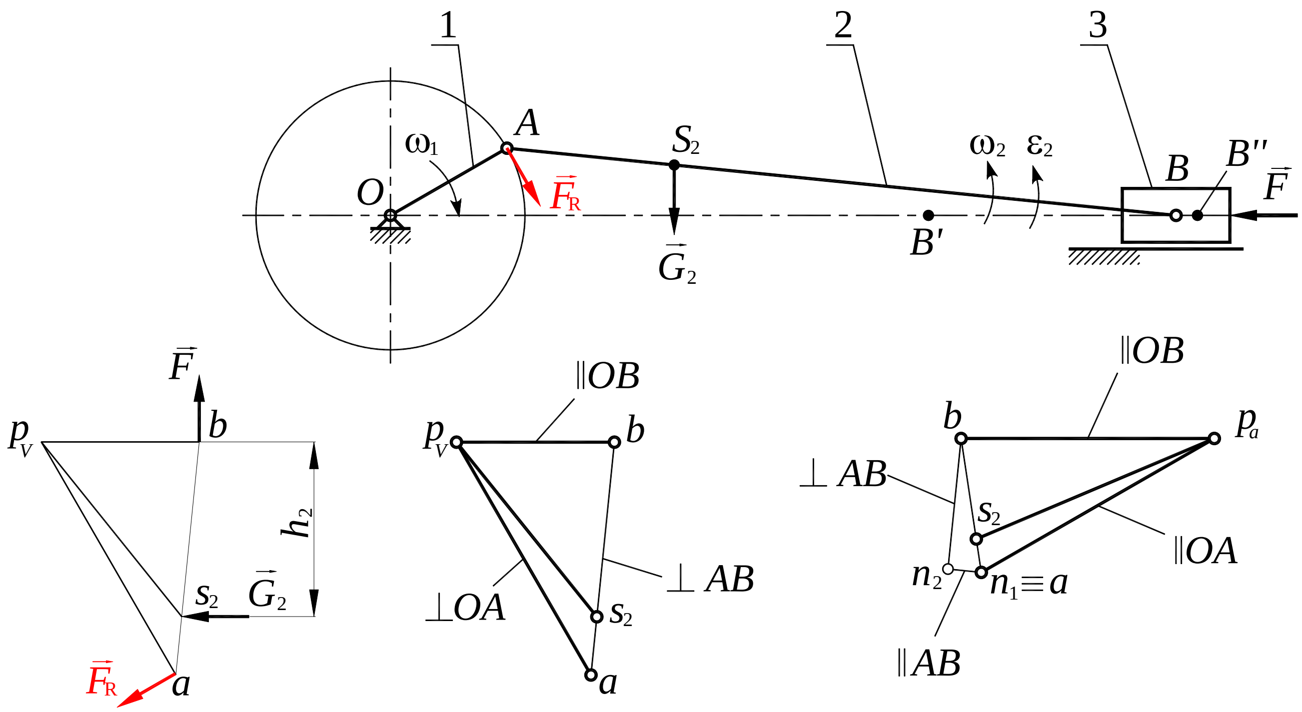


Fig. 30. Graphic method for definition of the Reduced Force (Zhukovsky Lever)

Reducing of masses

Mass is reduced to the link of the mechanism if the link with this mass has kinetic energy which is equal to the sum of kinetic energies of all links.

$$E_{kRM} = \sum_i E_{k_i}$$

Kinetic Energies

Translational motion: $E_k = \dfrac{mV^2}{2}$.

Rotational motion: $E_k = \dfrac{J\omega^2}{2}$.

Plane motion: $E_k = \dfrac{mV^2}{2} + \dfrac{J\omega^2}{2}$.

For example

For reduction link (crank): $E_{kRM} = \dfrac{m_R V_A^2}{2} = \dfrac{J_R \omega_1^2}{2}$,

For link 2 (connecting rod): $E_{k2} = \dfrac{m_2 V_{S2}^2}{2} + \dfrac{J_{S2}\omega_2^2}{2}$, For link 3 (slider): $E_k = \dfrac{m_3 V_B^2}{2}$.

Hence the reduced mass is

$$m_R = \frac{2}{V_A^2} \sum_i E_{k_i}$$

$$m_R = \frac{m_2 V_{S2}^2 + J_{S2}\omega_2^2 + m_3 V_B^2}{V_A^2}$$

if $J_{S2} = \dfrac{m_2 l_{AB}^2}{10}$ we can obtain using plan of velocities:

$$m_R = \frac{m_2 (p_V s_2)^2 + 0.1 \cdot m_2 (ab)^2 + m_3 (p_V b)^2}{(p_V a)^2}.$$

Reduced Inertia Moment

For the datum link, which produces turning motion, the reduced inertia moment is

$$J_R = m_R \cdot l_{OA}^2$$

$$J_R = \frac{m_2 (p_V s_2)^2 + 0.1 \cdot m_2 (ab)^2 + m_3 (p_V b)^2}{(p_V a)^2} \cdot l_{OA}^2$$

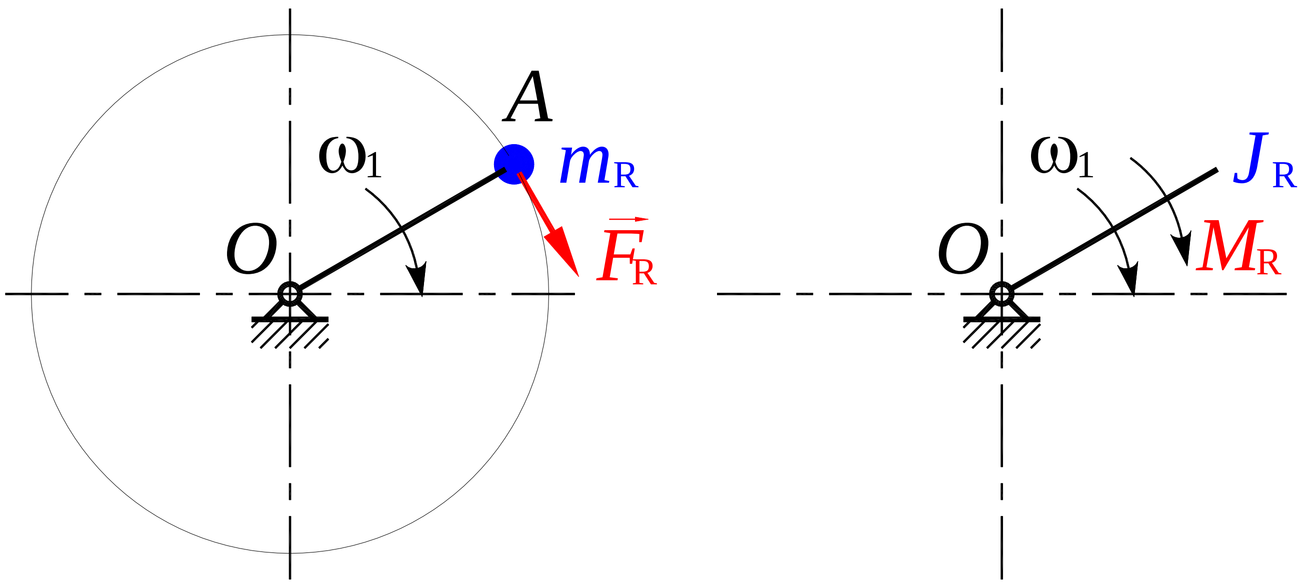


Fig. 31. Dynamic models of motion

Self-Study and Review

1 What is the concept of reducing force analysis in mechanical systems, and why is it important?

2 How do you resolve a force into its rectangular components in force analysis?

3 What is mass analysis in the context of mechanical systems, and why is it important?

4 How do you calculate the reduced mass of a mechanical system with multiple components?

5 How is the concept of the "parallel axis theorem" applied in calculating moments of inertia?

6 What role does mass analysis play in understanding the dynamic behavior of mechanisms and machines?

Lecture 8

KINETOSTATICS OF LEVER MECHANISMS

Force of inertia

$$\overrightarrow{F_{in}} = -m \cdot \overrightarrow{a_S},$$

where F_{in} – Force of inertia;

a_S – center of mass acceleration.

Inertial moment

$$\overrightarrow{M_{in}} = -J_S \cdot \vec{\varepsilon},$$

where M_{in} – Inertial moment;

J_S – Moment of inertia of a body relative to the center of mass;

ε – angular acceleration.

Equilibrium System in Statics

/the body is stationary or moves translationally at a constant speed /

Body in 3D space (6 degrees of freedom – 6 equations):

$$\begin{cases} \vec{R} = 0; \\ \vec{M} = 0. \end{cases}$$

Body in 2D space (3 degrees of freedom – 3 equations):

$$\begin{cases} \sum F_{iX} = 0; \\ \sum F_{iY} = 0; \\ \sum M = 0, \end{cases}$$

D'Alembert principle

The system of equations of equilibrium in dynamics

$$\begin{cases} \sum_i F_{iX} + \sum_i F_{iX}^{in} = 0; \\ \sum_i F_{iY} + \sum_i F_{iY}^{in} = 0; \\ \sum_i M_i + \sum_i M_i^{in} = 0, \end{cases}$$

where $\sum_i F_{iX}$ – the sum of the projections of external forces along the axis OX;

$\sum_i F_{iX}^{in}$ – the sum of the projections of inertial forces along the axis OX;

The section of dynamics in which the d'Alembert principle is used is called **kinetostatics**.

Since in TMM the links are in contact only with kinematic pairs, their mutual influence is replaced by reactions.

These unknown reactions are found from the system of equilibrium equations $\overrightarrow{R_i}$.

In the TMM discipline, reactions in kinematic pairs are denoted $\overrightarrow{R_{ij}}$,

where i, j – are the numbers of the contacting links. For example $\overrightarrow{R_{1-2}}$, $\overrightarrow{R_{0-1}}$, $\overrightarrow{R_{0-3}}$.

When analyzing the simplest kinematic chains – systems of 2 links (bodies), *Newton's third law* is used.

Newton's third law

The forces with which the bodies (links) interact with each other are equal in magnitude and directed along one straight line in opposite directions.

$$\overrightarrow{R_{1-2}} = -\overrightarrow{R_{2-1}}.$$

The force of action is equal to the force of reaction.

$$\begin{cases} \sum_i F_{iX} + \sum_i F_{iX}^{in} = 0; \\ \sum_i F_{iY} + \sum_i F_{iY}^{in} = 0; \\ \sum_i M_i + \sum_i M_i^{in} = 0, \end{cases}$$

Moment Equation Example

$$F_{in2} \cdot h_{2in} + G_2 \cdot h_2 - R_{12t} \cdot l_{AB} + M_{in2} = 0$$

Self-Study and Review

1 What are inertial forces in the context of mechanical systems, and how do they arise?

2 Can you explain the difference between mass forces and inertial forces in a mechanical system?

3 How are inertial forces affected by the acceleration of a body or a component in a mechanism?

4 How does angular velocity contribute to the generation of inertial forces in rotating components?

5 What are moments of inertia in the context of mechanical systems, and why are they important?

6 How do you define and calculate the moment of inertia of a rigid body?

7 Can you explain the role of the parallel axis theorem in finding moments of inertia for complex shapes?

8 What are the equilibrium equations for force systems, and why are they essential in mechanics?

9 How do you express the equations of equilibrium for a particle in three dimensions?

10 What is the significance of resolving forces into components when applying equilibrium equations?

11 How are moments or torques included in the equations of equilibrium for a rigid body?

12 What are the primary methods for calculating the forces acting on different components of a mechanism?

13 Can you provide examples of practical applications where force analysis is crucial?

Lecture 9

POWER TRANSMISSIONS

It is often a need to transform one type of energy into another in technic. For this ma-chines are used. Most of the existing machines consist of the following main parts:

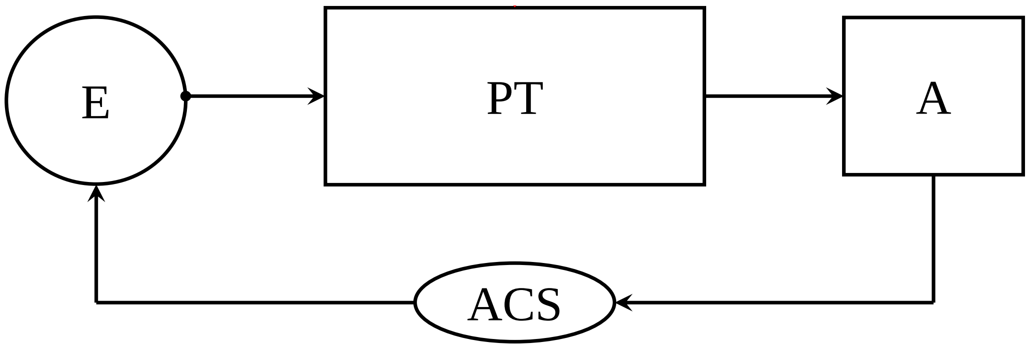


Fig. 32. Main elements of the machine:

E – engine, motor; PT – power transmission;

A – actuator, actuating device, executing (operating) mechanism;

ACS – automatic control system.

Power transmissions are devices used to transmit energy over a distance.

There are mechanical, electrical, pneumatic and hydraulic transmissions depending on the method of energy transmission.

In the course "Machine parts" only mechanical transmissions are studied, which are usually called simply transmissions.

Mechanical transmissions are called mechanisms that serve to transmit mechanical energy over a distance, as a rule, with the transformation of speeds and torques, sometimes with the transformation of the types and laws of motion.

The main reasons for using mechanical transmissions in machines:

1. The required speeds of movement of the working parts of the machine, as a rule, do not coincide with the optimum speeds of the engine. They are usually much lower.

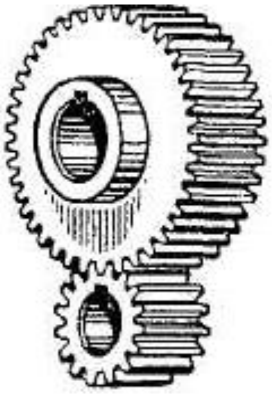
2. The need to regulate speed and torque. The process of decreasing the speed of rotation is called reduction and the transmission mechanism is called a speed reducer. The process of increasing the speed of rotation is called multiplying and the mechanism is called a speed multiplier.

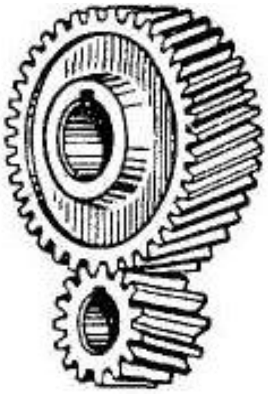
3. The need to transform rotary motion into translational motion.

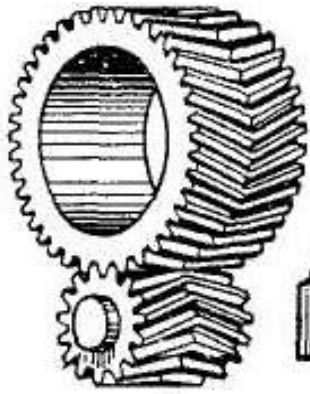
4. The need to drive several mechanisms from one engine.

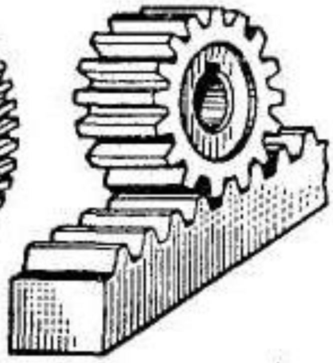
According to the principle of operation, mechanical transmission is divided into:

Transmission by mesh:

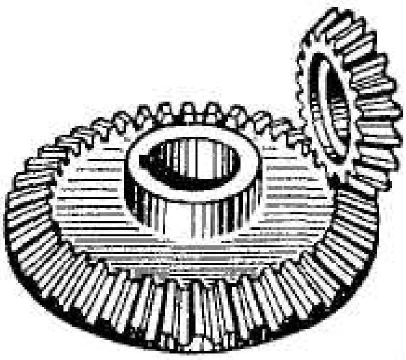


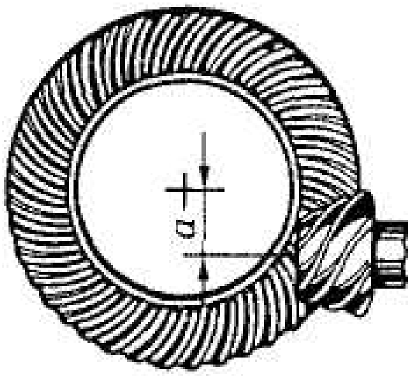


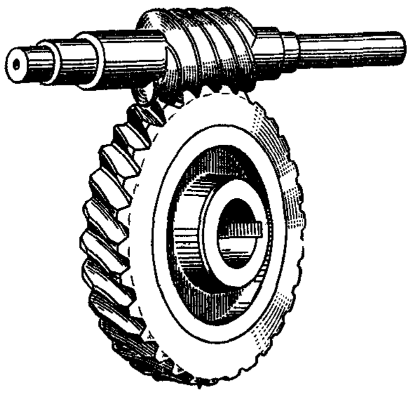




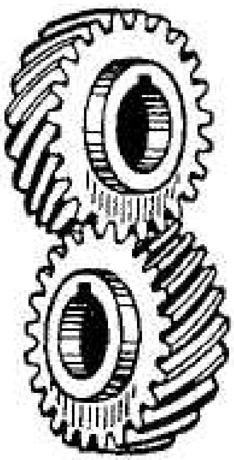
spur gear helical gear herringbone /chevron/ gear rack gear

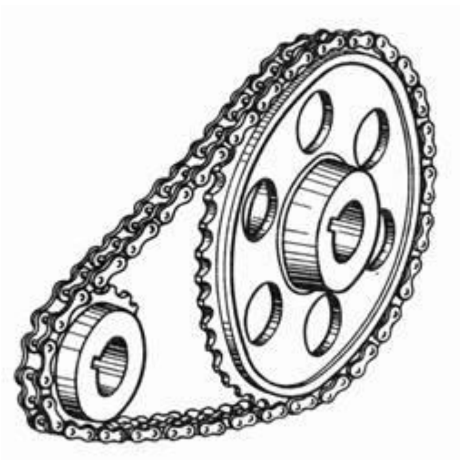


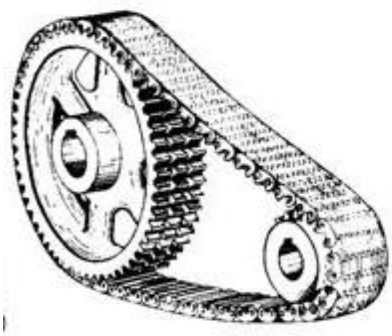




bevel gear hypoid gear worm gear







screw gear chain drive tooth-belt drive

Fig. 33. Types of gears

Transmission by friction:

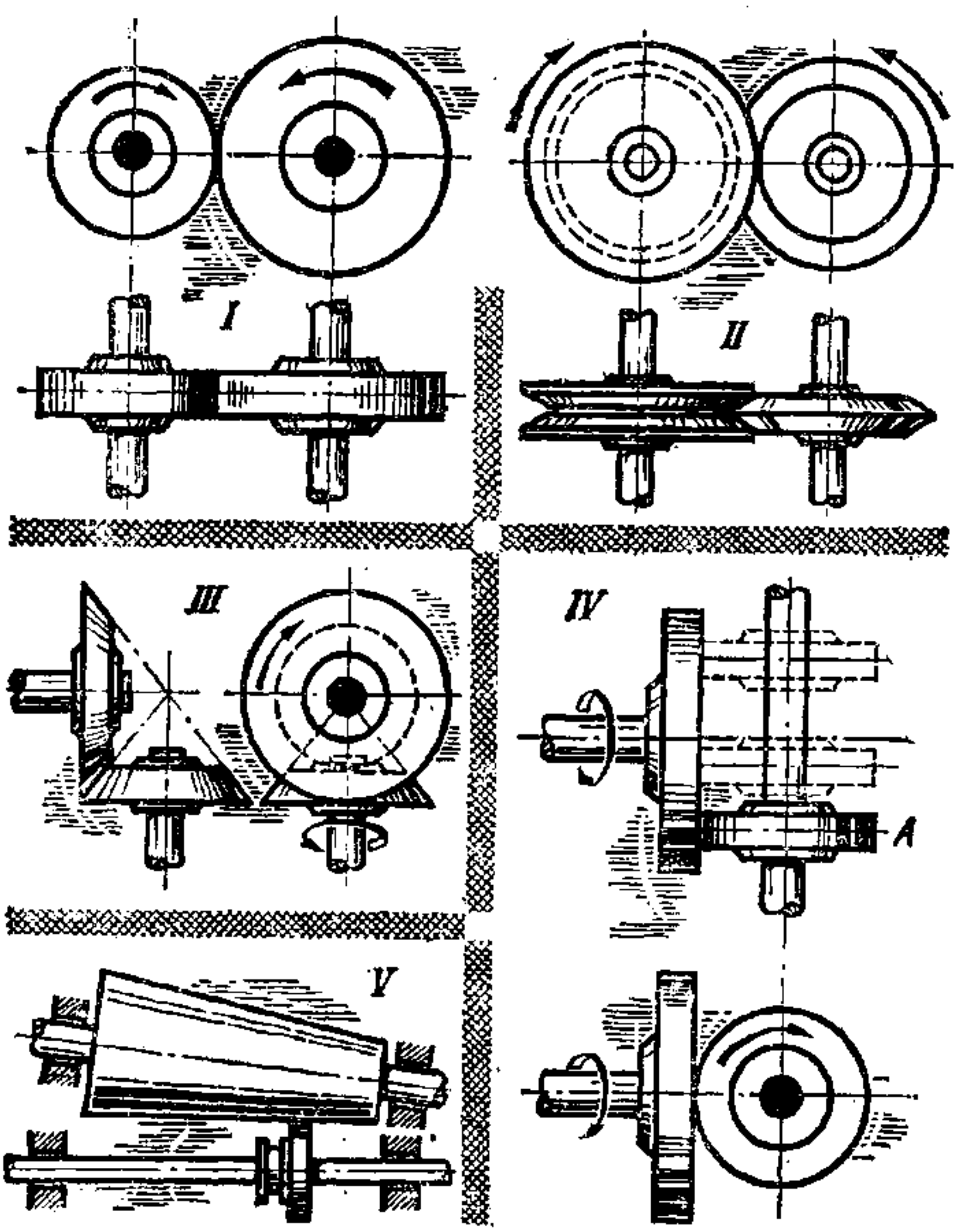


Fig. 34. Friction gears

Fig. 35. Belt drive

Fig. 36. Cable / rope drive

Also mechanical transmissions can have constant or variable speed ratio: stepped or stepless. Stepped regulation is cheaper and is carried out by simpler and more reliable mechanisms. Stepless regulation allows us to select the optimal process mode. They are easily automated, but more complex and less reliable.

Main parameters of mechanical transmissions

Angular velocity ω, rad/s.

Circumferential (peripheral) velocity V , m/s:

$$V = \omega \cdot r = \frac{\omega d}{2},$$

where: r – radius, m;

d – diameter, m.

Rotating frequency (revolutions per minute) n, rpm:

$$\omega = \frac{\pi \cdot n}{30}.$$

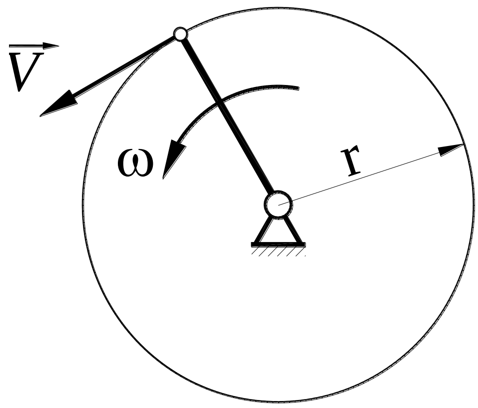


Fig. 37. Kinematic parameters of mechanical transmissions

Torque T, $N \cdot m$.

Peripheral force F_t, N.

The peripheral force F_t is related to the torque T transmitted by the rotating machine part by the dependence

$$F_t = \frac{2T}{d}.$$

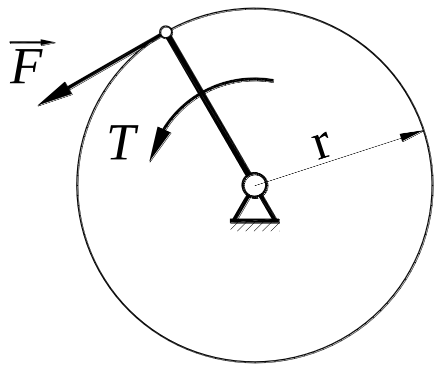


Fig. 38. Force parameters of mechanical transmissions

Efficiency factor

$$\eta = \frac{P_2}{P_1},$$

where: P_1 – input power,

P_2 – output power.

Angular velocities ratio (**gear ratio**) – the ratio of the angular velocities of the drive and driven rotating links:

$$i = \frac{\omega_1}{\omega_2},$$

where: ω_1 – angular velocity of the drive link,

ω_2 – angular velocity of the driven link.

Efficiency factor and gear ratio of compound mechanical transmission

The efficiency factor η and the speed ratio i of a drive consisting of several series-connected mechanical transmissions are determined as follows. Suppose the drive consists of four mechanical transmissions (Fig. 39).

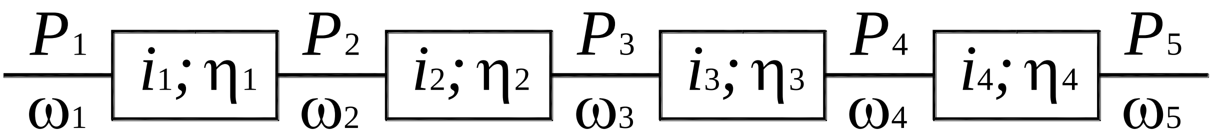


Fig. 39. Four-mechanical transmission drive

Efficiency factor of the entire drive:

$$\eta = \frac{P_5}{P_1}.$$

Let’s write down what are the powers on the output shafts:

$$P_2 = P_1\eta_1;$$

$$P_3 = P_2\eta_2 = P_1\eta_1\eta_2;$$

$$P_4 = P_3\eta_3 = P_1\eta_1\eta_2\eta_3;$$

$$P_5 = P_4\eta_4 = P_1\eta_1\eta_2\eta_3\eta_4.$$

Then

$$\eta = \frac{P_5}{P_1} = \frac{\cancel{P_1}\eta_1\eta_2\eta_3\eta_4}{\cancel{P_1}},$$

$$\eta = \eta_1\eta_2\eta_3\eta_4.$$

So the efficiency factor of a drive consisting of several series-connected mechanical transmissions is equal to the product of the efficiency factors of all its mechanical transmissions.

Angular velocity ratio of the entire drive:

$$i = \frac{\omega_1}{\omega_5}.$$

Let’s write down expressions for the angular velocities of the drive shafts:

$$\omega_4 = \omega_5 \cdot i_4;$$

$$\omega_3 = \omega_4 \cdot i_3 = \omega_5 \cdot i_3 \cdot i_4;$$

$$\omega_2 = \omega_3 \cdot i_2 = \omega_5 \cdot i_2 \cdot i_3 \cdot i_4;$$

$$\omega_1 = \omega_2 \cdot i_1 = \omega_5 \cdot i_1 \cdot i_2 \cdot i_3 \cdot i_4.$$

Then

$$i = i_1 \cdot i_2 \cdot i_3 \cdot i_4.$$

So the gear ratio of a drive consisting of several series-connected mechanical transmissions is equal to the product of the speed ratios of all its mechanical transmissions.

Self-Study and Review

1 What is the fundamental purpose of mechanical transmissions in engineering?

2 What are the primary components of a power transmission?

3 What are the main types of power transmissions commonly used in machinery?

4 How do you distinguish between the terms "driver" and "driven" in power transmissions?

5 Can you define the concept of gear ratio and its role in mechanical transmissions?

6 How do you calculate the gear ratio for a pair of meshing gears?

7 How are velocity and torque parameters calculated for mechanical power transmission?

8 How do you calculate the overall efficiency of a multi-stage mechanical power transmission?

9 How do you calculate the overall gear ratio of a multi-stage mechanical power transmission?

10 Can you list some common sources of energy loss in mechanical power transmissions?

Lecture 10

GEARS

Gears are widely used in machines and devices due to the following advantages:

1) The ability to transmit power in a wide range.

2) The smallest dimensions in comparison with other types of transmissions.

3) Constancy of the gear ratio regardless of the load transmitting.

4) High efficiency factor.

Classification of gears

Consider the classification of three-link gears according to the location of the geometric axes of the rotation.

a) With parallel shaft axes. In this case transmission is carried out by cylindrical gears (spur gear, helical gear, herringbone /chevron/ gear, rack gear).

b) With intersecting shaft axes. In this case transmission is carried out by bevel gear.

c) With crossing shaft axes. In this case transmission is carried out by hypoid gear, screw gear or worm gear.

Depending on the complexity gears can be:

a) The simple gears are three-link or single-stage. They include a fixed link (rack) and two moving links.

b) Compound gears can include several stages (and, accordingly, many links, they are called multi-link).

Gear ratio (angular velocity ratio)

The main transmission parameter of gears is the gear ratio i, which is the ratio of the angular velocity of the driving (input) link ω_1 to the angular velocity of the driven (output) link ω_n:

$$i_{1-n} = \frac{\omega_1}{\omega_n}$$

The gear ratio can be positive or negative (for mechanisms with parallel axes of rotation of the input and output shafts). The gear ratio is positive if the driving and driven shafts rotate in the same direction, and negative if they rotate in different directions.

The gear ratio can be more or less 1.

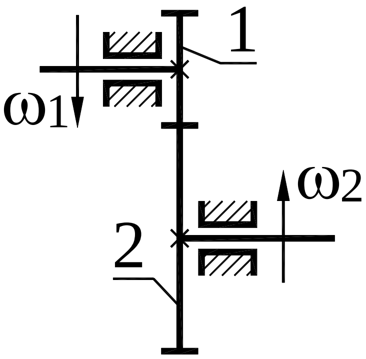
Gears with $|i| > 1$ are used to reduce the rotational speed. Such gears are called reducers. When $|i| < 1$, the gear increases the rotational speed and is called an accelerator or multiplier. There are also gears with $i \neq const$. If the gear changes i in steps, then it is called a gearbox. If it changes i smoothly in a certain range of values, then such a transmission is called a variator.

Gear ratio of three-link (single-stage) gears

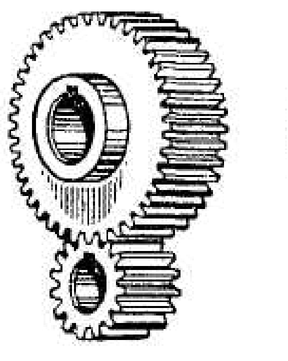
1. External meshing of round cylindrical wheels.

Gears in kinematic schemes are depicted in the form of disks or cylinders with the radii of the initial circles. In this case, the teeth of the wheels are not depicted.

The Fig. 40, *a* shows the kinematic scheme and Fig. 40, *b* shows the real image of external meshing with round cylindrical gear wheels.



a)



b)

Fig. 40. External meshing with round cylindrical gear wheels

The gear ratio of such a gear train is determined by the formula

$$i_{1-2} = \frac{\omega_1}{\omega_2} = -\frac{r_{w2}}{r_{w1}} = -\frac{z_2}{z_1},$$

where: ω_1 – angular velocity of the drive gear wheel,

ω_2 – angular velocity of the driven gear wheel,

r_{w1} – pitch circle radius of the drive gear wheel,

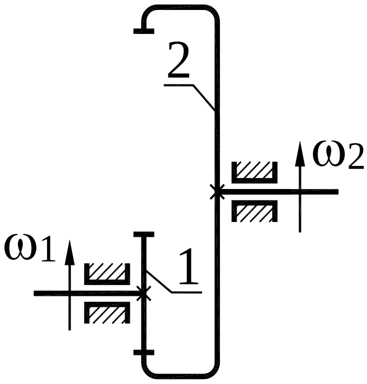
r_{w2} – pitch circle radius of the driven gear wheel,

z_1 – number of teeth of the drive gear wheel,

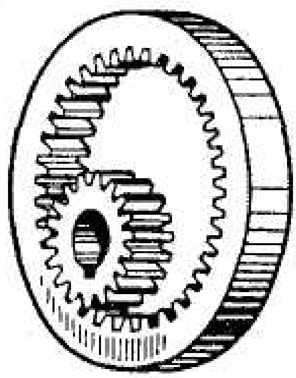
z_2 – number of teeth of the driven gear wheel.

i < 0, thus the input and output gears rotate in the opposite direction.

2. Internal meshing of round cylindrical gear wheels (Fig. 41, *a-b*).



a)



b)

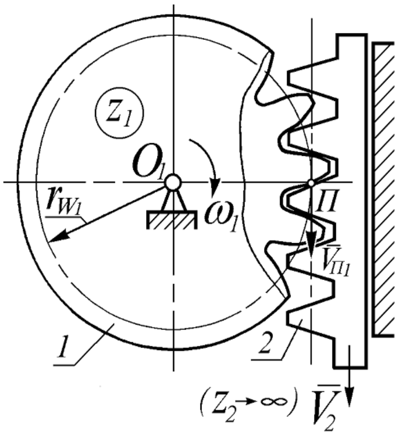
Fig. 41. Internal meshing with round cylindrical gear wheels

The gear ratio of such a gear train is determined by the formula

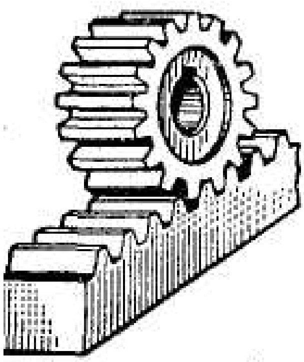
$$i_{1-2} = \frac{\omega_1}{\omega_2} = \frac{z_2}{z_1}$$

i > 0, thus the input and output gears rotate in the same direction.

Meshing of rack and pinion (Fig. 42, *a-b*).



a)



b)

Fig. 42. Meshing of rack and pinion

In meshing of rack and pinion the concept of a gear ratio is absent, because one link rotates and the second makes a translational motion:

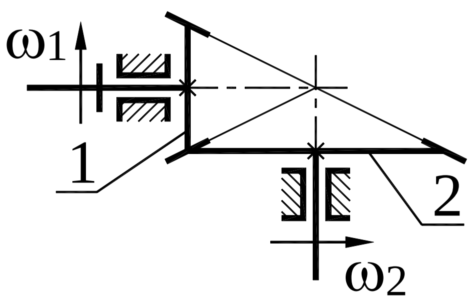
$$V_2 = V_{n_1} = \omega_1 \cdot r_{w1},$$

where: V_{n_1} – velocity of the rack,

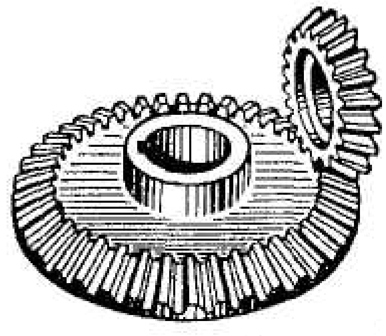
V_2 – peripheral velocity of the gear wheel.

Meshing of rack and pinion is designed to convert rotary motion into translational motion and vice versa.

4. The bevel gear (Fig. 43, *a-b*).



a)



b)

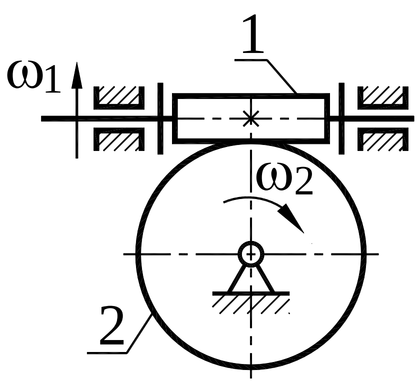
Fig. 43. The bevel gear

The gear ratio of such a gear train is determined by the formula

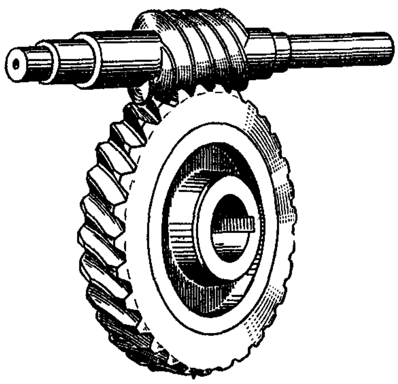
$$i_{1-2} = \frac{\omega_1}{\omega_2} = \frac{z_2}{z_1}$$

For the meshing of bevel gears, it is not customary to determine the sign of the gear ratio, because the rotation of the output and input shafts occurs in different planes.

5. Worm gear (Fig. 44, *a-b*).



a)



b)

Fig. 44. Worm gear

The gear ratio of such a gear train is determined by the formula

$$i_{1-2} = \frac{\omega_1}{\omega_2} = \frac{z_2}{z_1}$$

The gear ratio of the worm gear is equal to the ratio of the number of teeth of the worm wheel (z_2) to the number of starts of the worm (z_1).

For the worm gear, it is not customary to determine the sign of the gear ratio too.

Compound gear trains

Compound gear trains are multi-link (multistage) gear drives. They serve to obtain greater gear ratios than single-stage ones.

Compound gear trains differentiate into gear trains with shaft axes stationary in space (simple or non-planetary gear trains (Fig. 45, *a*)), as well as into gear trains with shaft axes moving in space (planetary or epicyclical gear trains (Fig. 45, *b*)).

a)

b)

Fig. 45. Compound gear trains

Compound gear trains with idler gear wheels

Idler gear wheels are those that are both driving and driven. On the Fig. 46, idler gear wheels are 2 and 3.

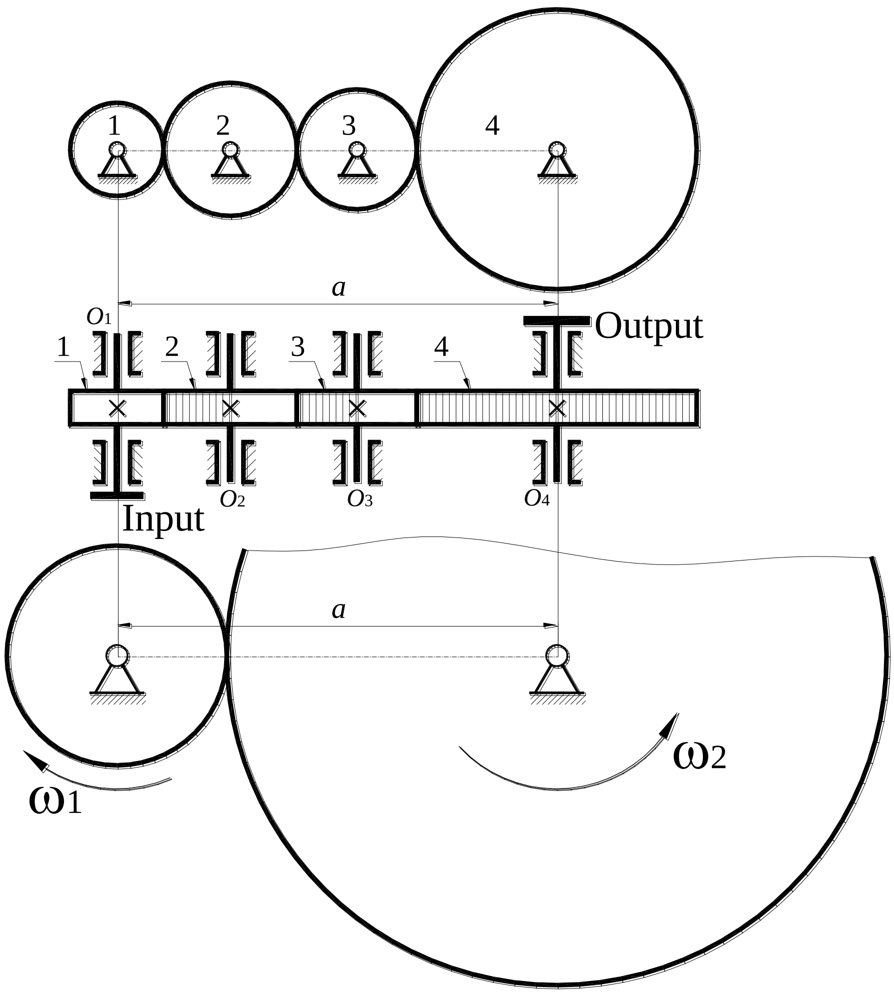


Fig. 46. Compound gear train with idler gear wheels

For each pair of gear wheels, we can write

$$\frac{\omega_1}{\omega_2}=-\frac{z_2}{z_1};\ \frac{\omega_2}{\omega_3}=-\frac{z_3}{z_2};\ \frac{\omega_3}{\omega_4}=-\frac{z_4}{z_3}$$

After multiplying separately the left and the right sides of the written equalities, then we get:

$$\frac{\omega_1\cdot\omega_2\cdot\omega_3}{\omega_2\cdot\omega_3\cdot\omega_4}=\left(-\frac{z_2}{z_1}\right)\cdot\left(-\frac{z_3}{z_2}\right)\cdot\left(-\frac{z_4}{z_3}\right)$$

After canceling and ordering the signs, we will have

$$\frac{\omega_1}{\omega_4}=\left(-1\right)^3\cdot\frac{z_4}{z_1}$$

Let us denote the number of external gears by the letter n (in this example, $n=3$). Let us also denote by the letter k the number of the output gear wheel of the row (in this case, $k = 4$). Then, for an arbitrary cylindrical row of gears with all intermediate idler gear wheels, we can write a formula to determine the value and sign of the gear ratio

$$i_{1-k} = (-1)^n \cdot \frac{z_k}{z_1}.$$

From the analysis of the gear trains with idler gear wheels the following conclusions can be concluded:

1. The number of teeth of the idler gear wheels does not affect the value of the gear ratio.

2. Each idler external gear reverses the direction of rotation of the output shaft.

3. The use of idler gear wheels allows changing the center distance between the input and output shafts in a wide range.

4. With a given (fixed) significant center distance between the input and output shafts, the use of idler gear wheels provides a large reduction in the dimensions of the gear train.

Multistage gear trains

Let us consider the definition of the total gear ratio of a multistage gear train shown on the Fig. 47.

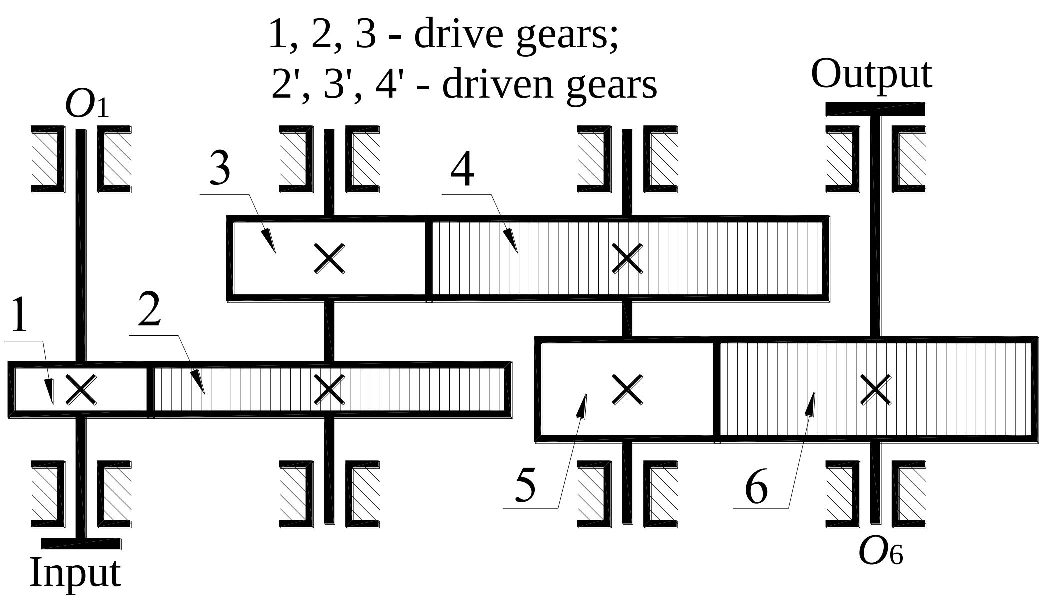


Fig. 47. Multistage gear train

For each stage we can write:

$$\frac{\omega_1}{\omega_2} = -\frac{z_2}{z_1}; \ \frac{\omega_3}{\omega_4} = -\frac{z_4}{z_3}; \ \frac{\omega_5}{\omega_6} = -\frac{z_6}{z_5}.$$

After multiplying separately the left and the right sides of the written equalities, then we get:

$$\frac{\omega_1 \cdot \omega_3 \cdot \omega_5}{\omega_2 \cdot \omega_4 \cdot \omega_6} = \left(-\frac{z_2}{z_1}\right) \cdot \left(-\frac{z_4}{z_3}\right) \cdot \left(-\frac{z_6}{z_5}\right).$$

Taking into account $\omega_2 = \omega_3$ (gear wheels 2 and 3 are mounted on the same shaft) and $\omega_4 = \omega_5$ (gear wheels 4 and 5 also are mounted on the same shaft), we can write:

$$i_{1-6} = \frac{\omega_1}{\omega_6} = (-1)^3 \cdot \left(\frac{z_2 \cdot z_4 \cdot z_6}{z_1 \cdot z_3 \cdot z_5}\right).$$

Notice, that

$$i_{1-6} = \frac{\omega_1}{\omega_6} = i_I \cdot i_{II} \cdot i_{III},$$

where i_I, i_{II}, i_{III} are the gear ratios of the individual gear stages, taken with their own sign.

We can also use the formula to determine the gear ratio of the multistage gear train in the form:

$$i_{1-k} = \frac{\omega_1}{\omega_k} = (-1)^n \cdot \frac{\prod z_{driven}}{\prod z_{drive}},$$

where k – is the serial number of the output shaft,

n – is the number of external gearing pairs,

$\prod z_{driven}$ – is the product of the numbers of teeth of the driven gear wheels,

$\prod z_{drive}$ – is the product of the numbers of teeth of the driving gear wheels.

Let's note that gear wheels 1, 3 and 5 are drive, and gear wheels 2, 4 and 6 are driven.

From the analysis of the multistage gear trains the following conclusions can be concluded:

1. Multistage gear trains allows to obtain a much larger gear ratio (speed reduction from the first shaft to the last shaft) with small gear wheels.

2. If a single-stage gear trains is used to give the same speed reduction, the last gear has to be very large.

Self-Study and Review

1 Can you list of advantages and disadvantages of gears?

2 How are gears classified according to the location of their geometric axes?

3 What gears are called reducers?

4 What are the names of gears that opposite to reducers?

5 Draw compound gear train scheme example with idler gear and write formula to determine value and sign of gear ratio.

6 Draw multistage gear train with two stages and write formulas to determine all gears ratios, angular velocites and torques.

Lecture 11

PLANETARY GEARS

Planetary gears are gears that have gear wheels that make a complex movement in space. Shafts of such gear wheels are moving in space.

We can say that the planetary gear has two housing links. One of them is fixed. The second housing is movable. It is called a carrier (Fig. 48) because this movable body drives gears, called planetary gears, in their spatial orbits.

Complex motion of the planetary gears causes an increased complexity of the kinematic analysis of planetary gears in comparison with the kinematic analysis of simple gears.

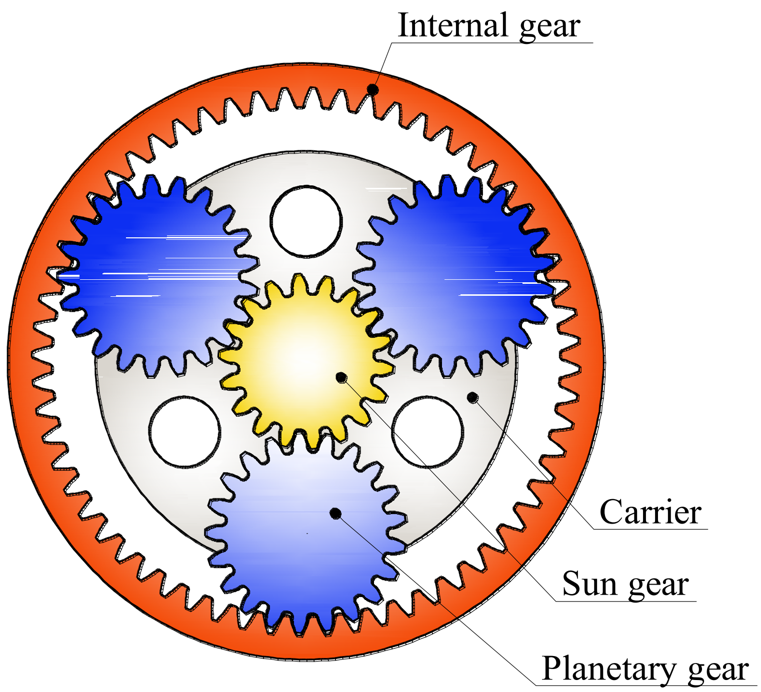


Fig. 48. The structure of planetary gears

Advantages of planetary gears

Planetary gears have the following advantages:

1. **The 2-stage planetary gear can provide almost any large gear ratio.** At the same time, with an increase in the value of the gear ratio, a significant decrease in the transmission efficiency is found, that must be taken into account when choosing a kind of planetary gear scheme for a power drive.

2. Possibility of integration (summation) of movements or differentiation (separation) of movements.

3. For the same amount of load capacity, a planetary gears has a higher service lifespan than the traditional gearboxes.

4. **Compact size and lightweight.** The modern planetary gears with the same torque output and gear ratio as the traditional gears weigh up to 50% less.

The simplest planetary gears

On the Fig. 49 are shown kinematic schemes of the simplest planetary gears of five types, which are widely used in machines, differing from each other by combinations of external and internal meshing.

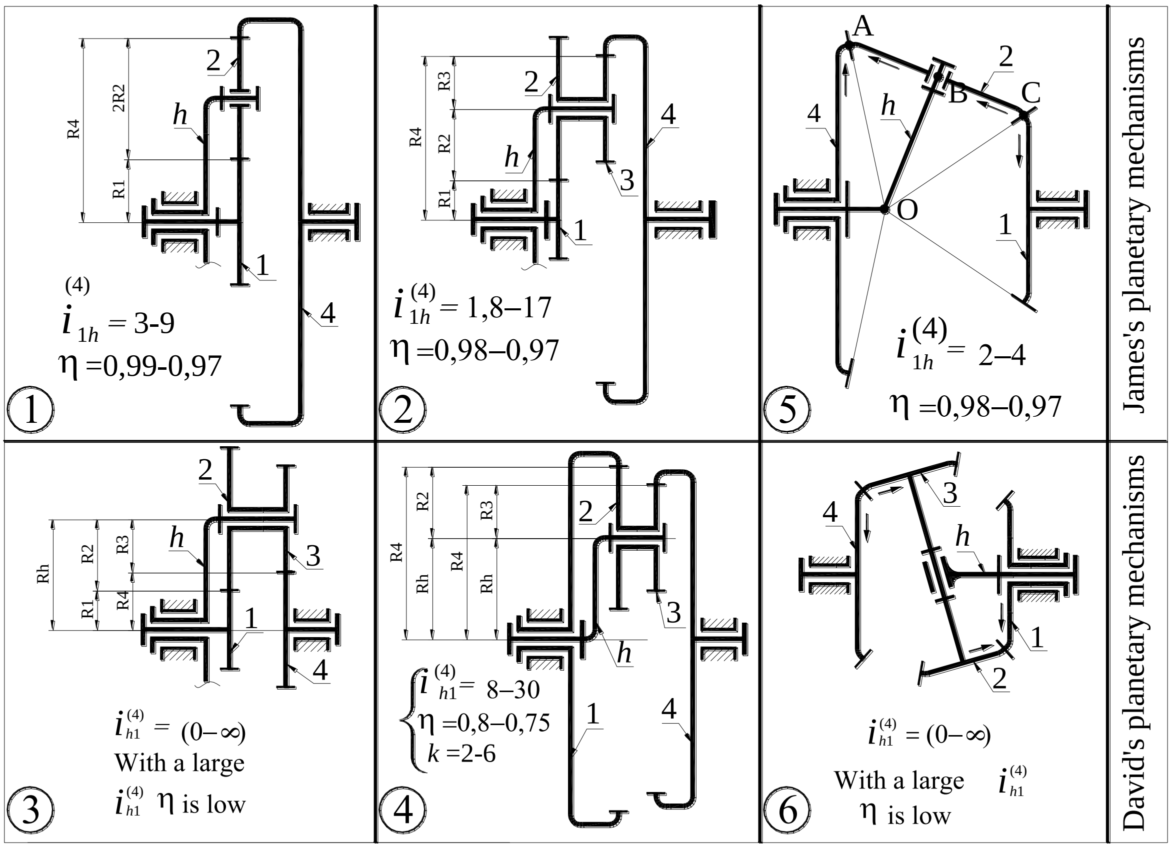


Fig. 49. The simplest planetary gears

Here it is denoted: 0 – rack, 1 and 4 – central gear wheels, 2 and 3 - planetary gear wheels, H – carrier.

Planetary gears 1, 2 and 5 represent the so-called James's planetary mechanism (with single planetary gear and double planetary gear). Schemes 3, 4 and 6 represent David's planetary mechanisms.

Kinematic analysis of planetary gears. Analytical method

Let us to consider kinematic scheme of the James's planetary mechanism with a single planetary gear:

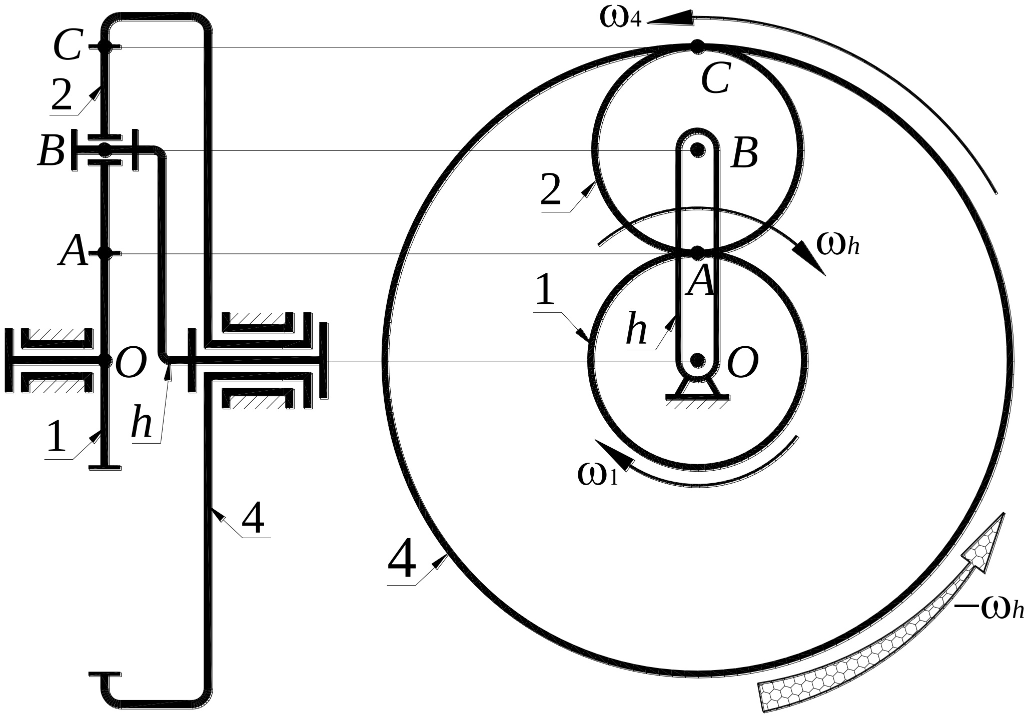


Fig. 50. James's planetary mechanism with a single planetary gear

Number of degrees of freedom can be calculated using the Chebyshev's formula:

$$w = 3n - 2p_L - p_H = 3 \cdot 4 - 2 \cdot 4 - 2 = 2$$

where $n = 4$ (1, 2, 4, *H*) – the number of moving links;

$p_L = 4$ (0-1, 2-*H*, 0-4, 4-*H*) – the number of lower kinematic pairs;

$p_H = 2$ (1-2, 2-4) – the number of higher kinematic pairs.

Since the mechanism has two degrees of freedom, a wide variety of combinations of angular velocities of all moving links can be observed in it. It is necessary to find an equation which allows us to obtain all these speeds.

To determine the gear ratio of the planetary mechanism, the Willis's method is used: let us mentally impart to the whole mechanism an additional angular velocity equal to ω_H, but directed opposite to it. In this case, we will observe the carrier stopped, and the mechanism is not planetary. All other links will be observed rotating relative to the stationary carrier with angular velocities, which can be determined by subtracting the magnitude of the angular velocity ω_H from the magnitude of the absolute angular velocity of each link.

The formula for the gear ratio of the planetary gear in inverse motion can be written as:

$$\frac{\omega_1 - \omega_H}{\omega_4 - \omega_H} = i_{1-4}^{(H)}$$

The fourth link is motionless, so $\omega_4 = 0$. Then:

$$-\frac{\omega_1}{\omega_H} + 1 = i_{1-4}^{(H)} \quad \Rightarrow \quad i_{1-H}^{(4)} = 1 - i_{1-4}^{(H)}.$$

This formula is valid for all five schemes of planetary mechanisms shown on Fig. 49, and may differ for individual schemes only in the sign and value of the gear ratio on the right side of equation.

For James's planetary mechanism with a single planetary gear (Fig. 49, a):

$$i_{1-H}^{(4)} = \frac{\omega_1}{\omega_H} = 1 - i_{1-4}^{(H)} = 1 + \frac{z_4}{z_1}$$

For James's planetary mechanism with a double planetary gear (Fig. 49, b):

$$i_{1-H}^{(4)} = \frac{\omega_1}{\omega_H} = 1 - i_{1-4}^{(H)} = 1 + \frac{z_2}{z_1} \cdot \frac{z_4}{z_3}$$

For David's planetary mechanism (Fig. 49, c-e):

$$i_{1-H}^{(4)} = \frac{\omega_1}{\omega_H} = 1 - i_{1-4}^{(H)} = 1 - \frac{z_2}{z_1} \cdot \frac{z_4}{z_3}$$

Kinematic analysis of planetary gears. Graphical method

In the planetary mechanism, the planetary gear in any position has a certain instant axis of rotation. The remaining links of the mechanism rotate around the constant axes of rotation.

The graphical method of kinematic analysis of the planetary mechanisms is based on the theory of the plane-parallel movement of the solid body. In accordance with it such movement can be represented as a rotational around the instant center of velocity.

The linear speed of any point of the link is equal to the product of the angular velocity of the link to the distance from this point to the instant axis of rotation. Thus, the dependence between the linear speeds of any point of the link and their distance to the instant axis of rotation is linear.

Consider the crank OA, which rotates around the point O (Fig. 51). Let’s draw a vector of peripheral velocity of point A. Then connect end of this vector with center of rotation O with a direct line at an angle α to the vertical. This line characterize the dependence of linear speeds of points of the link from distances of this points to center of their rotation O. So angular velocity of the link OA is tangent of an angle α.

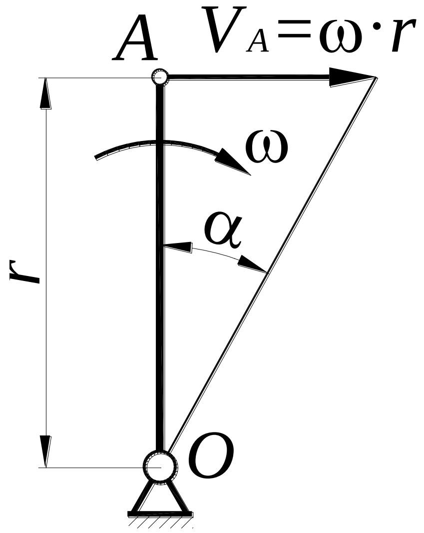


Fig. 51. Kinematic scheme of the crank

From Fig. 51 we can write $\omega_1 = V_A / r = \tan\alpha$.

Lets to show the application of the graphical method of kinematic analysis using for an example of a James's planetary mechanism with a single planetary gear (Fig. 51). We will consider that numbers of the teeth of the gear wheels and their diameters are known.

The mechanism must be drawn in scale. In the places of contact of the links we write the points *O, A, B, C*.

Next to the diagram of the mechanism in the projection connection we can draw the diagram of the dependence of the peripheral velocities of the points of the links on the radii of the wheels.

First, let's draw the linear velocity vector of point *A*.

From the end of this vector, that is, from point *a*, draw the line to the beginning of the coordinates (point *O*) and to point *c*, which is the end of the zero vector of the peripheral velocity of point *C* in the diagram, since wheel 4 is motionless.

The first line will be denoted by number 1, thus this line shows the dependence of the peripheral speeds of link 1 on the radius of rotation.

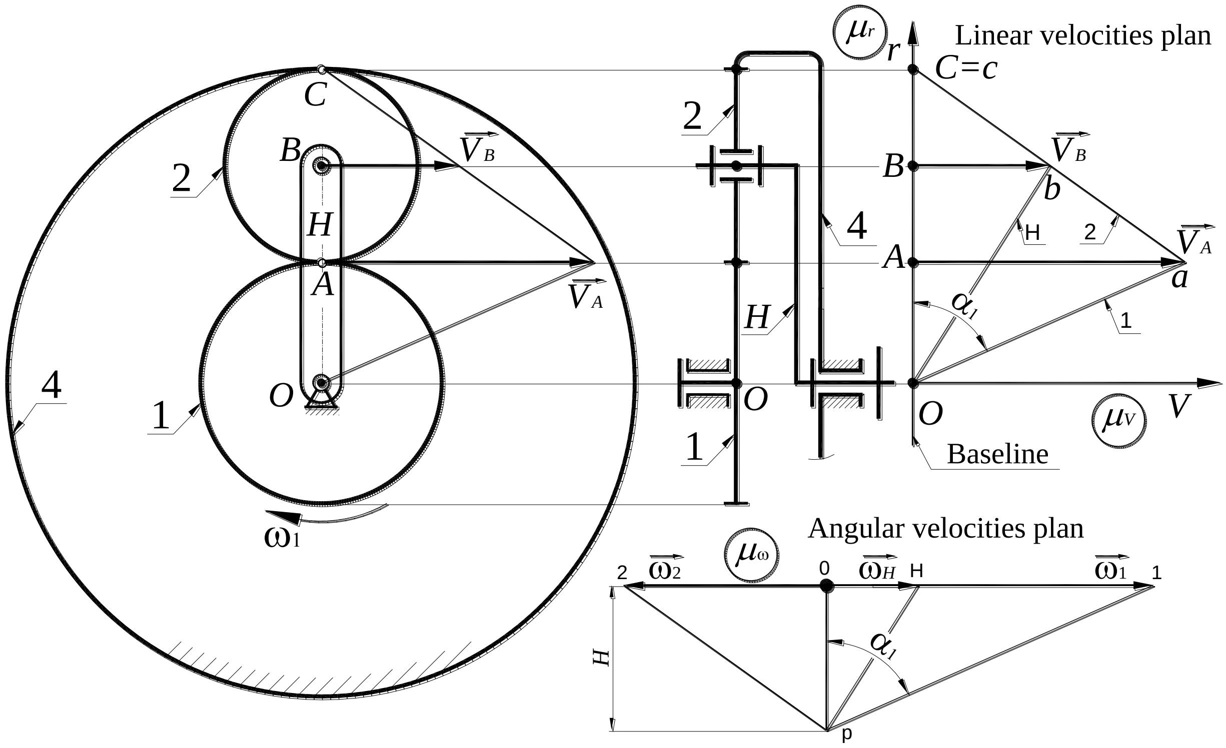


Fig. 52. Graphical method of kinematic analysis for James's planetary mechanism with a single planetary gear

The instant axis of rotation of wheel 1 always coincides with the central axis of the mechanism, which passes through point O.

The second line 2 shows the linear dependence of the peripheral velocity of link 2 on the distance to the instant axis of rotation passing through point C.

Now we can find the linear velocity vector of point B, which belongs to link 2.

For this, it is necessary to draw a horizontal line from point B to the crossing with line 2. The end of the vector of the peripheral speed of point B is denoted by a small letter *b*.

The movable axis of the planetary gear 2 also has peripheral speed V_B, that is, point B of carrier H. From the end of this vector, draw the line to the beginning of the diagram coordinates (point O).

This line shows the linear dependence of the carrier peripheral velocities on the radius of rotation.

The angular velocities ω_1, ω_H and ω_2 are proportional to the tangents of the slopes of the lines 1, H and 2.

This allows us to draw a plan of the angular velocities of the links of the mechanism in the following order.

Let's choose an arbitrary place for the pole p_ω of the plan of angular velocities.

Along the vertical line from p_ω we draw the auxiliary pole p with a certain pole distance. The scale of the plan of angular velocities will depend on the value of the pole distance. From the auxiliary pole p, draw lines 1, h and 2, parallel to the corresponding lines in the peripheral velocities diagram.

Lines 1, h and 2 will cut on the horizontal line passing through the pole p_ω, the lengths of the angular velocities vectors ω_1, ω_H and ω_2 in a scale.

The directions of the found vectors show the directions of rotation of the links of the mechanism.

So, links 1 and H rotate in the same direction, and link 2 rotates in the opposite direction. Now we can determine the value of the gear ratio of the mechanism:

$$i_{1-H}^{(4)} = \frac{\omega_1}{\omega_H} = \frac{|\, p_\omega 1\,|}{|\, p_\omega h\,|}$$

Self-Study and Review

1 Describe construction of simple planetary gear.

2 List the advantages of planetary gears.

3 Draw kinematic scheme of simple planetary gear and explain it construction.

4 Give an example of determination of number of degrees of freedom a planetary gear.

5 Use Graphical method of kinematic analysis for James's planetary mechanism with a single planetary gear to draw linear and angular velocites plans.

Lecture 12

GEOMETRY OF GEARS

1. Geometry of spur gears

The terminology of spur gears is illustrated in the Fig. 53.

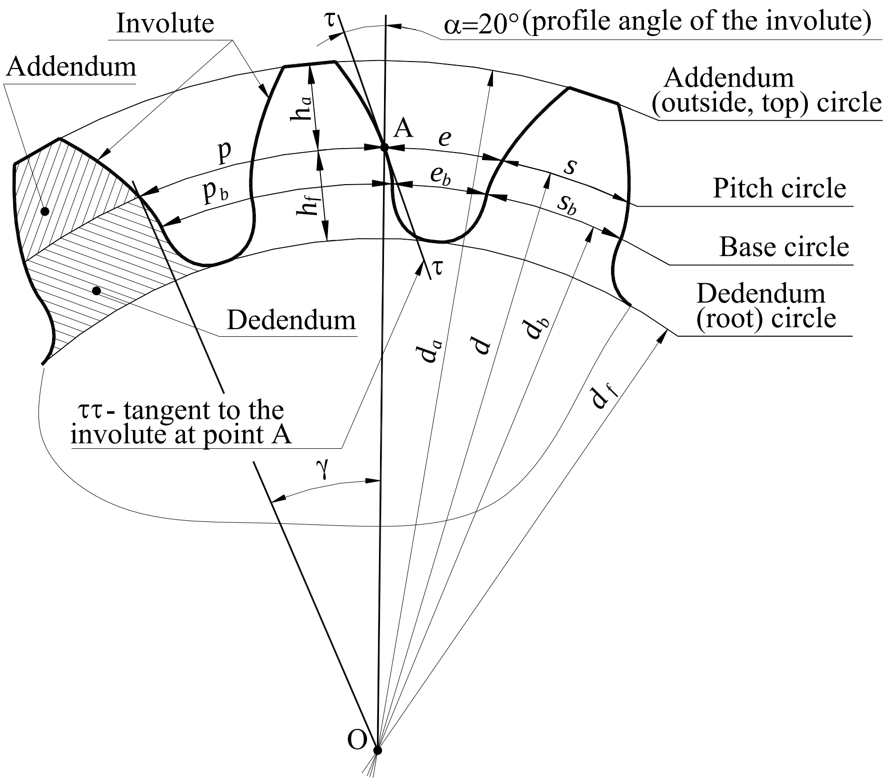


Fig. 53. Involute spur gear configuration

Addendum (ADD). Symbolized by “a”. It is the height of a tooth space beyond the pitch circle. Also it is the radial distance between the pitch circle and the addendum circle.

Addendum circle is the circle coincides with the tops of the teeth in a cross section.

Dedendum (DED). Symbolized by “f”. It is the depth of a tooth space below the pitch circle. Also it is the radial distance between the pitch circle and the dedendum circle.

Dedendum circle is the circle coincides with the bottom of the teeth in a cross section.

Pitch circle is the theoretical circle upon which all gear calculations are based and its diameter is called the “**pitch diameter**” (d).

The length of the pitch circle

$$L = \pi d = pz,$$

where z – is the number of teeth;

p – circular pitch (look at Fig. 53).

Circular pitch *" p "* is the distance measured on the pitch circle from any point on a tooth to the corresponding point on an adjacent tooth. The circular pitch is equal to the sum of "**tooth thickness (**s**)**" and "**width of space (**e**)**":

$$p = s + e.$$

The spacing of the teeth may also be expressed in terms of what may be called the angular pitch γ. This is the central angle per tooth equal to $2\pi/z$ or $360°/z$.

Angular pitch "γ " is the central angle equal to $2\pi/z$ or $360°/z$.

According to (1), the pitch diameter is

$$d = \frac{p}{\pi} z = m \cdot z$$

The value

$$m = \frac{p}{\pi}$$

is called the **module** and **is the base geometrical characteristics of gears** and is used in all calculations and measurements. For any given pair of meshing gears, the module must be same. Module is measured in millimeters and is chosen from a two standard module series:

Series 1: 1,0; 1,25; 1,5; 2,0; 2,5; 3,0; 4,0; 5,0; 6,0; 8,0; 10,0…

Series 2: 1,125; 1,375; 1,75; 2,25; 2,75; 3,5; 4,5; 5,5; 7,0; 9,0; 11,0…

In specifying the module, preference should be given to Series 1.

The use of standard modules provides the interchangeability of gears and the unification of gear-cutting tools.

According to (4), the circular pitch

$$p = m \cdot \pi.$$

Base circle is the circle from which **involute** tooth profiles are derived (look at the Fig. 54).

Involute is a curve that circumscribed by a point of a straight line, which rolls by circle without sliding (Fig. 54). Leonhard Euler offered to use involute as mating teeth profiles.

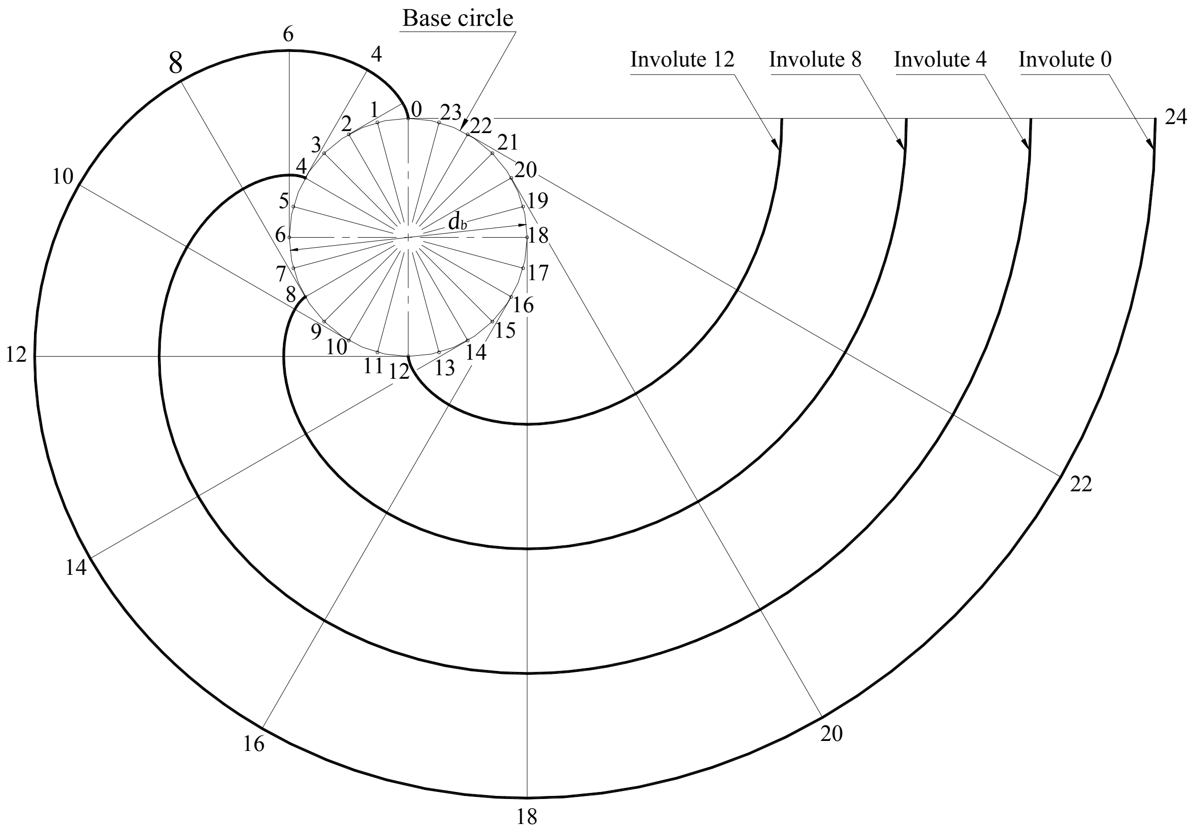


Fig. 54. Involute formation

The straight line, a point of which circumscribes an involute, is tangent to base circle, and at the same time it is a normal to involute profile. Points of tangency of the straight to the circle are centers of involute's curvature in the points. Thus, circle is the locus of centers of involute's curvature, i.e. **an evolute** of the involute.

The base circle diameter is (look at the Fig. 53)

$$d_b = d \cdot \cos\alpha,$$

where α – is the profile angle of the involute.

Profile angle "α" is the angle at a pitch point between a line tangent to a tooth surface and a radial line of a pitch circle (the standard value is $\alpha = 20°$).

Base pitch "p_b" is the pitch along base circle

$$p_b = p \cdot \cos\alpha.$$

The addendum

$$h_a = m \cdot h_a^*,$$

where h_a^* – is the addendum factor (the standard value is $h_a^* = 1{,}0$).

The dedendum

$$h_f = m \cdot (h_a^* + c^*),$$

Whole depth" h" is the total depth of a tooth space equal to the sum of the addendum and dedendum

$$h = h_a + h_f = m \cdot (2h_a^* + c^*) = 2{,}25m.$$

The addendum circle diameter

$$d_a = d + 2mh_a^* = d + 2m.$$

The dedendum circle diameter

$$d_f = d - 2m(h_a^* + c^*) = d - 2{,}5m.$$

The centre distance with $x_\Sigma = 0$ (x_Σ is the shift factor)

$$a = \frac{d_1 + d_2}{2} = m\frac{z_1 + z_2}{2},$$

where z_1; z_2 – are the number of teeth of gear wheels.

2. Geometry of helical gears

As compared with their spur (straight-tooth) counterparts, helical gears provide smoother engagement, are less noisy and have a higher load-carrying capacity. Helical gear drives are used in mechanisms operating at medium and high speeds. A major imperfection of helical gearing is the axial force.

Geometry. Look at the Fig. 55.

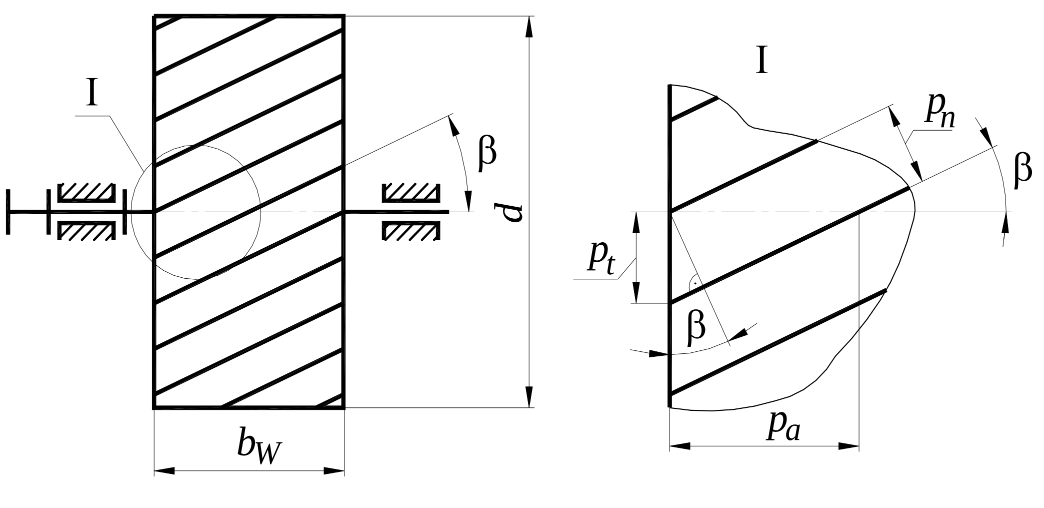


Fig. 55. The helical gear

There are:

p_n – is the normal pitch (equals the pitch of the tool rack);

p_t – is the transverse circular pitch;

p_a – is the axial pitch.

The transverse circular pitch is

$$p_t = \frac{p_n}{\cos\beta},$$

where β – is the helix angle on the pitch cylinder (usually β is from 8° to 22°).

$$p_a = \frac{p_n}{\sin\beta}; \; m_n = \frac{p_n}{\pi},$$

where m_n – is the normal module (equals the module of the tool rack).

At the normal section the helical tooth has the same profile as a straight tooth module $m = m_n$. Therefore, the normal module m_n must comply with standard.

$$m_t = \frac{p_t}{\pi} = \frac{p_n}{\pi\cos\beta} = \frac{m_n}{\cos\beta},$$

where m_t – is the transverse module.

$$m_a = \frac{p_a}{\pi} = \frac{p_n}{\pi\sin\beta} = \frac{m_n}{\sin\beta},$$

where m_a – is the axial module.

The pitch diameter of a helical gear (diameter of the pitch circle)

$$d = m_t z = \frac{m_n z}{\cos\beta},$$

where z – is the number of teeth on the gear.

The base circle diameter of a helical gear

$$d_b = d\cos\alpha_t,$$

where α_t – is the transverse angle and defined by formula

$$\alpha_t = \arctan\left(\frac{\tan\alpha}{\cos\beta}\right).$$

The addendum circle diameter

$$d_a = d + 2m_n h_a^* = d + 2m_n.$$

The dedendum circle diameter

$$d_f = d - 2m_n(h_a^* + c^*) = d - 2{,}5m_n .$$

The addendum equals the normal module

$$h_a = h_a^* m_n = m_n .$$

The clearance

$$c = 0{,}25m_n .$$

The dedendum

$$h_f = m_n(h_a^* + c^*) = 1{,}25m_n .$$

The centre distance with $x_\Sigma = 0$ (x_Σ is the shift factor)

$$a = \frac{d_1 + d_2}{2} = m_n \frac{z_1 + z_2}{2\cos\beta} .$$

The pitch diameter of helical gears can smoothly be changed by varying the helix angle β, which provides more accurate standard values of the centre distance.

Self-Study and Review

1 What are the key geometric parameters that characterize mechanical transmissions?

2 Can you define the term "pitch diameter" and explain its importance in gear design?

3 How is the number of teeth related to the pitch diameter in gears?

4 What is the purpose of the pressure angle in gear geometry, and how is it measured?

5 Can you explain the concept of the module in relation to gear design and analysis?

6 How are the addendum and dedendum of a gear tooth defined and measured?

7 What role does the center distance play in the geometric design of gear pairs?

8 How is the clearance or backlash between meshing gears calculated and specified?

9 Why is the face width or tooth width an important geometric parameter for gears?

Lecture 13

FRICTION IN MECHANISMS AND MACHINES

1. General information

On the one hand, friction is useful, and on the other hand it may be harmful. The advantage of friction is the following: contributes to the contact between bodies or within them without additional elements. (Examples: due to friction people and animals can walk, jump; objects remain stationary on a relatively inclined plane, etc.) The harmful effect means that 33% of the world's energy resources are spent on friction work. (Examples: overcoming resistance, wear in machines and mechanisms.)

Depending on the nature of the relative movement of bodies, there are: sliding friction and rolling friction.

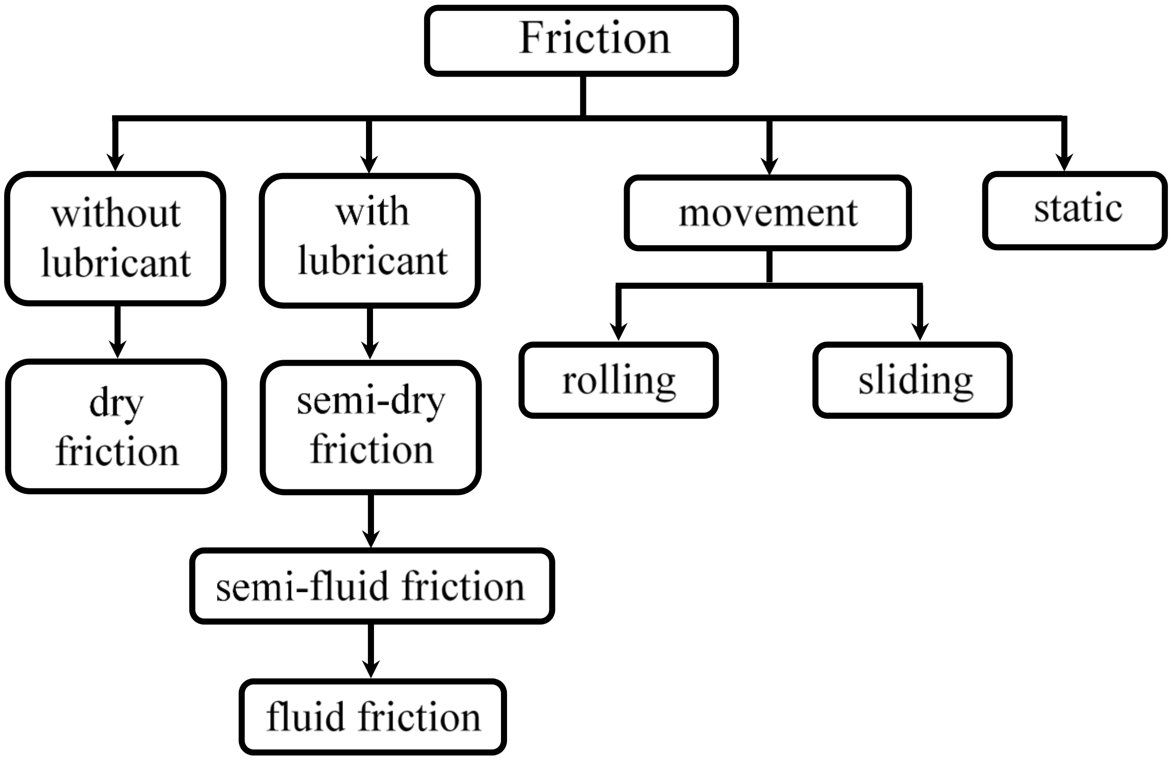


Fig. 56. Types of friction

In heir turn, the following types of sliding friction are distinguished: dry, semi-dry, semi-fluid and liquid friction. All these types of sliding friction are different in nature. In the future, we will keep in mind that in the kinematic pairs of mechanisms, there is mainly dry and semi-dry sliding friction.

The main patterns will be considered, according to the laws formulated by Amonton and Coulomb.

Fig. 57. Guillaume Amontons

Guillaume Amontons (1663−1705)

French physicist who made a significant contribution to the development of mechanics, thermodynamics, molecular physics. Being almost deaf from birth, he had no opportunity to study at the university, so he studied mathematics, physics, geodesy, architecture and other sciences independently. This did not prevent him from becoming a member of the French Academy of Sciences in 1699. In particular, he studied the properties of friction of solids, established the thermodynamic point – the boiling point of water, and also he came out with the idea of the existence of an absolute zero temperature, developed later by Lord Kelvin.

Fig. 58. Charles-Augustin de Coulomb

Charles-Augustin de Coulomb (1736–1806)

Prominent French physicist, military engineer, member of the Paris Academy of Sciences. He made a great contribution to the development of mechanics and the doctrine of electromagnetism. He formulated a number of fundamental laws and theories, named after him, which formed the basis of modern science: one of the classical strength theories using the maximum shear stress criterion, the law of dry friction, the law of interaction of electric charges and magnetic poles. His name is on the list of the most prominent French scientists, located at the base of the Eiffel Tower.

1) Basic laws of sliding friction are the following:

2) The sliding friction force is proportional to the normal pressure force;

3) The sliding friction force depends on the materials of the rubbing bodies and their roughness (this is taken into account by the sliding friction coefficient (f);

4) The sliding friction force is almost independent of the sliding velocity. It is directed opposite to the relative velocity;

5) The sliding friction force does not depend on the size of the rubbing bodies area;

6) The static friction force is greater than the sliding friction force, since the static friction coefficient is $f_0 > f$.

Thus, the sliding friction force is:

$$F = N \cdot f = const$$

The static friction force varies from zero to $F_{0\max} = N \cdot f_0$, i.e. $0 \le F_0 \le N \cdot f_0$

2. Angle and cone of friction

In Fig. 59 a) a load of weight G is located on the supporting surface. Under the action of the force P the load moves to the right at a constant speed. This makes it possible to ignore the force of the load inertia. The load is acted upon by the normal force $N = G$ and the tangential force – the sliding friction force $F = N \cdot f = const$. The overall reaction of the bearing surface is force $\vec{R} = \vec{N} + \vec{F}$. It is obvious that the force R is constant in magnitude and direction, since its components are constant. Let us denote the angle between the vector R and the normal nn to the support surface by φ. From the triangle formed by the vector and its components, the sliding friction force will be equal to:

$$F = N \cdot \tan\varphi$$

On the other hand $F = N \cdot f$, then, taking into account previous expression, we get:

$$\tan\varphi = f$$

where φ is angle of sliding friction.

The friction angle φ – is the angle by which the total reaction of the bearing surface R deviates from the normal "nn" in the direction opposite to the relative velocity.

If you change the direction of the force P to the opposite, then the load will move to the left and the vector R from the normal nn will deviate in the opposite direction. Thus, a number of possible positions of the vector form a friction cone. This is the cone of sliding friction. The friction cone at its vertex has an angle φ_0 ($\varphi_0 > \varphi$).

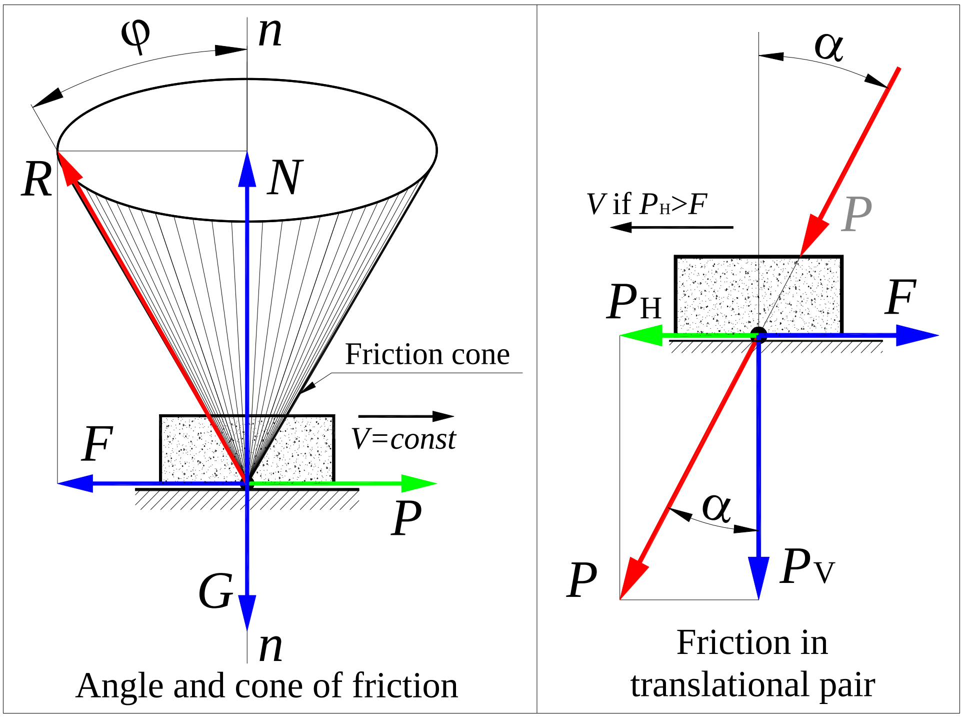


Fig. 59. Angle and cone of friction

The angle φ_0 is determined from the relation:

$$\tan\varphi_0 = f_0$$

3. Friction in a translational pair

In Fig. 59 b) force P is applied to the slider. The angle between the force vector and the normal is equal to α. It is necessary to determine the conditions under which the slider can move under the action of the force P. We transfer the origin of the force P along the line of action to the supporting surface and decompose it into components P_H and P_V. In this case, the normal compression force of the bodies is $N = P_V$, and under the action of P_H the movement of the slide is possible. The components of the force P are equal to:

$$P_H = P \cdot \sin\alpha,\ P_V = P \cdot \cos\alpha$$

Obviously, motion is possible if $P_H \geq F$. Here $F = N \cdot f = P \cdot \cos\alpha \cdot f$. After substituting the P_H and F, values into the motion condition, we get:

$P \cdot \sin\alpha \geq P \cdot \cos\alpha \cdot f$, or $\sin\alpha \geq \cos\alpha \cdot f$. Dividing both parts of the motion condition by $\cos\alpha$, we get:

$$\tan\alpha \geq f = \tan\varphi, \text{ or } \alpha \geq \varphi$$

Conclusion: movement under the action of force P is possible if it is located outside the friction cone.

4. Wedge slider

Fig. 60 shows a wedge slider. It is loaded with a force Q. The wedge angle is 2β. It is necessary to determine the total friction force *F*. This force arises on the side faces of the wedge slider.

From the equilibrium condition of the slider $\sum P_y = 0$ we can write:

$$2 \cdot N/2 \cdot \sin\beta = Q, \text{ i.e. } N \cdot \sin\beta = Q$$

Whence the total force of normal pressure on the side faces is equal to:

$$N = Q/\sin\beta$$

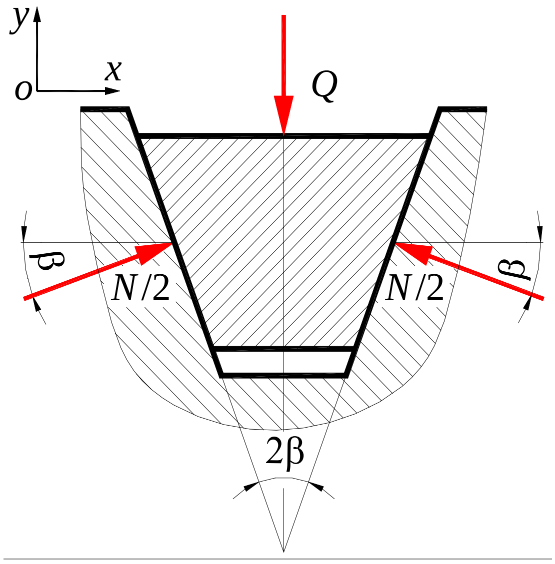


Fig. 60. Wedge slider

Then the total friction force will be equal to:

$$F = N \cdot f = Q \cdot \frac{f}{\sin\beta} \text{ denote:}$$

$$f' = \frac{f}{\sin\beta} \text{ – reduced coefficient of friction, then}$$

$$F = Q \cdot f$$

5. Inclined plane

a) General case (this case can be ignored).

This case is shown in Fig. 61. Here the driving force P is located at an arbitrary angle β relative to the inclined plane. α – angle of the plane inclination.

1) General case

$$\boxed{P = G\left(\sin(\alpha+\varphi)/\cos(\beta-\varphi)\right)}$$

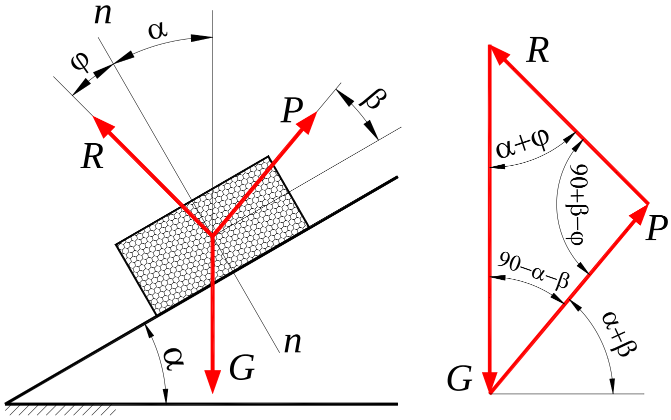


Fig. 61. General case

It is necessary to find the value of the driving force P for lifting the load at a constant velocity. From the said above, it is known that the total reaction of the support surface (inclined plane) R deviates from the normal nn by an angle φ in the direction opposite to the relative velocity. The value of the load gravity is G. From the condition $\sum P_i = 0$ using these data, we construct a force triangle. We indicate the angles between the vectors. By the sine theorem, we write down the relation to find the force P.

$$\frac{P}{\sin(\alpha+\varphi)} = \frac{G}{\sin\left(90^\circ + \beta - \varphi\right)} = \frac{G}{\cos(\beta-\varphi)}$$

Then the driving force will be equal to:

$$P = G \cdot \frac{\sin(\alpha+\varphi)}{\cos(\beta-\varphi)}$$

b) A special case when the driving force is directed horizontally, i.e. $\beta = -\alpha$.

This case is shown in Fig. 62.

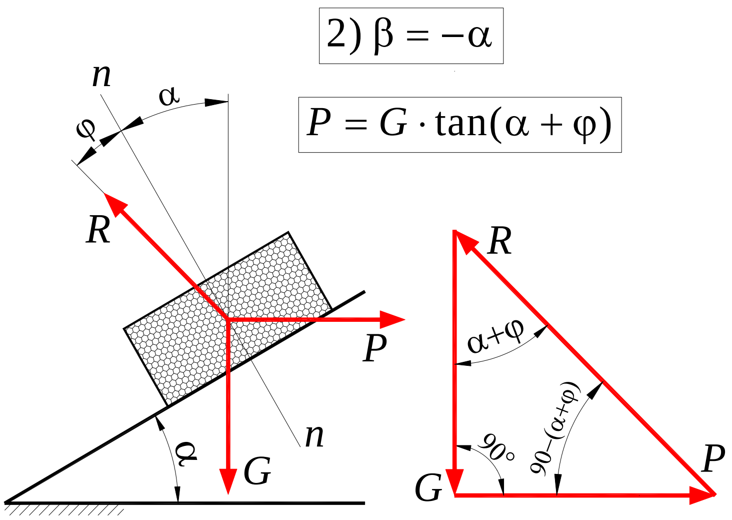


Fig. 62. Driving force is directed horizontally

We will not repeat all the preliminary discussion. We also define the force P by the sine theorem.

$$\frac{P}{\sin(\alpha+\varphi)} = \frac{G}{\sin\left(90^\circ - (\alpha+\varphi)\right)} = \frac{G}{\cos(\alpha+\varphi)}$$

From here the driving force for the case $\beta = -\alpha$, is equal to:

$$P = G \cdot \tan(\alpha+\varphi)$$

Often there is a case of lowering the load on an inclined plane. In this case, the direction of speed will be reversed. And accordingly, the force R will deviate in the opposite direction from the normal. Then the sign at the angle φ will change to the opposite.

The value of the holding force will be equal to:

$$P_f = G \cdot \tan(\alpha - \varphi)$$

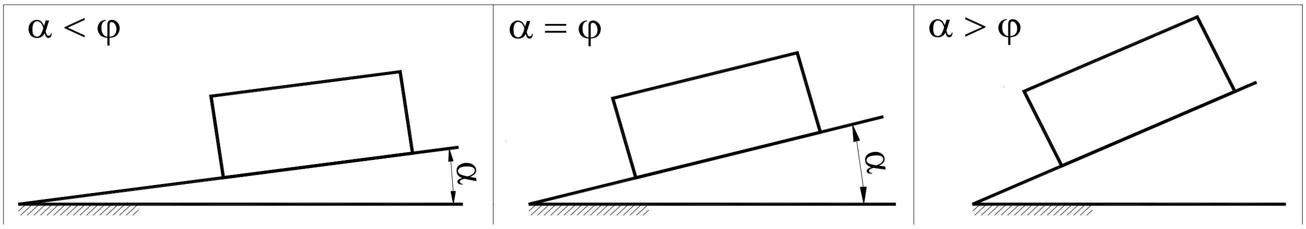
It follows that if $\alpha > \varphi$, then $P_f > 0$, i.e. the load must be kept from sliding down the inclined plane. And for $\alpha < \varphi$, $P_f < 0$, i.e. the load must be "pushed" to lower it along the inclined plane, since the load is "self-retaining" on the plane due to friction. The condition $\alpha \le \varphi$ is a self-braking condition.

Fig. 63 shows 3 cases:

1) $\alpha < \varphi$ – the case when the load self-brakes. Here, the shear force from the gravity of the load G is less than the static friction force F_0.

2) $\alpha = \varphi$ – the case of the boundary state. And if the load is at rest, then it will keep this state, since $f_0 > f$.

3) $\alpha > \varphi$ – the case when the load under the action of the force G_τ descends along an inclined plane. Here, the shear force from the gravity force of the load G is greater than the static friction force F_0.



The load is self-braking Boundary state The load doesn't self-brake

Fig. 63. Three possible states of load on inclined plane

6. Friction in a rotational pair

Fig. 64 shows two states of the shaft in the bearing:

1) The shaft does not rotate because the torque is $T = 0$;

2) The shaft rotates under the action of the torque T .

In both states, the shaft is loaded with a radial force Q. When the shaft does not rotate, the lines of the Q force action and the bearing reactions R coincide. Moreover, $R = N$.

When the shaft starts to rotate under the action of the torque T, then the shaft will roll onto the bearing. The shaft will stop at the point where the lift angle is equal to the friction angle φ (point A). The figure on the right shows the total bearing response R and its components N and F.

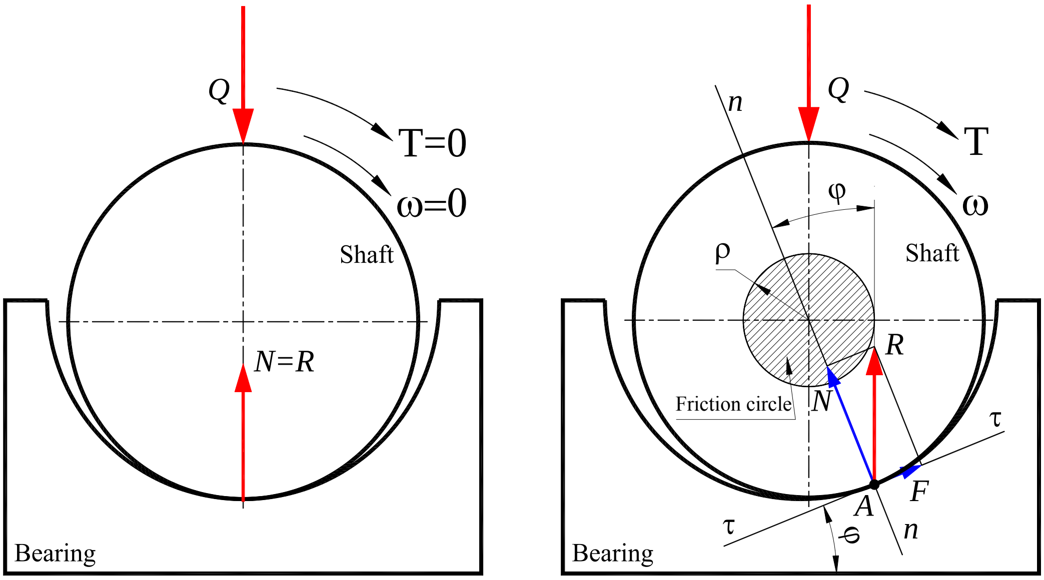


Fig. 64. Friction in a rotational pair

The normal reaction N is directed perpendicular to the common tangent $\tau\tau$, and the tangential force F – the sliding friction force is directed along the tangent, while the total reaction R is directed parallel to the force Q.

From the condition of equilibrium of the shaft

1) $\sum P_y = 0$, we have $R = Q$; 2) $\sum M_A = 0$, $T = Q \cdot \rho$.

Where ρ - is the radius of the friction circle. From triangle ABO we have:

$$\rho = OB = r \cdot \sin\varphi,$$

then the torque T will be equal to:

$$T = Q \cdot r \cdot \sin\varphi$$

Usually the angle of friction φ is small enough and $\sin\varphi \approx \tan\varphi = f$, can be taken, then

$$T \approx Q \cdot r \cdot f$$

7. Friction in the screw pair

An axial force Q is applied to a screw with a rectangular thread profile (Fig. 65). We have to determine how much torque T needs to be applied to the screw shaft. It is necessary to take into account the frictional force for lifting the screw. Elevation angle of the helix along the average diameter is λ.

Distributed normal and tangential forces are - friction forces. They act between the turns of the screw and nut.

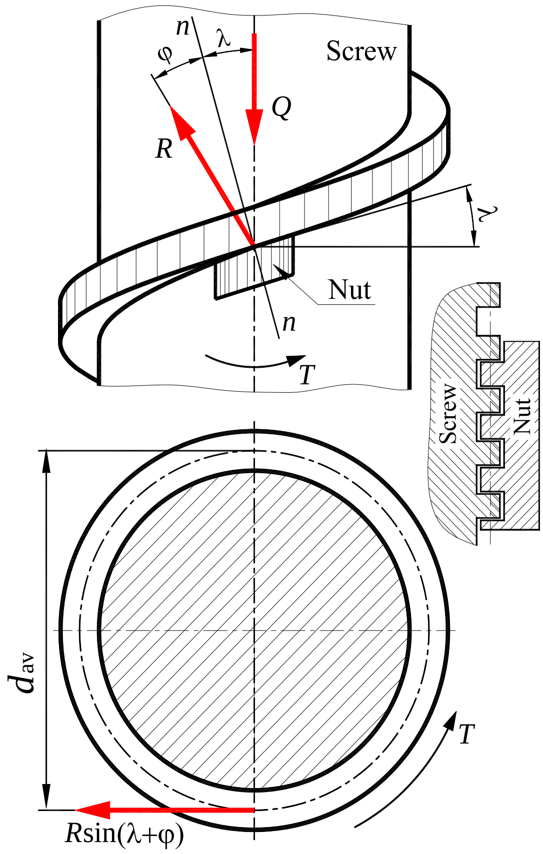


Fig. 65. Friction in the screw pair

Let us replace the indicated distributed forces with the total concentrated force R. As you know, R deviates from the normal nn by an angle φ in the direction opposite to the relative velocity, i.e. to the left. From the equilibrium condition of the screw 1) $\sum P_y = 0$ we can write:

$$R \cdot \cos(\lambda + \varphi) = Q$$

From expression 18), the total reaction of the turn of the nut R is equal to:

$$R = Q / \cos(\lambda + \varphi)$$

From equation 2) $\sum M_y = 0$ we have (see the top view):

$$T = 0{,}5 \cdot d_m \cdot R \cdot \sin(\lambda + \varphi)$$

After substitution in 20) the value of the force R we get:

$$T = 0{,}5 \cdot d_m \cdot Q \cdot \tan(\lambda + \varphi)$$

We have considered the case of the propeller lifting. When the propeller is lowered, the direction of the relative speed will be reversed. And the force vector R will deviate from the normal nn in the opposite direction. The sign at the angle φ will change to the opposite. For the case of lowering the screw, we define the holding moment. This moment arises from the action of the axial force Q.

$$T_f = 0{,}5 \cdot d_m \cdot Q \cdot \tan(\lambda - \varphi)$$

Note that at $\lambda < \varphi$ the holding moment $T_f < 0$, i.e. it is necessary to apply a moment to the screw shaft to tighten it.

Equation $\lambda < \varphi$ – is the condition for the self-locking of the propeller.

Now consider a screw with a triangular thread profile. In this case, the interaction of the turns of the screw and nut is similar to the interaction of a wedge slider with a wedge guide (see Fig. 60).

In this case, in formulas 21) and 22) it is necessary to apply the concept of the reduced coefficient of friction $f' = f/\sin\beta$ and substitute angle φ'. Next, you need to find the reduced coefficient of friction for a thread with a triangular profile. The sum of the angles of triangle *ABC* (see Fig. 66) is equal to:

$$2\alpha + 2\beta = 180°$$

We find: $\beta = 90° - \alpha$, then:

$$f' = f/\sin(90° - \alpha) = f/\cos\alpha = f/\cos 30° = f/0{,}866 = 1{,}15 f$$

Thus, the sliding friction coefficient for a triangular thread is 15% higher than for a rectangular thread. In this case, the angle of friction will be equal to:

$$\tan\varphi' = f'$$

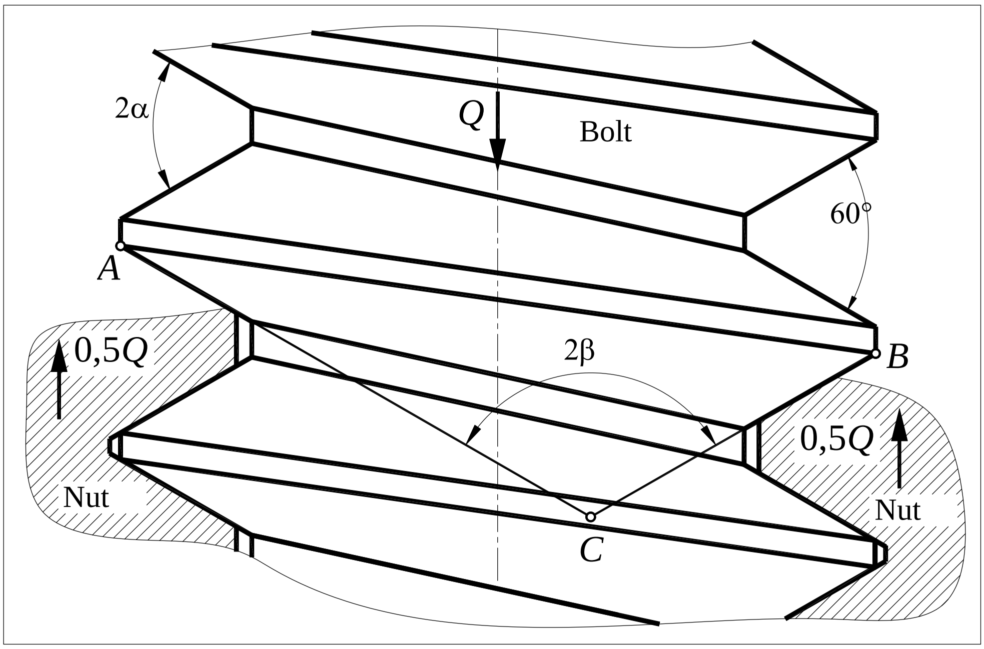


Fig. 66. Thread with a triangular profile

Accordingly, the moment T when lifting the screw is equal to: $T = 0{,}5 \cdot d_m \cdot R \cdot \sin(\lambda + \varphi')$, and when lowering $T_f = 0{,}5 \cdot d_m \cdot R \cdot \sin(\lambda - \varphi')$.

Determine the efficiency of the screw pair:

a) The case of lifting the screw. By definition, efficiency is the ratio of output work to input work, i.e. $\eta = A_{out} / A_{in}$. In the case of lifting the screw, the work at the input is the work of the moment T, and the work at the output is the work of the force Q. Then $\eta = A(Q) / A(T)$. Let's find these works in one turn of the screw. The work of the moment $A(T) = T \cdot 2\pi$. After substituting the value of the moment T, we get:

$$A(T) = Q \cdot \pi \cdot d_m \cdot \tan(\lambda + \varphi)$$

The work of the force Q is equal to:

$A(Q) = Q \cdot p$, here p is the pitch of the screw

To determine the pitch of the screw, consider the sweep of the helix along the average diameter.

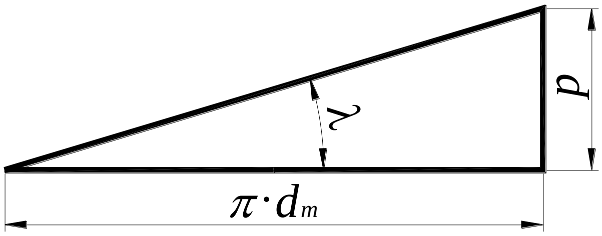


Fig. 67. Sweep of the helix along d_m

From this figure we will write: $\tan\lambda = p/\pi \cdot d_m$. Whence the pitch of the screw is equal to:

$$p = \pi \cdot d_m \cdot \tan\lambda$$

After substitution in the expression for $A(Q)$, the value of the pitch of the screw will be:

$$A(Q) = Q \cdot \pi \cdot d_m \cdot \tan\lambda$$

Then the efficiency of the screw pair in the case of lifting the screw will be equal to:

$$\eta = \frac{A(Q)}{A(T)} = \frac{Q \cdot \pi \cdot d_m \cdot \tan\lambda}{Q \cdot \pi \cdot d_m \cdot \tan(\lambda + \varphi)} = \frac{\tan\lambda}{\tan(\lambda + \varphi)}$$

Fig. 68 shows a graph of the change in the efficiency of the screw, extraneous according to previous formula for the case when $\varphi = 5°$. The figure shows the values of the friction angles φ and φ'. They define the positions of the boundaries of self-braking (area I or I') and non-self-braking screws (area II).

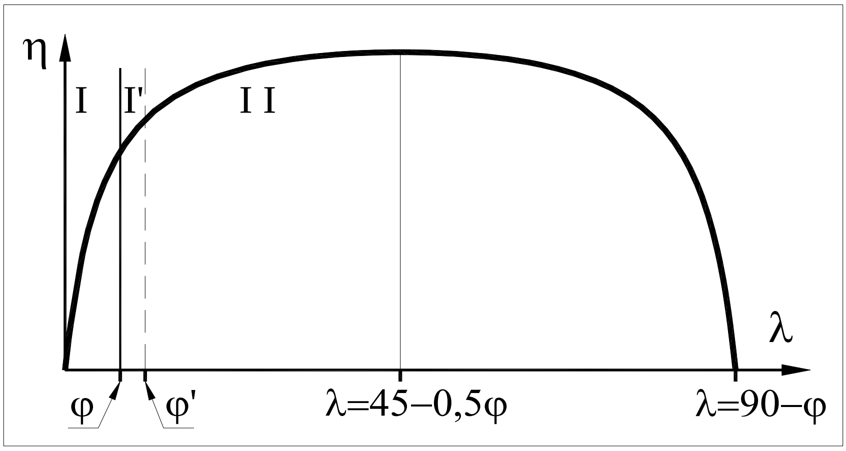


Fig. 68. Change in the efficiency of the screw

b) For the case of lowering the screw, the work at the input will be the work of the force Q, and the work at the output will be the work of the moment T_f. Then $\eta = A(T_f)/A(Q)$.

Here $A(T_f) = Q \cdot d_m \cdot \pi \cdot \sin(\lambda - \varphi)$, $A(Q) = Q \cdot d_m \cdot \pi \cdot \tan\lambda$, then we obtain:

$$\eta = \frac{A(T_f)}{A(Q)} = \frac{Q \cdot \pi \cdot d_m \cdot \tan(\lambda - \varphi)}{Q \cdot \pi \cdot d_m \cdot \tan\lambda} = \frac{\tan(\lambda - \varphi)}{\tan\lambda}$$

From this formula, it can be seen that at $\lambda < \varphi$ the efficiency is negative, *i.e. The efficiency of self-motorized mechanisms is negative.*

In all the above formulas, we must use the concept of the reduced coefficient of friction and instead of the angle φ substitute the angle φ', if the thread profile is triangular or trapezoidal.

8. Friction of flexible thread (Euler's formula)

A flexible, weightless, inextensible thread is thrown over a fixed cylinder (Fig. 69, a)). These assumptions make it possible to ignore:

a) redistribution of the weight of the weighty thread at its ends;

b) the work spent on bending and unbending a non-flexible thread;

в) the movement of the stretchable thread over the entire girth arc α would not be the same.

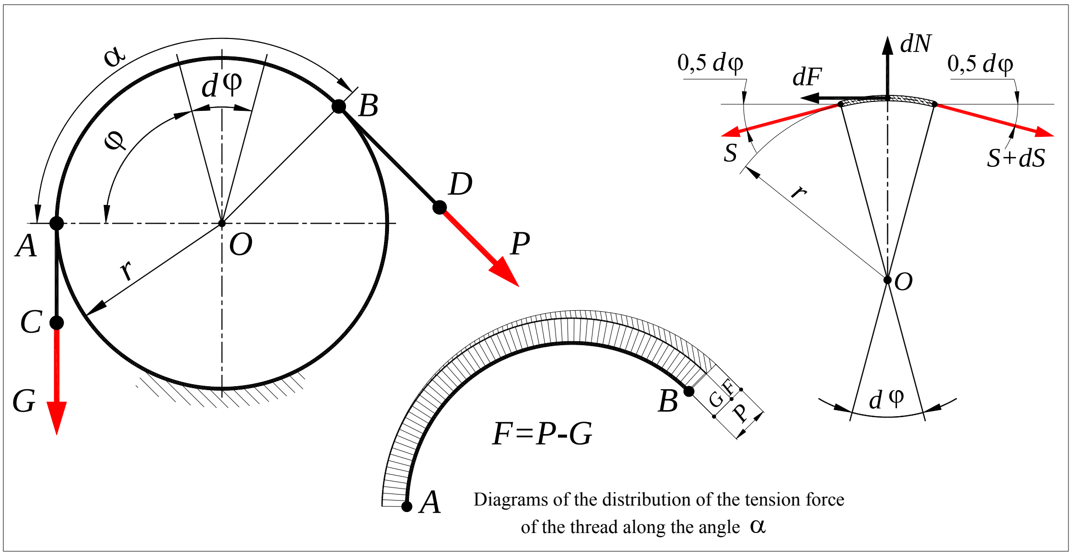


Fig. 69. Friction of flexible thread

A load of weight G is suspended to one of the ends of the thread (Fig. 69, a)). We must determine the magnitude of the force P. The force P is applied to the second end of the thread to lift the load with the speed $V = const$.

The thread angle of the cylinder is α. It is obvious that the sliding friction force is equal to $F = P - G$. It is also obvious that in the section of the AC thread its tension is equal to G, and in the BD section the thread tension is P. Fig. 69, b) shows the diagram of the distribution of the thread tension along angle α. We see that at an angle α the thread tension changes from $S = G$ when the angle $\varphi = 0$ to $S = P$ at $\varphi = \alpha$.

To solve the problem, we will cut out a section of the thread with an infinitely small angle $d\varphi$. From the equilibrium condition of the cut-out section of the thread 1) $\sum P_Y = 0$, we can write:

$$2S \cdot \sin(0{,}5d\varphi) + dS \cdot \sin(0{,}5d\varphi) = dN$$

Because the angle $d\varphi$ is infinitely small, then $\sin(0{,}5d\varphi) \approx 0{,}5d\varphi$. The quantity $dS \cdot \sin(0{,}5d\varphi)$ is the product of two infinitesimal quantities, so we can conclude that $dS \cdot \sin(0{,}5d\varphi) \approx 0$. Then expression 31) gets the form:

$$S \cdot d\varphi = dN$$

From the equation 2) $\sum M_y = 0$ we can write $(S + dS) \cdot r = (S + dF) \cdot r$, whence we get:

$$dS = dF$$

The elementary sliding friction force is equal to:

$$dF = dN \cdot f$$

After substituting in 34) the values dN from 32) and dF from 33) we get:

$$dS = S \cdot d\varphi$$

We represent this differential equation with separable variables in the form:

$$\frac{dS}{S} = d\varphi \cdot f$$

We integrate expression 36) bearing in mind that when the angle $\varphi = 0$ the tension force of the thread is $S = G$, and at $\varphi = \alpha$, the tension force is $S = P$.

$$\int_G^P \frac{dS}{S} = \int_0^\alpha d\varphi \cdot f \quad \text{or} \quad \ln\frac{P}{G} = \varphi \cdot f$$

From 37) we get:

$$P = G \cdot \mathrm{e}^{f\alpha} \text{ (Euler's formula)}$$

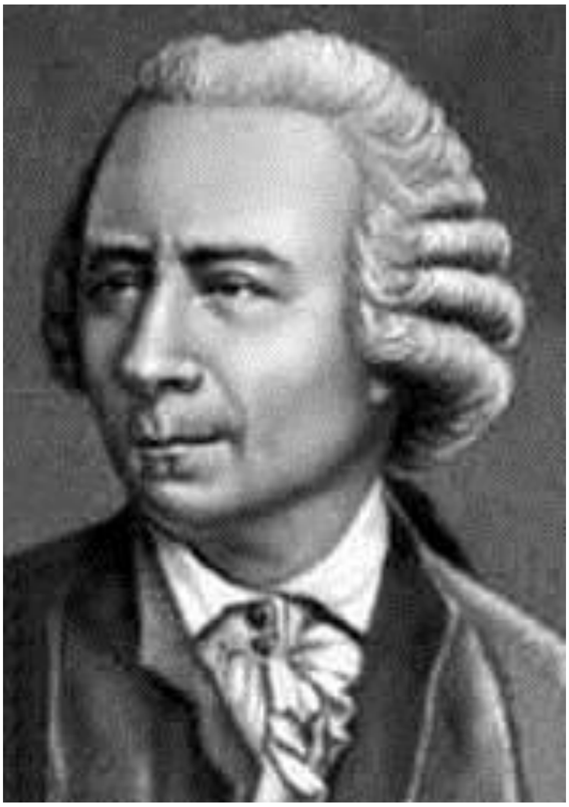


Fig. 70. Leonhard Euler

Leonhard Euler (1707–1783)

Prominent Swiss scientist, member of many European academies. He worked in Basel, Berlin, St. Petersburg, published over 850 scientific papers on mathematical analysis, differential geometry, mathematical physics, optics. He deeply studied chemistry, botany, medicine, music theory, knew many languages, including ancient ones. He is rightly considered one of the most prominent encyclopaedic scientists of all time, who laid the foundations of modern science.

9. Rolling resistance. (Rolling friction)

We will consider three cases. Case 1 is shown in Fig. 71.

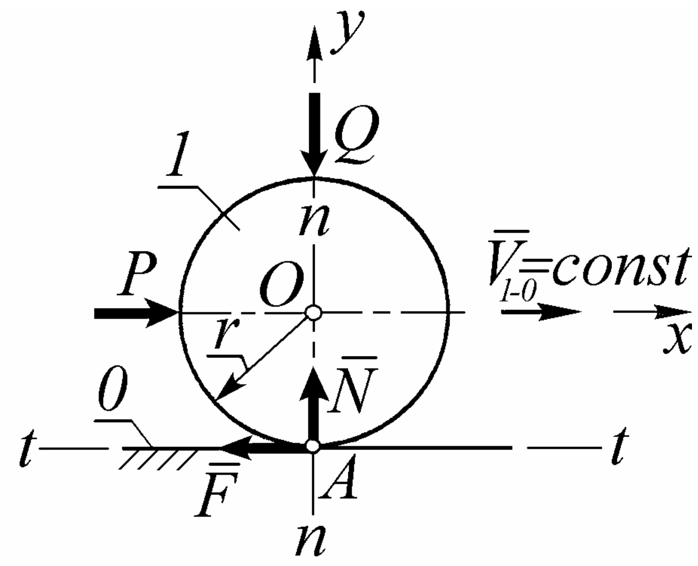


Fig. 71. Rolling of an absolutely rigid roller 1 on an absolutely rigid base 0

Here 1 is absolutely rigid (inelastic) round roller; 0 is absolutely rigid horizontal base; r is the radius of the roller; $\vec{Q}$ is the vertical force; $\vec{P}$ is the pushing force; $\vec{N}$ is normal reaction; $\vec{F}$ is tangential reaction.

The force $\vec{F}$ is usually called *the rolling friction force*.

We use the equations of statics. These equations are valid if the condition is met $\vec{V}_{1-0} = const$.

From the equation $\sum F_{iY} = 0$ we will find $N = Q$.

From the equation $\sum F_{iX} = 0$ we will find $F = P$.

From the equation $\sum M_O = 0$ we will find $F \cdot r = 0$.

Since $r \neq 0$, then $F = P = 0$.

We come to the conclusion that it is impossible to determine the value of the rolling friction force, because the deformations of the rolling bodies are not taken into account.

By the way, we can draw another conclusion: the higher the rigidity of the rolling bodies, the lower the rolling friction force. This is confirmed by experiments.

For example: the rolling friction is low on rail vehicles, where rigid steel wheels and rails are used.

There are cases when one person could move a 60-ton carriage on horizontal rails. Case 2 is shown in Fig. 72.

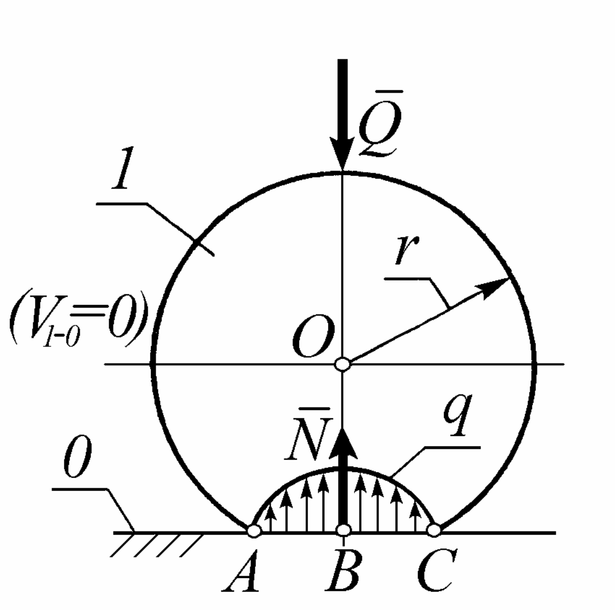


Fig. 72. Non-rolling resilient roller under load

Here 0 is rigid base; 1 is elastic roller; $\vec{Q}$ is external load; q is distributed normal reaction or specific forces in contact; $\vec{N}$ is total normal response; $\vec{V}_{1-0} = 0$ is relative velocity.

Fig. 73. Elastic wheel on a rigid base

Let the roller be homogeneous (isotropic). Then the diagram of specific forces in contact (q) will have a symmetric elliptical shape. The famous scientist Hertz proposed this form.

Heinrich Rudolf Hertz 1857 – 1894

German physicist. Graduated from the University of Berlin, where his teachers were Hermann von Helmholtz and Gustav Kirchhoff. From 1885 to 1889 he was professor of physics at the University of Karlsruhe. Since 1889 – Professor of Physics at the University of Bonn. Since 1933, the name of Hertz has been the name of the unit for measuring the frequency of the hertz, which is included in the International System of Units (SI).

We can see that the resulting response of specific forces $\vec{N}$ is opposite to the force $\vec{Q}$. Moreover, $N = Q$.

Case 3 is shown in Fig. 74.

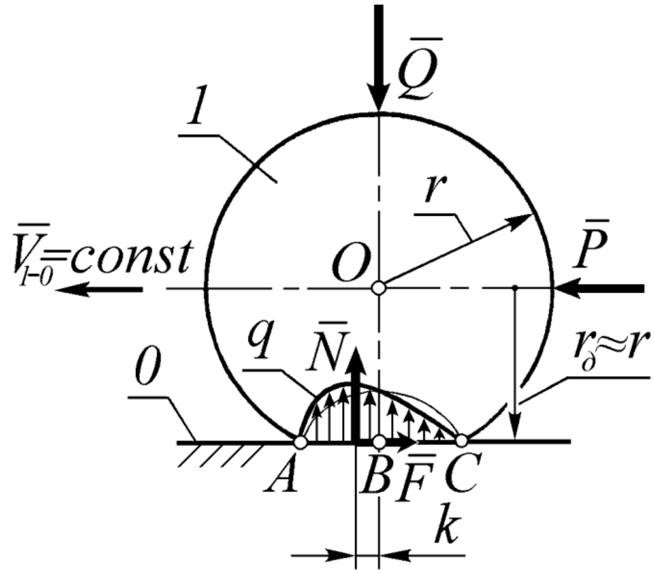


Fig. 74. Rolling of the elastic roller on a rigid base

Here, in contrast to case No. 2, the relative velocity $\vec{V}_{1-0} \neq 0$ and $\vec{V}_{1-0} = const$. In addition, here exists the horizontal pushing force $\vec{P}$. In this case, the friction force of the second kind $\vec{F}$ – arises in the contact plane.

Let's take a look at the rolling process.

The roller body 1 will experience progressive compression deformation at the front of the contact zone (from A to B). Internal friction prevents deformation from changing. This is known from the physics of an elastic body.

Therefore, the forces of internal friction will be added to the elastic forces in the contact zone (from A to B).

This force will increase the plot of specific pressures q. Conversely, the body of the roller 1 is straightened by elastic forces at the rear of the contact (from B to C). The forces of internal friction will also prevent deformation here. They will be subtracted from the elastic forces. The plot of specific pressures (q) will decrease.

Thus, the resulting plot of specific pressures in the contact zone q will change according to Fig. 74. Reaction force N will move by distance k.

According to the equations of statics, in this case the force $N = Q$, and the force $F = P$.

From the equation $\sum M_O = 0$, taking into account $r_d \approx r$, we find the expression, $F \cdot r = N \cdot k$, whence

$$F = \frac{k}{r} N$$

Here r_d – is the dynamic radius of the roller, depending on its deformation under load, r is the static radius of the roller (in the absence of load).

The shoulder of the forward drift of the normal reaction vector $\vec{N}$ in the direction of motion is *the coefficient of friction of the second kind*.

Experiments have established that the value of the rolling friction coefficient k depends primarily on the material of the rolling elements and little depends on the curvature of the roller.

In the reference literature, you can find a table of approximate values of the coefficient k for various pairs of rolling elements. For example, for rolling bodies made of non-hardened steel $k \approx 0{,}005\,\text{cm}$, for rolling bodies made of hardened steel $k \approx 0{,}001\,\text{cm}$, for rolling bodies made of wood $k \approx 0{,}05\,\text{cm}$.

Usually, the dimensionless ratio k/r turns out to be much less than the sliding friction coefficient f; therefore, rolling is accompanied by significantly less power losses than it is observed during sliding.

There are case when both the roller and the base are deformed (Fig. 75). Then the roller leaves a track. In this case the description of the rolling phenomenon becomes more complicated. Elastic deformations appear not only in the vertical, but also in the horizontal direction. In front of the roller, an additional resistance may arise as a risen. Rolling resistance rises sharply.

Fig. 75. Elastic wheel on a deformed base

Nevertheless, in this case the roller rolling model proposed above can be used. But the shoulder (k) of the drift of the normal component $\vec{N}$ must be determined experimentally under conditions close to real ones.

Self-Study and Review

1 Is friction usefull? What do you think about that?

2 What types of friction do you know?

3 What basic laws of sliding do you know?

4 What are the factors that influence the magnitude of frictional forces?

5 Can you describe the relationship between normal force and frictional force?

6 How does friction affect the efficiency of mechanical systems?

7 What do you know about angle and cone of friction?

8 Describe wedge slider construction.

9 What do you know about friction in a rotational pair?