UDC 624

GEOMETRIC MODELLING AND ITS APPLICATION TO DESIGN HIGHWAYS

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Abstract. Mathematical models to solve optimization connection problems in nonsimply connected regions under typical technological restrictions on geometric and topological parameters of routes, first of all, on curvature and the number of bends, have been investigated and developed. The models are linked with the extant and prospective topogeodesic models of the polygonal images of territories.

Key words: mathematical model, optimization problem, restriction, topological parameter, construction norm and rule, homotopy, accuracy.

The object of the research is the mathematical models of problems to search for optimal networks and routes that come into existence during the automation of design and control taking into consideration a landscape under the restrictions on the form, mutual location and other connection parameters. The subject of the research is the development of a mathematical model to solve the problem of the search of optimum routes and connecting networks in nonsimply connected regions under restrictions on curvature, the number of bends and other geometric topological connection parameters that reflect typical technological requirements. This paper uses the following methods of research: optimization techniques, computational approaches, computational geometry, mathematical modelling and combinatorial topology approaches. As of topological parameters, this paper provides their decomposition as a system of basic and standard problems; basic optimization problems are posited.

The most effective approach to solve analogous problems, which are NP-complete, even for a case of bends in a convex area, is developed in the papers of Yu. H. Stoyan and S. V. Smelyakov [1 – 4]. It is the synthesis of combinatorial and variational optimization methods within a framework of a hierarchical system of models. Low-level models are oriented to solve basic problems for the search of an optimum route, using variational methods, which have been developed by M. M. Moiseyev, S. V. Smelyakov and N. Z. Shor. High-level models are oriented to examine the homotopic families of highways and networks on the basis of the discrete optimization methods that have been developed in the papers of V. I. Mykhalievych, I. V. Sergiyenko, Yu. H. Stoyan, S. V. Yakovlev and others [5 – 7]. The impossibility of the complete formalization of requirements for CN&S – construction norms and standards – (as a consequence of their inconsistency and not complete strictness of formulae) and the subordinate character of connection problems in relation to the problems of the normalization of regions causes a need for their solution in two modes – optimization and imitation when a decision-making person is involved in the process of the interactive solution of connection problems.

The objective of this paper is the development of a mathematical model to solve the optimization connection problems in nonsimply connected regions under typical technological restrictions on geometrical and topological route parameters, first of all, on curvature and the number of bends. The model should be in conformity with the extant and prospective topogeodesic models of a polygonal territory image; optimization methods should guarantee the effective solution of the main classes of applied problems and allow natural integration in the extant and prospective systems of design and control.

To attain the objective, it is necessary to solve the following tasks:

- to sort out the major criteria and restrictions that are linked with geometric and topological route parameters that will be under consideration to design connections and on this basis to posit the main optimization problem;

- to develop the general model of connection problems that ensures the statement of the main types of route optimization and modelling problems using the functional classes of lines and allows augmentation and integration into existing systems.

In accordance with requirements for the accuracy of obtainable solutions and the computational effectiveness of the methods of their construction as well as with extant world tendencies to use polygonal terrain models in topogeodesic provision systems, the region models and optimization methods to solve connection problems should use the polygonal models of a nonsimply connected region that is specified for route connections rather than the grid models that are used in modern systems like ReCAD, CREDO. So, the region is calculated as follows:

$$F = Cl\left[\frac{F_0}{\left(\bigcup_{i=1}^{n_F} F_i\right)}\right], \tag{1}$$

where CI – a closing operation; F_0 – the overall area that is used to examine modelling problems; F_i, $(i = 1, 2, \ldots, n_F)$ – the simply connected regions that are mutually crossed ("prohibited zones" for routing) in the simply connected region F_0 on the plane.

The general mathematical feature of even routes (highways, railroads and tramlines) is a requirement for the continuity and limitedness of curvature. The requirement has to be fulfilled to guarantee transport traffic safety and the lack of impact blows. In connection with it, the construction norms and standards read that highway center lines will be presented as splines

$$p = S_1\, r_1\, C_1\, R_1\, S_2\, r_2\, C_2\, R_2 \ldots S_m\, r_m\, C_m\, R_m\, S_{m+1}, \tag{2}$$

where S_i – legs; C_i – arcs of circles; r_i, R_i – the connecting curves that can be presented by the fragments of clotoids and cubic parabolas, m – the number of the fragments of respective curves.

The analysis of CN&S shows that the majority of technological restrictions for transport and engineer networks under investigation can be reduced to the following main classes of geometric and topological restrictions: Q_1 – restrictions on maximum length of straight legs; Q_2 – restrictions on the angle of turn α, $\alpha \in \left(-90^\circ, 90^\circ\right)$, in vertices; Q_3 – restrictions on a functional class of smooth curves: $\tilde{W}$, SC, SKC, SPC; Q_4 – conditions on ends (to determine tangents and their length in the points of *A* and *B*); Q_5 – junctions with allowable sections; Q_6 – junctions to coordinate flows in terms of direction; Q_7 – junctions to provide their reach; Q_8 – junctions to boundaries and routes; Q_9 – the topological structure of a sought network (connectivity, cycles).

According to CN&S, the choice of the route of a pipeline, highway etc. should be made with the help of mathematical methods on the basis of one or some optimum criteria (for example, curves should be designed with as large radii as possible). In so doing, criteria, as a rule, are expressed by the geometric parameters of routes and the topological parameters of networks, and some of them can be nonadditive. Among the main geometric parameters of the route p, it is necessary to indicate length $l(p)$, the quantity $m(p)$ of circle insertions and their curvature radii $\{p_i\}_{i=1,m(p)}$ (or – for a bandy line – the quantity of bends $n(p)$ and turning angles $\{\varphi_i\}_{i=1,m(p)}$), the location of the route p in the region F. They influence the cost of the construction $c(p)$, maintenance charges $e(p)$, manufacturability $t(p)$ and reliability $b(p)$ that is one of the most important and the product of the possible analogues of reliability

$$\begin{cases} b_m(p) = \prod_{i=1}^{m} \left(1 - \frac{1}{p_i}\right), (p_i \geq 1) \text{ for } m > 0; \\ b_d(p) = \prod_{j=1}^{d} (1 - g_i)^{k_j}; \\ b_l(p) = \frac{1}{l(p)}, (l(p) \geq 1). \end{cases} \tag{3}$$

To summarise, the general problem to optimize connections (GPOC) taking into consideration the introduced criteria and restrictions can be posited as follows.

General problem to optimize connections. The given region F has both a set of points $\{A_i\}_{i=1,N}$ and some networks $\{S_j\}_{j=1,,M}$. It is required to connect these points and networks with the connecting network s^*, which consists of the lines of the given functional class $S\Theta C$ to meet the specified restrictions Q of the type $Q_1 - Q_9$, and it will be the most effective one in the meaning of the given optimum principle $R(s)$.

Taking into account the above-mentioned effectiveness of the decomposition of GPOC, which is based on the reduction of this problem to a system of basic problems on the continual families of routes that allow an exact solution, we can have the main typical optimization problem (MOP) to search for an optimum way in the class of route equivalence $[\tau]$ on a given functional class of lines $P(A,B)$ with the set ends of A and B under respective reduction of restrictions Q^*:

Main optimization problem. Calculate

$$\underset{p \in P_{tQ}(A,B)}{arg\,extr}\, R(p). \tag{4}$$

This paper further posits informational and computational requirements to models to calculate the specified problems.

The problem of the global and local decomposition and regularization of GPOC on the basis of its reduction to a system of the homogeneous basic problems of type (4) is considered. It guarantees constructive regularization and an effective computational approach to solve a general problem to optimize connections, using discrete and continual models and methods. The two-level FL model by Yu. H. Stoyan and S. V. Smelyakov is used to do it. On the top topological level the structure of this model is determined by an algebraic exfoliation of lines on the classes of the equivalence of routes and networks and on the geometric level by the consideration of the demanding functional class of lines in each class. Restrictions can be imposed on components on both levels. Since the variety F in form (1) has the homotopical nature of the type of disk with $n_F \geq 0$ holes, the structure of the homotopical model of lines consists of the free group $C_{(n_F)}$, which discretizes the continuous families of routes.

The network $\Gamma = \{\gamma_i\}_{i=1,m}$ in the region *F* is a set of rectifiable curves that are limited in number. They are non-selfintersecting and have no common points except ends. The review of the connected combinations of the types of networks, graphs etc. brings about the problems of three classes: the search of optimum networks on the topological level with no account taken of the geometry of the region *F*; the optimization of the contributions of an abstract model in the region *F* and the structure of an optimum network in the region *F*. To solve these problems, we can introduce the concept of the abstract network $s^* = (V^*, R^*)$ as the end connected abstract simplical complex $K = (V^*, R^*)$. The implementation of the abstract network by the geometric complex $s = (V, R)$ is a geometric network. The networks $s_1 = (V_1, R_1)$ and $s_2 = (V_2, R_2)$ are homotopical in F if their abstract analogues have the isomorphism $\omega : s_1 \to s_2$ under which the correspondence of 0-simplexes means the combination of apexes in F and the correspondence of 1-simplexes means their belonging to one class of route equivalence in *F*; the networks s_1 and s_2 are free homotopical if they are homotopical with accuracy to the shift of 0-simplexes in *F* and deformatively equivalent if they are isomorphic and homotopical and their various edges can have common points only in the basic apexes which they connect. It should be noted that network isomorphism saves algebraic invariants for basic apexes but it does not take into account the structure of the region *F* with enclosures in it. Homotopy saves network homotopy but it does not contribute to their isomorphism due to the emergence of additional apexes. Network deformity keeps homotopical invariants.

The geometric model of routes is the functional classes of lines $\Lambda = \{S, SC, SKC, SPC, W, \tilde{W}\}$ in the region F that have a spline image (2). Topological restrictions can be applied to them as well. The introduction of the models of networks and lines allows us to reduce the GPOC to the system of the basic problems of type (2) on various functional classes of lines and standard problems on the same classes that reflect the combination of restrictions that are typical for applied problems and are reduced to basic problems.

The hierarchical mathematical model of a common connection problem has been developed. The essense of it is the search of optimum connections (routes and connecting networks) in nonsimply connected regions. The problem comes into existence when designing transport and engineer networks and controlling vehicle traffic on cross-country terrain. In the framework of the model this problem is reduced to a system of basic and standard problems to search optimum routes. The use of this hierarchical structure of models in the systems of decision-making allows us to solve the problem of the adequate modelling of connections in nonsimply connected regions in terms of accuracy, computational effectiveness and absence of informational redundancy for all functional classes of bendy and even lines $\{S, SC, SKC, SPC\}$, which are regulated by CN&S, under restrictions on curvature and other geometric and topological parameters of routes.

References

1. Плехова А. А., Смеляков С.В. Моделирование коммуникационных сетей с учетом ландшафта при разнородных критериях и ограничениях. *Информационные системы. Сборник научных трудов*. 1998. Вып. 3 (11). С. 143-146.

2. Стоян Ю. Г., Яковлев С. В. Математические модели и оптимизационные методы геометрического проектирования. К.: Наук. думка, 1986. 268 с.

3. Смеляков С. В., Стоян Ю. Г. Моделирование пространства путей в задачах построения оптимальных траекторий, *Ж. вычисл. матем. и матем. физ.* 1983. Том 23, № 1. С. 73-82.

4. Смеляков С. В., Плехова А. А. Построение кратчайшей трассы в неодносвязной области при ограничениях на кривизну и класс кривых. *Информационные системы. Сборник научных трудов*. 1999. Вып. 1 (12). С. 170-175.

5. Сергиенко И. В., Шило В. П. Задачи дискретной оптимизации: проблемы, методы исследования, решения. К.: Наук. думка, 2003. 264 с.

6. Плехова А. А. Модель и метод решения задачи поиска оптимального соединения при ограничении на кривизну. *Радиоэлектроника и информатика*. 1998. № 3. С. 56-59.