

Ye. Ye. Aleksandrov, V. O. Bohomolov, V. I. Klimenko, D. M. Leontiev

APPLIED THEORY OF VIBRATIONS
for Students of Automotive
Specializations at Higher
Education Institutions

Study Guide



Ministry of Education and Science of Ukraine
Kharkiv National Automobile and Highway University

**Ye. Ye. Aleksandrov, V. O. Bohomolov,
V. I. Klimenko, D. M. Leontiev**

**APPLIED THEORY OF VIBRATIONS
for Students of Automotive Specializations
at Higher Education Institutions**

Study Guide

Kharkiv
KhNAHU
2026

UDC 629.3

BBK 30

A 46

Recommended by the Academic Council (Approval No. 80/25/6.22 dated 11/25/2025)
as a study guide for students of higher education institutions.

Reviewers:

O.O. Larin – *Corresponding Member of the National Academy of Sciences of Ukraine, D.Sc. (Eng.), Professor, Director of the Educational and Scientific Institute of Computer Modeling, Applied Physics and Mathematics, National Technical University "Kharkiv Polytechnic Institute"*

K.V. Avramov – *Corresponding Member of the National Academy of Sciences of Ukraine, D.Sc. (Eng.), Professor, Head of the Department of Nonlinear Mechanics and Mathematical Modeling of the Institute of Energy Machines and Systems, National Academy of Sciences of Ukraine*

R.O. Kaidalov – *D.Sc. (Eng.), Prof., Head of the Department for Organization of Educational and Scientific Activities of the Professional Training Directorate (J-7) of the Main Directorate of the National Guard of Ukraine*

M.A. Podryhalo – *D.Sc. (Eng.), Prof., Head of the Department of Mechanical Engineering Technology and Machine Repair, Kharkiv National Automobile and Highway University.*

Authors:

Ye.Ye. Aleksandrov, D.Sc. (Eng.), Professor, V.O. Bohomolov, D.Sc. (Eng.), Professor, V.I. Klymenko, D.Sc. (Eng.), Professor, D.M. Leontiev, D.Sc. (Eng.), Professor, KhNADU.

Ye. Ye. Aleksandrov

A 46 Applied theory of vibrations for students of automotive specializations at higher education institutions : 2nd Edition / Ye. Ye. Aleksandrov, V. O. Bohomolov, V.I. Klimentko, D. M. Leontiev – Kharkiv : 2nd Edition ФООП Бровін О.Б., 2026. – 160 p.

ISBN 978-617-8587-36-9

The study guide covers free, forced, and random vibrations of the sprung part of a vehicle body, as well as self-oscillations of the sprung part with friction shock absorbers, and self-oscillations of the engine coolant temperature using the method of harmonic linearization of nonlinearities. It also explores free and forced elastic vibrations of a gun barrel mounted on the chassis of a military vehicle, and the influence of liquid sloshing on the directional stability of a fuel tanker truck.

This study guide is intended for students of automotive specialties at higher education institutions of the III and IV accreditation levels. It can also be used as a self-study resource by engineers in the fields of mechanical engineering and transportation.

Fig. 56. Table 5. Page 4. Bibliogr. 16 titles.

UDC 629.3

BBK 30

ISBN 978-617-8587-36-9

© Ye. Ye. Aleksandrov, V. O. Bohomolov,
V.I. Klimentko, D. M. Leontiev, 2026

© KhNAHU, 2026

CONTENTS

Foreword	5
Chapter 1. Oscillations of linear dynamic systems with one degree of freedom	8
1.1 Free oscillations of linear systems with one degree of freedom	8
1.2 Forced oscillations of linear systems with one degree of freedom	16
1.3 Random oscillations of linear systems with one degree of freedom	19
1.4 MATLAB software package for investigating random oscillations of dynamic systems.....	31
Control questions for the chapter 1	41
Chapter 2. Oscillations of linear dynamic systems with two degrees of freedom	42
2.1 Free oscillations of linear systems with two degrees of freedom	42
2.2 Forced oscillations of linear systems with two degrees of freedom	46
2.3 Random oscillations of linear systems with two degrees of freedom.....	50
Control questions for the chapter 2	57
Chapter 3. Oscillations of nonlinear dynamic systems	58
3.1 Main nonlinearities in dynamic systems.....	58
3.2 Harmonic Linearization Method for Nonlinearities	63
3.3 Oscillations of a vehicle body with torsion bar suspension and friction dampers	69
3.4 Oscillations of the coolant temperature in a vehicle engine	75
3.5 Oscillations of non-stationary dynamic systems, or parametric oscillations	82
Control questions for the chapter 3	86
Chapter 4. Torsional vibrations of automotive engine and transmission shafts	87
4.1 Equations of free torsional vibrations of shafts	87
4.2 Frequency equation of free torsional vibrations of shafts	89

4.3 Normal modes of torsional vibrations of shafts.....	91
4.4 Forced torsional vibrations of shafts.....	92
Control questions for the chapter 4.....	97
Chapter 5. Vibrations of discrete-continuous dynamic systems	98
5.1 Free vibrations of an elastic gun barrel on a military vehicle chassis.....	98
5.2 Forced vibrations of an elastic gun on a military vehicle chassis.....	108
5.3 The refueling truck as a discrete-continuous system. Forced oscillations of the free surface of the transported liquid and their influence on the vehicle's directional stability	110
Control questions for the chapter 5.....	117
Chapter 6. Stability of linear dynamic systems.....	118
6.1 Definition of dynamic system stability according to Lyapunov	118
6.2 Stability of linear dynamic systems.....	120
6.3 Algebraic stability criteria.....	124
6.4 Frequency stability criteria	128
6.5 Constructing stability regions of linear dynamic systems in the plane of variable parameters	131
Control questions for the chapter 6.....	133
Chapter 7. Stability of nonlinear dynamic systems	134
7.1 Lyapunov's theorems on stability in the first approximation	134
7.2 Theorems of the second or direct method of Lyapunov	137
7.3 Lyapunov's theorem on the instability of motion of dynamic systems.....	141
Control questions for the chapter 7.....	142
Appendix 1. Methodological guidelines for completing calculation-graphic work No. 1.....	144
Appendix 2. Methodological guidelines for completing calculation-graphic work No. 2.....	150
Bibliography.....	156

Bibliography.

1. Aleksandrov Ye., Aleksandrova T., Kostianyk I., Morgun Ya. (2023). Simulation of Random External Disturbance Acting on the Car Body in the Urgent Braking Mode. *Advanced Information Systems*. 7(1). 14 – 17.
2. Aleksandrov, E. E., Bohomolov, V. O., Klymenko, V. I., & Leontiev, D. M. (2025). *Prykladna teoriia kolyvan dlia studentiv avtomobilnykh spetsialnostei vyshchiv: navchalnyi posibnyk* [Applied Theory of Oscillations for Students of Automotive Specialties at Higher Education Institutions: A Textbook].
3. Aleksandrov, E. E. (2010). *Osnovy avtomobil'noi avtomatiki* [Fundamentals of automotive automation]. KhNADU.
4. Aleksandrov, E. E., Aleksandrova, T. E., & Ovcharenko, Yu. E. (2019). *Pidvyshchennia tekhnichnykh ta erhnomicznykh kharakterystyk rukhomykh ob'ektiv viys'kovoho pryznachennia* [Improvement of technical and ergonomic characteristics of military mobile objects]. KhNADU
5. Alexandrov, Ye., Alexandrova, T., & Morgun, Ya. (2019). Parametric Synthesis of the Electronic Control Unit of the Course Stability System of the Car. *Easter-European Journal of Enterprise Technologies*, 6/9(102), 39–45.
6. Aleksandrov, E. E., Aleksandrova, T. E., Hryhor'iev, O. L., & Morhun, Ya. Yu. (2020). Stiikist' ta avtokolyvannia elektronnoi zamknienui systemy stabilizatsii kursu avtomobilia z tsystemoiu [Stability and self-oscillations of the electronic closed-loop car course stabilization system with a tank]. *Bulletin of NTU "KhPI". Series: "Mathematical Modeling in Engineering and Technologies"*, 1, 44–63.
7. Aleksandrov, E. E., Aleksandrova, T. E., Hryhor'iev, O. L., & Morhun, Ya. Yu. (2021). Pro vplyv kolyvan' vil'noi poverkhni ridyny na kursovu stiikist' avtomobilia-palyvozapravnyka [On the influence of free liquid surface oscillations on the directional stability of a refueling truck]. *Ozbroiennia ta viys'kova tekhnika*, 1, 36–43.
8. Aleksandrov, E. E., Aleksandrova, T. E., Hryhor'iev, O. L., & Morhun, Ya. Yu. (2021). Pro vplyv kolyvan' transportuiemoi ridyny na oblast' stiikosti zamknienui systemy avtomatychnoho keruvannia kursom avtomobilia [On the influence of transported

- liquid oscillations on the stability region of the closed-loop automatic car course control system]. *Bulletin of NTU "KhPI". Series: "System Analysis, Control and Information Technologies"*, 1(5), 29–41.
9. Ihdalov, I. M., Kuchma, L. D., Poliakov, M. V., & Sheptun, Yu. D. (2004). *Rakety-nosii i kosmichni stupeni raket yak ob'iekt keruvannia* [Launch vehicles and space rocket stages as a control object]. ART-PRESS.
 10. Eswaran, M., Parulekar, Y.M., Reddy, G.R. (2019). Introduction to Structural Dynamics and Vibration of Single-Degree-of-Freedom Systems. In: Reddy, G., Muruva, H., Verma, A. (eds) *Textbook of Seismic Design*. Springer, Singapore. https://doi.org/10.1007/978-981-13-3176-3_3
 11. Kerschen, G., Vakakis, A.F. (2020). *Modal Analysis of Nonlinear Mechanical Systems*. In: Allemang, R., Avitabile, P. (eds) *Handbook of Experimental Structural Dynamics*. Springer, New York, NY. https://doi.org/10.1007/978-1-4939-6503-8_35-1
 12. Linge, S., Langtangen, H.P. (2017). Vibration ODEs. In: *Finite Difference Computing with PDEs. Texts in Computational Science and Engineering*, 16. Springer, Cham. https://doi.org/10.1007/978-3-319-55456-3_1
 13. Povstenko, Y. (2024). Essentials of Fractional Calculus. In: *Fractional Thermoelasticity. Solid Mechanics and Its Applications*, 278. Springer, Cham. https://doi.org/10.1007/978-3-031-64587-7_1
 14. Volterra, E., Zachmanoglou, E.C., & Kolsky, H. (1965). *Dynamics of vibrations*.
 15. Chang, Zongyu & Ali, Rai & Ren, Ping & Zhang, Guangbin & Wu, Peixin. (2015). Dynamics and Vibration Analysis of Delta Robot. *Proceedings of the 5th International Conference on Information Engineering for Mechanics and Materials*. 1408-1417. <https://doi.org/10.2991/icimm-15.2015.257>
 16. Genta, G. (2009). Vibration of Beams. In: Genta, G. (eds) *Vibration Dynamics and Control. Mechanical Engineering Series*. Springer, Boston, MA. https://doi.org/10.1007/978-0-387-79580-5_12

Навчальне видання

АЛЕКСАНДРОВ Євген Євгенович
БОГОМОЛОВ Віктор Олександрович
КЛИМЕНКО Валерій Іванович
ЛЕОНТЬЄВ Дмитро Миколайович

ПРИКЛАДНА ТЕОРІЯ КОЛИВАНЬ
ДЛЯ студентів автомобільних
спеціальностей вищів

Навчальний посібник
друге видання
(англійською мовою)

Відповідальний за випуск *В.І. Клименко*

Редактор *Д.М. Леонтъев*

Комп'ютерна верстка *Д.М. Леонтъев*

Дизайн обкладинки *Д.М. Леонтъев*

Видавець **ФОП Бровін О.В.**
Свідоцтво про внесення суб'єкта до Державного реєстру
видавців та виготовників видавничої продукції серія ДК 3587 від 23.09.09 р.
Формат 60x84/16 Ум. друк. арк. 9.3. Тир. 100 прим. Зам. 856.

Надруковано з макету замовника **ФОП Бровіна І.П.**
61022, м. Харків, вул. Трінклера, 2, корп.1, к.19. Т. (066) 822-71-30

СТИЛЬ®
-ІЗДАТ
ДРУКАРНЯ
www.stil-izdat.com