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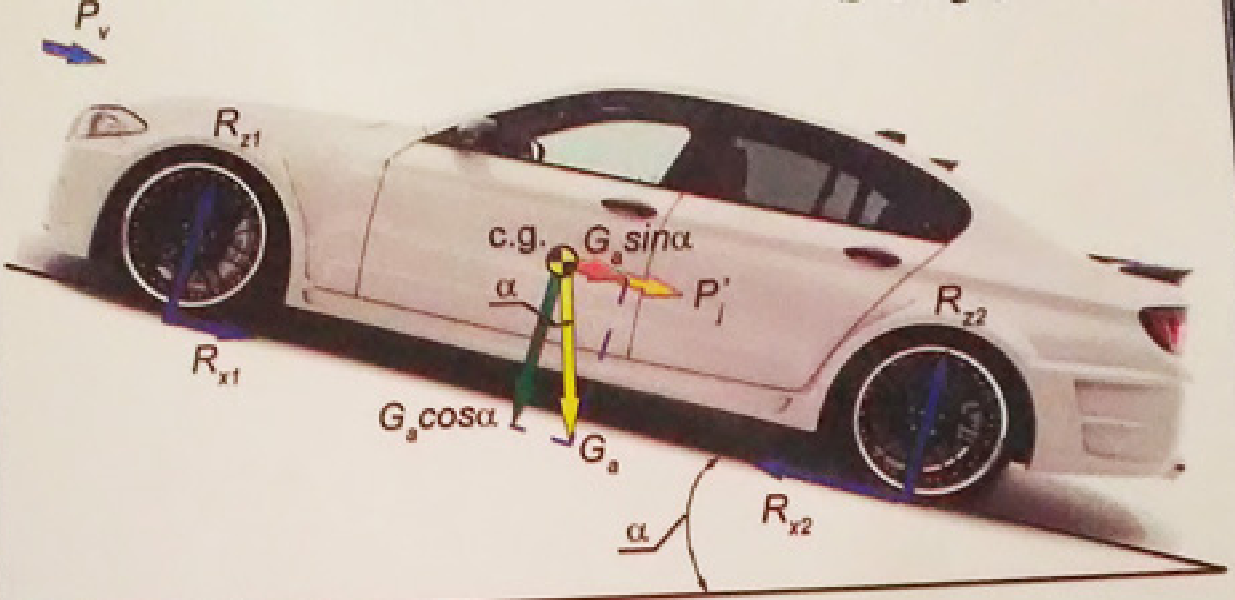
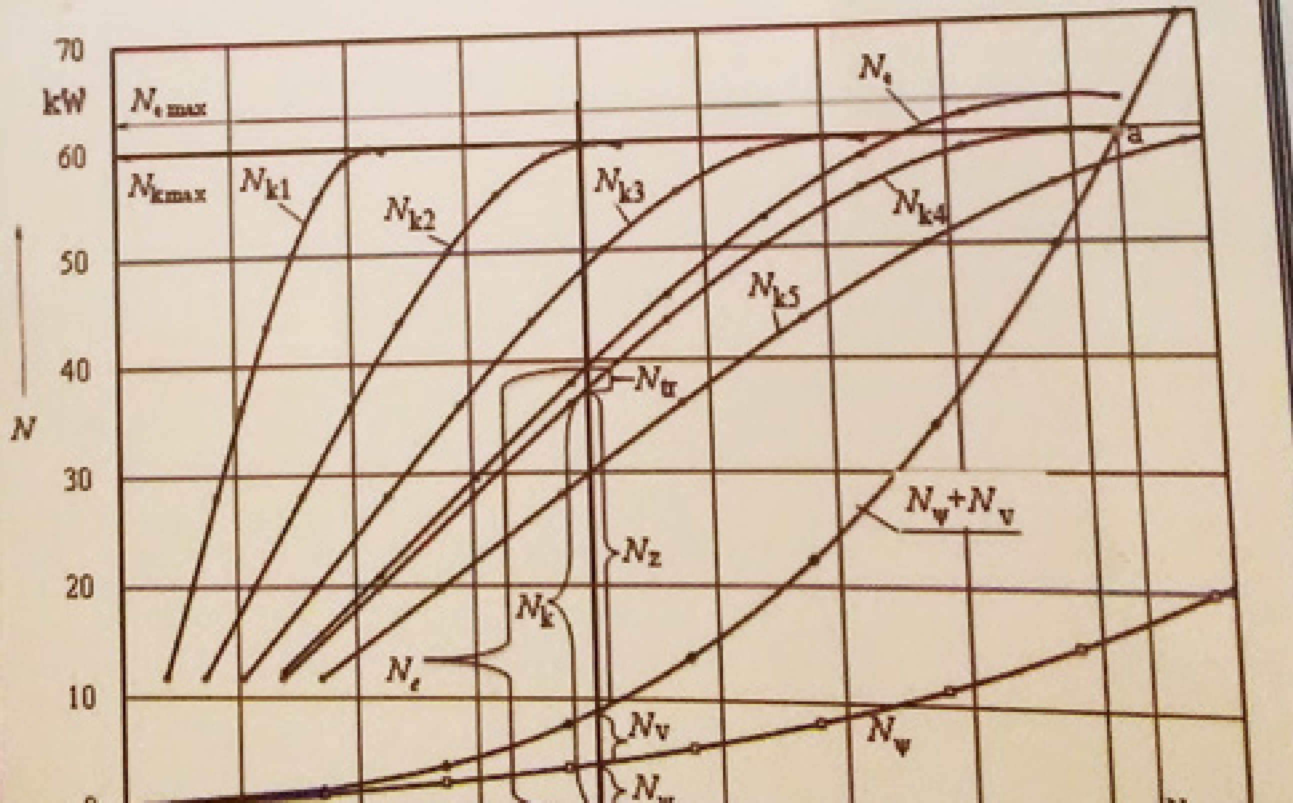
AUTOMOBILE. THEORY AND OPERATIONAL PROPERTIES

Study guide





Ministry of Education and Science of Ukraine
Kharkiv National Automobile and Highway University (KhNAHU)

**S.M. SHUKLINOV, V.I. KLYMENKO,
D.M. LEONTIEV, M.M. ALOKSA**

AUTOMOBILE.
THEORY AND OPERATIONAL PROPERTIES

Study guide

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Ш 95

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as a study guide for students of higher education institutions*

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Shuklinov S.M.
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The basics of the "Theory and operational properties of the vehicle" section are outlined. Considered phenomena that occur during the interaction of the vehicle with the supporting surface and air, the mechanics of its aggregates and nodes that provide a controlled change in the vehicle's speed vector, its operational properties. Study guide intended for students of technical higher and will also be useful for specialists working in the industry design, testing and operation of vhicle.

Викладено основи розділу «Теорія та експлуатаційні властивості автомобіля». Розглянуто явища, що виникають при взаємодії автомобіля з опорною поверхнею та повітрям, механіку його агрегатів і вузлів, що забезпечують керовану зміну вектора швидкості автомобіля, його експлуатаційні властивості. Навчальний посібник призначений для студентів технічних вищих навчальних закладів, а також буде корисним для спеціалістів, які працюють у галузі проектування, випробувань та експлуатації транспортних засобів.

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From the authors

The Department of Automobiles of the KhNAHU, which turned 90 years old in October 2021, is one of the oldest in Ukraine and has extensive experience in teaching special disciplines to students of automotive specialties. The educational disciplines "Automobiles", "Vehicles", "Rolling stock of motor vehicle", "Theory, operational properties and design of a vehicle" have sections devoted to the study of the processes of interaction of the vehicle with the supporting surface, air, as well as determining the operational properties of the vehicle. Depending on the training plans of the students, these sections are called "Theory of the vehicle", "Theory of the operational properties of the vehicle" or "Operational properties of the vehicle".

This study guide is the result of the generalization of the experience of teaching the educational discipline "Theory of the vehicle" and "Theory of the operational properties of the vehicle" at the department of automobiles named after A.B. Hredeskul. Lectures on the specified disciplines were given in different years by the staff of the department: professor A.B. Hredeskul, professor S.Y. Lomaka, professor A. M. Turenko, associate professor M.O. Bulgakov, senior lecturer Yu.V. Troshchii, senior lecturer L. Ya. Lagunov, senior lecturer A.M. Khoroshilov, professor V.M. Alekseenko, professor V.P. Volkov, professor M.A. Podrigalo, professor E.B. Reshetnikov, professor S.Ya. Khodyrev, professor L.O. Ryzhikh, associate professor A.V. Uzhva, associate professor A.I. Shilov, professor M.M. Aloksa, professor S.M. Shuklinov.

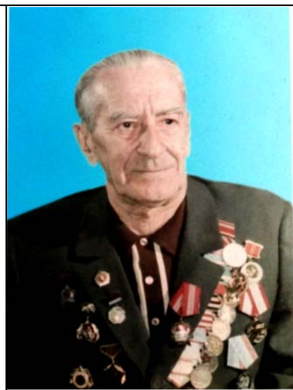
In study guide takes into account the experience of Professor V.P. Volkov during his writing of the textbook "Theory of operational properties of the vehicle", the materials obtained during the upgrading of the qualifications of teachers of the Department of Automobiles of KhNAHU from such scientists as Professor A.S. Lytvynov, Professor Ya.E. Farobin, Professor V.K. Vakhlamova.

The authors are grateful to the reviewers: Dr. Sc (Eng.), Prof. M.A. Podrigalo , Dr. Sc (Eng.), Prof. D. O. Volontsevich, Dr. Sc (Eng.), Prof. A.A. Kashkanov and Dr. Sc (Eng.), Prof. M.L. Shuliak - for useful comments that were taken into account during the preparation of the manuscript of the study guide for printing.

We are sure that the edition is not without flaws, and we will be grateful to everyone who will send their comments and wishes. Our address: 61002, Kharkov, st. Yaroslava Mudrogo, 25, KhNAHU, the department of automobiles named after A.B. Hredeskul

The study guide is the result of the generalization of the department's experience in teaching questions that reveal the processes of the interaction of the vehicle with the supporting surface and the assessment of the level of compliance of the vehicle design with specific operating conditions.

All these years the department of automobiles was managed by:

			
1931 - 1933 A.I. Voeikov	1933 - 1935 M.P. Denisenko	1935 - 1941 I. Yu . Lubarsky	1946 - 1956 B.V. Reshetnikov
			
1956 - 1962 1964 - 1986 A.B. Hredeskul	1962 - 1964 O.R. Sukhorukov	1986 - 1997 V.M. Alekseenko	since 1997 V.I. Klymenko

INTRODUCTION

At the Kharkiv National Automobile and Highway University, students are trained in the automotive specialties 133 - Industrial mechanical engineering (educational and professional program "Automotive engineering") and 274 - Road transport (educational and professional program "Road transport"), as well as educational programs in related specialties which are implemented in the KhNAHU.

The educational disciplines "Automobiles", "Vehicles", "Rolling stock of motor vehicle", "Theory, operational properties and design of a vehicle" in the listed specialties have sections devoted to the study of general information of the theory, processes of interaction of the vehicle with the supporting surface, air, as well as determination of its operational properties. Depending on the training plans of the students, these sections are called "Theory of the vehicle", "Theory of the operational properties of the vehicle" or "Operational properties of the vehicle".

The sections "Theory of the vehicle", "Theory of the operational properties of the vehicle" are intended to give students knowledge about the phenomena that occur during the interaction of the vehicle with the environment, the dynamics of its movement and the skills of forming the operational properties of the vehicle during its design, as well as to evaluate the design of existing vehicles.

The content of the study guide contains 11 topics. The first topic presents general information about the theory of the vehicle. The main parameters that characterize the engine, chassis and vehicle as a whole are considered, and an analysis of the external speed characteristics of various types of vehicle engines is given.

The second topic is devoted to the study of the main parameters of the vehicle wheel and its dynamics. The processes of rolling of a vehicle wheel on a non-deformable road surface under load conditions that act in the plane of rotation of the vehicle wheel, as well as when a lateral force acts on the wheel, are considered. The concept of the coefficient of rolling resistance of a vehicle wheel and its adhesion to the road surface is highlighted, as well as the factors that affect the coefficient of rolling resistance of a vehicle wheel and the coefficient of adhesion utilizat.

In the third topic of the study guide, the forces and reactions acting on a vehicle moving in the longitudinal plane of the road are considered.

The definition of forces and reactions, the analysis of their changes depending on the mode of movement of the vehicle is presented.

The fourth topic is devoted to the study of traction dynamics of the vehicle and its traction-speed properties. Indicators and methods of evaluating the traction and speed properties of the vehicle are presented. To evaluate the traction dynamics of the vehicle, its motion equations, traction balance, dynamic characteristics, dynamic passport and power balance are used. Examples of solving problems of vehicle traction dynamics are given.

The fifth topic deals with the fuel economy of the vehicle. The equations for determining the fuel consumption of a vehicle is given, and the parameters of its fuel economy characteristics and factors affecting it are analyzed.

In order to gain knowledge on the design of the traction-speed properties of the vehicle being designed, the traction calculation of the vehicle is considered in the sixth topic. The topic covers the issue of choosing and determining the parameters of the vehicle.

In the seventh topic, indicators of braking properties of the vehicle and its braking dynamics are considered. Methods of evaluating and improving the vehicle's braking properties are also considered in the topic.

The content of the eighth and ninth topics is closely related, they highlight the factors that determine the stability and controllability of the vehicle during its movement. The stability indicators of the vehicle were determined and the kinematics and dynamics of the vehicle movement along a curved trajectory were considered.

The topic ten describes the parameters that determine the vehicle's ability to overcome increased traffic resistance and form its operational property - passability. Information is given on the formation of parameters that ensure the increase in the passability of the vehicle.

The last, eleventh, topic is devoted to the study of vehicle vibrations and parameters that determine the smoothness of its movement. Both free and forced oscillations of the sprung mass and unsprung masses of the vehicle are considered. Mathematical models of oscillations of vehicle elements are presented.

The study guide does not consider the dynamic properties of vehicle with hydromechanical transmissions, because a separate study guide "Theory and traction dynamics of a vehicle with hydromechanical transmission" is being prepared.

ACCEPTED DESIGNATIONS

Mass and moments of inertia

m_a – total weight of the vehicle, kg;

m_{a1}, m_{a2} – the total mass of the vehicle, which falls on the front and rear axle, respectively, kg;

m_0 – equipped mass of the vehicle, kg;

m_{01}, m_{02} – equipped mass of the vehicle, which falls on the front and rear axle, respectively, kg;

m_{0p} – equipped mass of the analogue vehicle, kg;

m_{gr} – load weight (carrying capacity), kg;

m_{ch} – weight of the driver or passenger, kg;

m_b – baggage weight, kg;

m_{bg} – mass of the balancing load (balancing mass), kg;

m_k – mass of the wheel, kg;

m_p – sprung mass of the vehicle, kg;

m_{p1}, m_{p2} – sprung mass of the vehicle, which falls on the front and rear suspensions, respectively, kg;

m_n – unsprung mass of the vehicle, kg;

m_{n1}, m_{n2} – unsprung mass of the vehicle, which falls on the front and rear suspensions, respectively, kg;

α_{m0} – specific equipped mass of the analogue bus, kg/person;

α_{m0c} – the average value of the specific equipped mass of analogue buses, kg/person.

q – carrying capacity, t;

q_p – carrying capacity of the analogue vehicle, kg;

M_{pr} – reduced mass of the vehicle, kg;

M_1, M_2, M_3 – reduced sprung masses of the oscillating system model of the vehicle, kg;

J_k – moment of inertia of the wheel, kg · m ^{2};

J_{k1}, J_{k2} – the total moment of inertia of the wheels of the front and rear axles , respectively , kg · m ^{2};

J_e is the moment of inertia of the rotating parts of the engine and transmission , kg · m ^{2}.

Linear dimensions

a – front longitudinal coordinate of the center of gravity of the vehicle, m;

a_{sh} – wear of the normal reaction of the road to the wheel in the longitudinal plane (shoulder of resistance to rolling of the wheel), m;

a_x – wear of the lateral reaction of the road to the wheel in the longitudinal plane, m;

a_y – wear of the longitudinal reaction of the road to the wheel in the lateral plane, m;

a_p – front longitudinal coordinate of the center of gravity of the sprung mass (body) of the vehicle , m;

b – rear longitudinal coordinate of the center of gravity of the vehicle, m;

b_p – rear longitudinal coordinate of the center of gravity of the sprung mass (body) of the vehicle , m;

b_k – turning width of the vehicle, m;

b_{sf} , b_{sr} – wheel track width, respectively, of the front and rear axles, m;

b_{sh} – tire profile width, m;

B – vehicle track, m;

B_1, B_2 – track of the vehicle, respectively front and rear wheels, m;

B_a, H_a, L_a – overall dimensions of the vehicle, respectively width, height, length, m;

ΔB – displacement of the center of gravity of the vehicle body during its roll , m;

L – vehicle base, m;

h_g - height of the center of gravity of the vehicle, m;

h_b - the height of the center of windage (height of the metacenter), m;

h_p – height of the center of gravity of the sprung mass of the vehicle (body), m;

h_{kr} – height of the roll center of the vehicle , m;

h_z – normal tire deflection, m;

h_y – side deflection of the tire, m;

h_{sh} – tire profile height, m;

h_{pod} - the height of the road rise, m;

h_{aqua} – the height of the water layer, m;

h_{α} is the height of the front part of the vehicle when the steering wheel turns, m;

h_{ec} – height of the vertical wall, m;

h_{d} – ground clearance, m;

Δh_{z} – tire hysteresis, m;

Δh_{p} – decrease in the height of the center of gravity of the sprung mass of the vehicle when the body rolls over , m;

Δ_{uk} – toe-in of steered wheels, mm;

e_{st} – eccentricity module of the wheel, m;

r – radius of rotation, m;

r_{0} – free wheel radius , m;

r_{c} – static wheel radius, m;

r_{d} – dynamic radius of the wheel , m;

r_{k} – rolling radius of the wheel , m;

r_{ko} – rolling radius of the wheel in the controlled rolling mode, m;

r_{kc} – rolling radius of the wheel at $R_{x} = 0$ – free rolling mode, m;

r_{Θ} – turning radius of a vehicle with rigid wheels, m;

r_{δ} – turning radius of a vehicle with elastic wheels, m;

r_{bg} – the distance from the wheel axis to the point of attachment (on the rim) of the balancing load, m;

r_{vn}, r_{n} – turning radii, respectively internal and external, m;

ρ_{1}, ρ_{2} – longitudinal and transverse radii of passage, m;

d_{0} – diameter of the central running track, m;

d_{sh} – tire mounting diameter, m;

l_{k} – the length of contact of the tire with a solid surface, m;

l_{0} – the length of the circumference of the tire running track, m;

l_{cf}, l_{cr} – front and rear overhang of the vehicle, m;

l_{pod} – the length of the road rise, m;

l_{tsb} – the shoulder of the centrifugal force acting on the wheel, m;

l_{α} – running-in shoulder of the controlled wheel (transverse displacement of the center of the footprint from the center of rotation), m;

l_{β} – the arm of the lateral reaction of the controlled wheel, m;

S_{k} – the distance displacement by the wheel, m;

S_{r} – acceleration distance of the vehicle , m;

S_{p100} – the control path of acceleration of passenger vehicle – the path of acceleration to a speed of 100 km/h, m;

S_{p60} – the control path of acceleration of trucks – the path of acceleration to a speed of 60 km/h, m;

s_p – the distance displacement by the vehicle during gear shifting, m;

s_1, s_2,... s_{n_k} – the distance displacement by the vehicle during acceleration in each gear in the box, m;

$s_{n(1-2)}$, $s_{n(2-3)}$ – the distance displacement by the vehicle during gear shifting, respectively, from the first to the second, from the second to the third, m;

S_a – the distance displacement by the vehicle, km;

S_T – braking distance, m;

F – Midel area, m ^{2};

F_a – frontal resistance area of the vehicle , m ^{2};

F_k – the area of the spot of contact of the wheel with the road, m ^{2};

F_{kpr} – the area of the tread protrusions in the spot of contact of the wheel with the road, m ^{2}.

Corner dimensions

α – angle of rise (inclination), degree;

α_{max} – angle of maximum rise (inclination) that the vehicle can overcome, degree;

α_1, α_2 – overhang angles of the vehicle, respectively front and rear, degrees;

α_o – critical angle for longitudinal overturning, degree;

α_{shk} – angle of longitudinal inclination of the axis of rotation (pivot) of the steered wheel, degrees;

β – angle of transverse slope (slope) , degrees;

β_{sh} – angle of rotation of the tire , degrees;

β_{shk} – angle of transverse inclination of the steering axis (pin) of the steered wheel, degrees;

β_k – camber angle of steered wheels, degrees;

β_φ – the critical angle of the transverse downhill (slope) for lateral sliding, degrees;

β_{op} – critical angle of transverse downhill (slope) after overturning, degrees;

β_v, β_g – angle of flexibility of the road train, respectively, in the vertical and horizontal planes , degrees;

$\Delta \beta_k$ – change of the angle of installation of the plane of rotation of the wheel, degrees;

β_m – bridge skew angle, degrees;

β_{body} – roll angle of the body , degrees;

δ – the angle of the wheel's side deflection , degrees;

δ_k – angles of convergence of steered wheels, degrees;

Θ_k – angle of rotation of the wheel;

Θ_n, Θ_v, Θ – turning angles of the outer and inner wheels and the average turning angle of the steered wheels , deg.

Forces, reactions, moments and powers

G_a – the force of gravity of the vehicle of full mass m_a, H;

G_{ap} – the force of gravity of the sprung mass m_n, N;

G_k is the weight of the wheel , N;

G – vehicle weight , N;

G_0, G_{100} – weight of the vehicle, respectively, without load and with full load (load 0% and 100%) , N;

G_1 and G_2 – the weight of the vehicle with a full load, which falls on the front and rear wheels, respectively, H;

G_{01} and G_{02} – weight of the vehicle without load, which falls on the front and rear wheels, respectively, N;

G_φ – adhesion weight, N;

$G_{\varphi 1}$, $G_{\varphi 2}$ – adhesion weight of the front and rear wheels of the vehicle, N;

G_n – weight of the sprung mass m_n, H;

G_t – useful load on the vehicle when carrying out transport work (tons of cargo or number of passengers);

P_z – normal load on the wheel, N;

P_{side} – lateral force, N;

$P_{j\Sigma}$ – the total inertia force of the vehicle, N;

P_{jy} – lateral component of inertia force, H;

P_{jkh} – longitudinal component of inertial force, N;

P'_{jy} – centrifugal force, N;

P''_{jy} – lateral force, which appears as a result of turning the steered wheels, H;

P'''_{jy} – lateral force, which appears as a result of a change in the speed of the vehicle, H;

P_j – the force of inertia of the forwardly moving masses of the vehicle, N;

P_{Je} – the inertial force of the rotating masses of the engine and transmission, reduced to the driving wheels , N;

$P_{J\text{k}}$ – force of inertia of the rotating masses of vehicle wheels , N;

P_{j_a} – vehicle acceleration resistance force , N;

$P_{j_\text{т}}$ – force of inertia of translationally moving vehicle masses during braking, N;

$P_{J_\text{к}+J_e}$ – the total inertial force of the rotating masses of the vehicle, reduced to the driving wheels, N;

$P_{j_\text{т}\max}$ – inertial force arising at the maximum deceleration of the vehicle, H;

P_{yk} – wheel inertia force, N;

P_x – longitudinal load on the wheel , H;

P_{k} – full traction force, N;

P_{v} – air resistance force , N;

P_{f1}, P_{f2} – the rolling resistance force of the wheels of the front and rear axles, H;

P_α – lifting resistance force , N;

P_{cb} – free traction force of the vehicle , N;

P_φ – force of adhesion of the wheels to the supporting surface , N;

$P_{\varphi\text{ v}}$ – adhesion utilized force of the driven wheels with the support surface, N;

$P_{\varphi 1}$, $P_{\varphi 2}$ – the adhesion utilized force of the wheels of the front and rear axles of the vehicle , N;

P_y – lateral force, N;

P_{y1}, P_{y2} – lateral force acting on the front and rear axles, H;

P_{tsb} – centrifugal force, N;

P_{tsb1} , P_{tsb2} – centrifugal force, which falls on the front and rear axles, H;

P_{t} – braking force , N;

$\Sigma\ P$ is the sum of the forces of resistance to the movement of the vehicle, H;

P_{t1}, R_{t2} – braking force on the wheels, respectively, on the front and rear axles of the vehicle , N;

P_{tmax} – the maximum total braking force of the vehicle, N;

P_{t1max} , P_{t2max} – the maximum braking force, respectively, on the wheels of the front and rear axles of the vehicle , H;

P_{τ} – braking force of the engine, applied to the driving wheels, N;

P_{p} – constant value of force on the brake pedal, N;

P_{0} – insensitivity force of the braking system, N;

[$P_{p\ max}$] – the maximum permissible force on the brake pedal, N;

$P_{p\ max}$ – the recommended value of the force on the brake pedal, at which the maximum deceleration of the vehicle is achieved, N;

P_{a} – the resistance force of the shock absorber, H;

P_{aqua} – the lifting force of the water layer , H;

R – total reaction on the wheel, H;

R_{1} , R_{2} – total reaction, respectively, on the wheels of the front and rear axles of the vehicle, H;

R_{1n} , R_{1v} – total reaction, respectively, on the outer and inner wheel of the front axle, H;

R_{2n} , R_{2v} – total reaction, respectively, on the outer and inner wheel of the rear axle, H;

R_{z} – normal reaction on the wheel, H;

$R_{z\,1}$, $R_{z\,2}$ – total normal reaction, respectively, on the front and rear axles of the vehicle, H;

$R_{z\,l}$, $R_{z\,r}$ – normal reaction, respectively, on the left and right wheels of the vehicle, N;

$R_{z\,n}$, $R_{z\,p}$ – total normal reaction, respectively, on the loaded and unloaded side of the vehicle, N;

R_{x} – longitudinal reaction on the wheel, H;

$R_{x\,L}$, $R_{x\,R}$ – longitudinal reaction, respectively, on the left and right wheels of the vehicle, H;

$R_{x\,1}$, $R_{x\,2}$ – total longitudinal reaction, respectively, on the front and rear axles of the vehicle, H;

$R_{x\,1n}$, $R_{x\,1v}$ – longitudinal reaction, respectively, on the outer and inner wheels of the front axle , H;

$R_{x\,n}$, $R_{x\,p}$ – longitudinal reaction, respectively, on the loaded and unloaded sides of the vehicle, H;

$R_{x\,1max}$, $R_{x\,2max}$ – the maximum longitudinal reaction, respectively, on the front and rear axles of the vehicle, H;

$R_{x\,max}$ – the maximum total longitudinal reaction on the axles of the vehicle, H;

R_y – lateral reaction on the wheel, H;

$R_{y\,L}$, $R_{y\,R}$ – lateral reaction, respectively, on the left and right wheels of the vehicle, H;

R_{v1}, R_{v2} – total lateral reaction, respectively, on the front and rear axles of the vehicle, H;

$R_{y\,n}$, $R_{y\,p}$ – total lateral reaction, respectively, on the loaded and unloaded side of the vehicle, N;

$R_{v\,1n}$, $R_{v\,1b}$ – lateral reaction, respectively, on the outer and inner wheels of the front axle , H;

M_e – the current value of the effective torque on the motor shaft, N·m;

$M_{e\,max}$ – the maximum effective torque on the motor shaft, N·m;

M_k – torque (driving) moment applied from the transmission to the driving wheel , N·m;

M_T – braking torque applied from the braking mechanism to the wheel, N·m;

M_N – torque on the engine shaft at maximum power, N·m;

M_{jd} – the moment of inertia of the rotating parts of the engine, N·m;

M_{ik} – moment of inertia of the wheel , N·m;

M_{jk1}, M_{jk2} – moment of inertia of the wheels of the front and rear axles of the vehicle, respectively, N·m;

M_{P_y} – overturning moment created by centrifugal force , N·m;

M_{G_a} – the moment created by the force of gravity of the vehicle, N·m;

M_{podv} – elastic torque of the suspension , N·m;

M_r – moment of resistance in the engine , N·m;

M_{hyrg}, M_{hyrv} – gyroscopic moment of the steered wheels, respectively, in the horizontal and vertical planes, N·m;

M_d – destabilizing moment, N·m;

M_β – weight stabilizing moment of the controlled wheel, N · m;

M_α – high-speed stabilizing moment of the controlled wheel, N · m;

$M_{c\,sh}$ – stabilizing moment of the tire of the driven wheel, N · m;

N_e – current effective power of the engine, kW;

$N_{\max}$ – maximum engine power, kW;

$N_{e\max}$ – the maximum effective power of the engine, kW;

N_{k} - power supplied to the driving wheels of the vehicle (supplied from the engine through the transmission), kW;

N_{ψ} – the power used to overcome the total road resistance, kW;

N_{v} – power spent to overcome air resistance, kW;

N_{j} – power spent on acceleration (acceleration) of the vehicle , kW;

N_{tr} – power lost in the transmission, kW;

N_{z} – power reserve on the driving wheels, kW.

Velocities, accelerations and time

n_{e} – current engine crankshaft rotation frequency, rpm;

$n_{\max}$ – maximum engine crankshaft rotation frequency, rpm;

$n_{e\min}$ – the minimum stable engine crankshaft rotation frequency at full fuel supply, rpm;

n_{0} – engine crankshaft speed at which the speed limiter turns on, rpm;

n_{N} – engine crankshaft rotation frequency, at maximum engine power, rpm;

n_{N}^{*} – the theoretical engine crankshaft speed at which the maximum effective engine power would be achieved in the absence of a maximum engine speed limiter, rpm;

n_{m} – crankshaft rotation frequency at maximum torque, rpm;

n_{v} – crankshaft rotation frequency at the maximum speed of the vehicle, rpm;

$n_{ge\min}$ – crankshaft rotation frequency at which the engine operates with the lowest specific fuel consumption , rpm;

ω_{k} – angular speed of the wheel, rad/s ;

ω_{k1} – angular speed of the wheels of the front axle, rad/s;

ω_{k2} – angular speed of the wheel of the rear axle, rad/s;

ω_{e} – angular speed of the engine crankshaft, rad/s;

ω_{a} – angular speed of the vehicle, rad/s ;

$\frac{d\beta_{\kappa}}{dt}$ – angular rate of change of the plane of rotation of the steered wheels, rad/s;

v – vehicle speed, m/s;

v_w – vehicle speed relative to air space, m/s;

v_a – speed of the vehicle, km/h;

v_{amax} – the maximum speed of the vehicle, km/h;

v_{amin} – the minimum steady speed of the vehicle with full fuel supply, km/h;

v_k – gradual speed of the wheel, m/s;

v_t – theoretical progressive speed of the wheel, m/s;

v_{aqua} – aquaplaning speed, m/s;

v_n, v_{kj} – speed, respectively, at the beginning of the section and at the end of the acceleration section, m/s;

v_{sr} – average speed of movement on the section, m/s;

$v_{a\varphi}$, v_{φ} – critical speed for side slip, respectively km/h, m/s;

v_{aop}, v_{op} – critical speed after overturning in km/h, m/s, respectively ;

$v_{\varphi y}$ – critical speed under controllability conditions, m/s;

$v_{a\delta}$, v_{δ} – critical speed on the lateral lead , respectively km/h, m/s;

$v_{a\delta x}$ – characteristic speed on the lateral lead, km/h;

v_{kz} – the vector of the instantaneous speed of swinging the wheel around the pole, m/s;

v_1, v_2 – speed of the front and rear axles of the vehicle, respectively, m/s;

v_{1x}, v_{2x} – the speed of the front and rear axles of the vehicle, respectively , along the x axis , m/s;

v_{z1} , v_{z2} – skidding speed of the front and rear axles along the y axis, m/s;

v_{an} – initial braking speed, km/h;

v_{ac} – final braking speed, km/h;

v_{ap}, v_p – speed of the vehicle before gear shifting, respectively, km/h, m/s;

v_{app}, v_{pp} – speed of the vehicle after gear shifting, respectively, km/h, m/s;

$v_{a\,ek}$ – vehicle speed at which it is desirable to have the lowest road fuel consumption, km/h;

v_{opt} – optimal vehicle speed , m/s;

V_p – shock absorber piston movement speed, m/s;

V_a, V_{ash} – the average speed of the vehicle on off-road and highway, respectively, km/h;

$\varepsilon_k = d\omega_k/dt$ – angular acceleration of the wheel, s^{-2};

j_k – wheel acceleration, m/s^2;

j_a – acceleration of the vehicle , m/s^2;

j_n and j_k – acceleration of the vehicle at the beginning of the section and at the end of the section , m/s^2 , respectively ;

j_{sr} – average acceleration on the section , m/s ^{2};
j_p – acceleration of the vehicle when switching gears , m/s ^{2};
j_T – vehicle deceleration during braking, m/s ^{2};

$j_{t0\ max}$, $j_{tp\ max}$ – the maximum deceleration of the vehicle during braking, respectively, without load and with full load, m/s ^{2};

$\Delta t_1, \Delta t_2, \Delta t_3, \ldots \Delta t_k$ – section passage time (k – section number during acceleration on this gear), s;
t_p – acceleration time of the vehicle , s;
$\tau_1, \tau_2, \ldots \tau_{n_к}$ – acceleration time in each gear in the gearbox, s;
t_n – gear shifting time in the gearbox, s;
$t_{n(1\text{-}2)}$, $t_{n(2\text{-}3)}$ – the time of switching gears from the first to the second and from the second to the third, respectively, s;
t_{p100} – control time of acceleration of passenger vehicle, s (acceleration time to a speed of 100 km/h);

t_{p60} – control time of truck acceleration, s (acceleration time to a speed of 60 km/h);
t_T – braking time, s;
τ_p – driver reaction time, s;
τ_z – delay time, s;
τ_n – time of increase of deceleration, s;
τ_s – activation time, s;
τ_y – braking time with constant deceleration, s;
τ_{rast} – deceleration time, p.

Coefficients

k_v – coefficient of air resistance, N · s ^{2}/m ^{4};
k_e – empirical coefficient of reduction of adhesion efficiency ;
c_x – coefficient of aerodynamic resistance (dimensionless);
c_w – dimensionless coefficient of total aerodynamic force;
f – coefficient of rolling resistance;
f_v – coefficient of rolling resistance at maximum speed;
f_0 – rolling resistance coefficient at low speed;
f_1, f_2 – the rolling resistance coefficient of the wheels, respectively, of the front and rear axles;
i – lift resistance coefficient (longitudinal slope of the road);

k_M, k_ω – coefficient of adaptability of the engine in terms of torque and revolutions, respectively;

k_δ – resistance coefficient of the wheel to the lateral deflection, N/degree;

$k_{\delta 1}$, $k_{\delta 2}$ – the coefficient of resistance to the lateral deviation of the wheels, respectively, of the front and rear axles, N/degree;

k_{n} – coefficient of longitudinal force;

k_{t} – traction coefficient;

$k^0_{\text{ту}}$, $k^{\text{п}}_{\text{ту}}$ – coefficient of efficiency of braking control of the vehicle, respectively, without load and with full load, (m/s ^{2}) / N;

k_φ – adhesion mass coefficient;

k_h – tire profile coefficient;

k_u – coefficient that takes into account the dependence $g_e = f(u)$;

k_n is a coefficient that takes into account the dependence $g_e = f(n)$;

k_{g} – coefficient of carrying capacity;

k_{hs} – the average value of the carrying capacity coefficient;

s_{b} – slip coefficient;

s_{bkr} – the critical slip coefficient;

s – coefficient of longitudinal slip;

s_{kr} – coefficient of critical longitudinal slip;

$c_{\text{sh}\,z}$ – coefficient of normal tire stiffness, N/m;

$c_{\text{sh}\,x}$ – coefficient of longitudinal stiffness of the tire, N/m;

$c_{\text{sh}\,\beta}$ – coefficient of torsional rigidity of the tire , N·m/rad;

$c_{\text{sh}\,\Theta}$ – coefficient of angular stiffness of the tire , N·m/rad;

c_{sh1}, c_{sh2} – tire stiffness coefficients of the front and rear axles respectively , N/m;

c_{p1}, c_{p2} – stiffness coefficient of the front and rear suspension , respectively , N/m;

c_{pr} – coefficient of reduced stiffness of the suspension, N/m;

c_{pr1}, c_{pr2} – coefficient of reduced stiffness of the front and rear suspension , respectively , N/m ;

m_{z1}, m_{z2} – coefficient of change of the normal reaction on the front and rear axles of the vehicle , respectively ;

m_F – coefficient of filling the frontal area of the vehicle;

u_N – coefficient of engine power utilization;

β_{t} – coefficient of distribution of braking forces of the vehicle;

β_{opt} – optimal coefficient of distribution of braking forces of the vehicle;

β_{opt1}, β_{opt2} – the optimal coefficient of distribution of braking forces of the vehicle, respectively, the first and second values;

β_{const} – constant value of the braking force distribution coefficient;

γ_t – coefficient of specific braking force;

δ_{vr} – coefficient of consideration of rotating masses;

δ_{vr0} – coefficient of consideration of rotating masses during coasting motion;

φ_x – coefficient of longitudinal adhesion of the wheel to the supporting surface;

φ_y – coefficient of lateral adhesion of the wheel to the supporting surface;

$\varphi_{x\ aqua}$ – coefficient of longitudinal adhesion of the wheel to the support surface in the presence of a liquid film;

φ_{x1}, φ_{x2} – coefficient of longitudinal adhesion of the wheels, respectively, of the front and rear axles of the vehicle;

φ_{x0}, φ_{xn} – coefficient of longitudinal adhesion of vehicle wheels, respectively without load and with full load;

φ_{opt} – the optimal value of the adhesion coefficient ;

λ_{sh} – tire radial deformation coefficient;

η_{tr} – transmission efficiency coefficient;

η_n – coefficient of lateral stability of the vehicle;

η_c is the coincidence coefficient of the tracks of the front and rear wheels;

ψ – total road resistance coefficient;

Ψ_v – the total coefficient of road resistance at the maximum speed of the vehicle.

Transmission parameters

u_0 – gear ratio of the main gear;

u_k – transmission ratio of the gearbox;

u_{k1}, u_{k2}, u_{k3}, ... $u_{k\,i\,k}$ are transmission numbers of the gearbox (where 1, 2, 3, ... *and* $_k$ are gear numbers);

i_k – serial number of transmission in the gearbox;

$u_{k\,v}$ is the higher calculated transmission ratio of the gearbox, at which the given maximum speed of the vehicle is reached;

$u_{k\,\psi}$ – gear ratio of the first gear under the condition of ensuring the possibility of overcoming the maximum road resistance;

$u_{k\,v}$ – gear ratio of the first gear under the condition of ensuring the possibility of maneuvering at the specified minimum speed;

$u_{k\,\varphi}$ – gear ratio of the first gear, provided there is no skidding of the driving wheels when overcoming the maximum road resistance;

n_{k} – the number of gears (stages) in the box (with the exception of the accelerating gear and reverse gear.);

m_{v} – the serial number of the gear in the gearbox on which v_{max} is reached (this is the transmission number to which the transmission number $u_{k\,v}$ corresponds);

u_{zx} – gear ratio of the reverse gear ;

u_{p} – gear ratio of the additional gearbox ;

$u_{p\,v}$ – gear ratio in the additional gearbox, at which the maximum speed of the vehicle is reached;

u_{rn} – gear ratio of the lower gear of the additional gearbox;

u_{rv} – gear ratio of the higher gear of the additional gearbox;

u_{rkn} – gear ratio of the lower gear of the transfer box;

u_{rkv} – gear ratio of the highest gear of the transfer box ;

n_{bk} – the number of gears of the base box;

u_{bkv} – higher gear in the base box ;

u_{bkp} – gear ratio of the direct transmission of the base box ;

$u_{bk\,n}$, $u_{bk(n-1)}$ – transmission numbers of the last and penultimate transmission of the base box , respectively ;

i_{b} – serial number of the transmission in the base box;

q_{bk} – the denominator of the geometric series of the base box;

D_{k} – gearbox range;

D_{dr} – range of additional gearbox;

D_{rk} – transfer box range;

k_{m} – an empirical coefficient for calculating v aqua;

z_{k} – the number of wheel revolutions.

Oscillations of the vehicle

f_{st} – static deformation of the suspension, m;

f_{n} – static deformation of the elastic element of the suspension, m;

f_{sh} – static deformation of the tire, m.

$z, \dot{z}, \ddot{z}$ – deformation, speed and acceleration of the deformation of the elastic element of the suspension, m, m/s, m/s ^{2};

$\xi, \dot{\xi}, \ddot{\xi}$ – deformation, speed and acceleration of tire deformation, m, m/s, m/s ^{2};

η_{01}, η_{01} – adhesion coefficient of free oscillations, respectively, of the front and rear suspensions;

ε_y – coefficient of distribution of sprung masses;

ω_1, ω_2 – partial frequencies of free oscillations, s $^{-1}$;

ω_0 – the partial frequency of oscillations of the sprung mass m_n with a stationary unsprung mass m_n, s $^{-1}$;

ω_{k0} – partial frequency of oscillations of the unsprung mass m_n with a stationary unsprung mass m_n, s $^{-1}$;

ω_k is the partial frequency of oscillations of the unsprung mass m_n with a stationary unsprung mass $m_{n \text{ and}}$ at $c_ш = 0$, c $^{-1}$;

k – coefficient of inelastic resistance of the shock absorber, N/(m/s) = kg/s;

k_1, k_2 – coefficient of inelastic resistance of the shock absorbers, respectively, on the front and rear axles, N · m/s = kg/s;

h – the coefficient of inelastic resistance of the suspension;

h_0 – partial coefficient of damping of oscillations of the sprung mass m_n; c $^{-1}$;

h_k – partial damping coefficient of unsprung mass oscillations, s $^{-1}$;

ω_h – frequency of oscillations with decay, s $^{-1}$;

ω_{0h} – frequency of oscillations of the sprung mass with damping, s $^{-1}$;

ω_{kh} – frequency of oscillations of the unsprung mass with damping, s $^{-1}$;

ω_d – frequency of road bumps, s $^{-1}$;

q_0 – the amplitude of the road unevenness wave, m;

$l_д$ – wave length of road irregularities, m;

x – the abscissa of the point for which the ordinate q , m is determined;

z_a – amplitude of forced oscillations of the vehicle, m;

φ_n – the initial phase angle of oscillations of the sprung mass , rad;

φ_n – the initial phase angle of oscillations of the unsprung mass, rad;

φ_v – phase angle of forced oscillations, rad;

ψ_q is the relative damping coefficient of the suspension;
ψ_{q0} is the relative damping coefficient of the sprung mass;
$\psi_{q\kappa}$ is the relative damping coefficient of the unsprung mass.

Different

p_v – tire pressure, kPa;
r_k – average pressure in the wheel contact spot, MPa;
r_{kpr} – the average pressure of the tread protrusions in the wheel contact patch, MPa;
Δp_1, Δp_2 – pressure increase in the front and rear brake circuits, MPa;
Δ_k – wheel imbalance, kg·m;
q_v – high-speed air pressure, kg/(m·s^2);
ρ_v – air density, kg/m^3;
ρ_t – fuel density, kg/l;
ρ_b – gasoline density, kg/l;
ρ_d – fuel density for diesel, kg/l;
W_v – the vehicle's smoothness factor, N · s^2/m^2;
D_0, D_{100} – dynamic factor of the vehicle, respectively without load and with full load (load 0% and 100%);
$D_{\varphi_{100}}$, D_{φ_0} – the dynamic factor of the vehicle's clutch with full load and without load;
D_n – dynamic factor of the vehicle when switching gears;
m_{D0} – the scale of the dynamic factor of the vehicle without load;
m_{D100} – the scale of the dynamic factor of the vehicle with a full load;
Q_t – hourly fuel consumption, kg/h;
Q – the amount of spent fuel, kg;
g_e – specific engine fuel consumption, g/(kW · h);
$g_{e\,min}$ – minimum specific engine fuel consumption, g/(kW · h);
g_N – specific fuel consumption at maximum engine power, g/(kW·h);
q_s – road fuel consumption, kg/100km;
q_{s1} – road fuel consumption, *l*/100 km;
q_t – specific cost of cargo transportation, g/tkm;
q_n – specific cost of passenger transportation, g/km;
o – center of roll of the suspension;
p – wheel rocking pole;

O – the instantaneous center of rotation of the vehicle;

O_{Θ} – instantaneous center of rotation of a vehicle with rigid wheels;

O_{δ} is the instantaneous center of rotation of a vehicle with elastic wheels ;

n – passenger capacity, people;

n_{n} – the nominal number of passengers of the analogue bus;

n_{c} - the number of seats for passengers;

n_{r} – number of places for standing passage;

x , z – longitudinal and vertical axes.

TOPIC 1

GENERAL INFORMATION ABOUT THE THEORY OF THE VEHICLE

1.1. Vehicle theory as a scientific discipline

The theory of the vehicle is a science that studies the mechanics of its movement.

The subject of studying the theory of the vehicle is:

– phenomena that occur during the interaction of the vehicle with the supporting surface and air;

– the mechanics of its aggregates and nodes, which ensure a controlled change of the vehicle's speed vector;

- operational properties of the vehicle.

The operational properties of the vehicle characterize the possibility of its effective use in certain conditions, allowing to assess the extent to which its design meets the operational requirements.

Depending on the parameters characterizing the operational properties of the vehicle , they can be divided into two groups:

- properties of the vehicle related to its movement: traction and speed properties, braking properties, fuel efficiency, controllability, maneuverability, maneuverability, stability, passability, smoothness of movement, environmental friendliness, traffic safety;

– properties of the vehicle, not related to its movement: capacity, strength, durability, suitability for maintenance and repair, suitability for loading and unloading operations, suitability for boarding and disembarking passengers.

The second group of operational properties is characterized by the fact that the parameters of their evaluation can be determined on a stationary vehicle. The values of certain operating properties of the vehicle are determined by its type and class, purpose and operating conditions.

In the theory of the vehicle, its operational properties are considered, which determine the possibility, nature and regularities of the vehicle movement. Let us define these properties.

Traction-speed properties are the properties of the vehicle that characterize the ranges of changes in driving speeds and maximum acceleration accelerations in different road conditions. These properties are manifested in the traction mode of the vehicle, in which the power and torque necessary for movement are supplied from the engine to the drive wheels through the transmission.

Braking properties are the properties of a vehicle to perform braking with maximum efficiency and with the efficiency necessary to control traffic, to stay in a braked state in place, and to move at a uniform speed on long descents.

Traction-speed and braking properties determine the dynamics of the vehicle.

Fuel efficiency is the ability of a vehicle to rationally use fuel energy when carrying out transport work.

Vehicle controllability is a set of properties that determine the characteristics of the vehicle kinematic and power reactions to the driver's control influence, which form the trajectory of movement.

The maneuverability of the vehicle is its ability to change the kinematic parameters of the turn under the action of external lateral forces at a fixed value of the angle of rotation of the steered wheels.

Maneuverability is the ability of a vehicle to turn in a minimum area and fit into the road dimensions.

Stability of the vehicle is a set of its properties that ensure movement in the desired direction without skidding, sliding or overturning.

The stability of the vehicle together with its braking properties and controllability determine the safety of its movement.

Permeability is a property of the vehicle that determines the ability to move on bad roads, off-road and when overcoming various obstacles.

The smoothness of the ride is a property of the vehicle that provides the ability to drive for a long time on various roads without fatigue or burdensome feelings for the passengers and the driver, while ensuring high speeds of movement and preservation of cargo.

The theory of the vehicle studies the following problems :

1 – selection of engine power;
2 – selection of the type of transmission and its parameters;
3 – reducing the resistance of the vehicle movement;
4 – improvement of dynamism, controllability and stability;
5 – reduction of fuel consumption during vehicle operation;
6 – increase in smoothness of movement and passability;
7 – decrease in the weight of the vehicle;
8 – rational methods of driving a vehicle.

1.2. The main parameters of the vehicle

For the theory of the vehicle, the following parameters are considered basic.

1 – Parameters characterizing the vehicle as a whole:

1.1. Overall dimensions (Fig. 1.1) – width B_a, m; height H_a, m; length L_a, m.

1.2. Vehicle track (Fig. 1.1) - front wheels B_1, rear wheels B_2, m.

1.3. The coordinates of the center of gravity (Fig. 1.1): a, b, h_g, m.

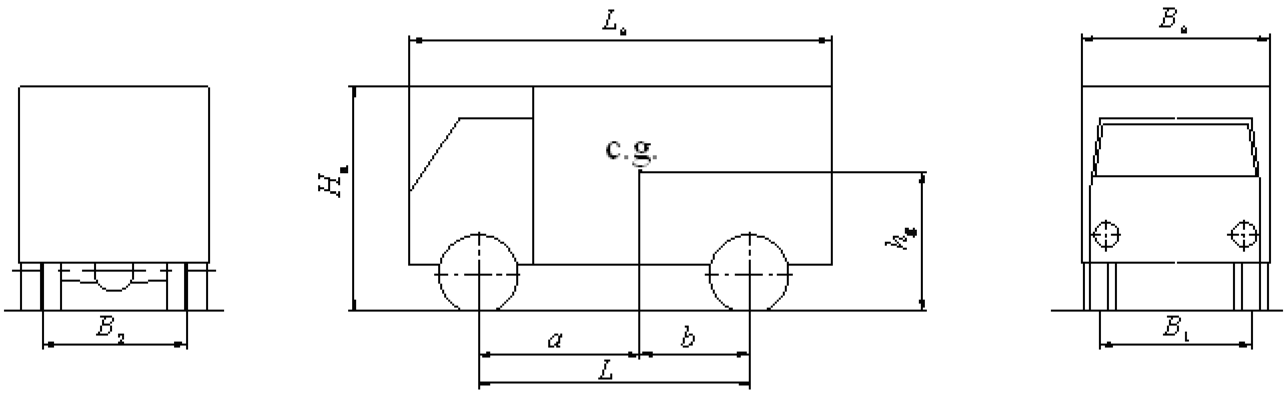
1.4. Base of the vehicle (Fig. 1.1) L, m.

1.5. Coefficient of air resistance k_v, N · s ^{2}/m ^{4}.

1.6. Total weight of the vehicle m_a, kg.

1.7. The mass that falls on the front axle m_{a1}, kg.

1.8. Mass that falls on the rear axle m_{a2}, kg.



a b c

Fig. 1.1. **The main geometric parameters of the vehicle:**
a – rear view; b – view from the left; c – front view;
c.g. - the center of gravity of the vehicle

2 – Parameters characterizing the engine:

2.1. Maximum power N_{max}, kW.

2.2. Maximum crankshaft rotation frequency n_{max}, rpm.

2.3. Maximum torque $M_{e\,max}$, N·m.

2.4. Crankshaft rotation frequency at maximum torque n_m, rpm.

2.5. Specific fuel consumption g_e, g · kW/h.

2.6. A comprehensive characteristic of the engine properties is the external speed characteristic of the engine (ESChE).

3 – Parameters characterizing the chassis of the vehicle:
3.1. Gear ratios of the gearbox (transfer box) u_{k1}, u_{k2}, u_{k3}, ... u_{ki}, $u_{рк}$.
3.2. Gear ratio of the main gear u_0.
3.3. Static wheel radius r_c, m.
3.4. Transmission efficiency coefficient η_{tr}.

1.3. Coordinates of the center of gravity of the vehicle

The coordinates of the center of gravity of the vehicle depend on its layout, as well as on the size, location and density of loads, which change significantly during the operation of the vehicle.

The coordinates of the center of gravity of the vehicle can be determined analytically or graphically based on the given values of the weight and the coordinates of the centers of gravity of its individual parts or by weighing the vehicle in horizontal and inclined positions and by further calculation.

When weighing a vehicle to find the coordinates a and b (corresponding to the distance between the projections on the road of the center of gravity and the front and rear axles), the reactions R_{z1} and R_{z2} are found with the help of scales installed under the wheels of the vehicle on a horizontal surface (Fig. 1.2). From the equilibrium equations, we find the sum of the moments relative to the axis of the front and rear wheels

$$\Sigma M_1 = 0;\ R_{z2} \cdot L - G_a \cdot a = 0 \rightarrow a = \frac{R_{z2}}{G_a} \cdot L; \tag{1.1}$$

$$\Sigma M_2 = 0;\ R_{z1} \cdot L - G_a \cdot b = 0 \rightarrow b = \frac{R_{z1}}{G_a} \cdot L. \tag{1.2}$$

To determine the height of the center of gravity of the vehicle, it is weighed in an inclined state, placing a stand with a height h_n under the wheels of the front axle (Fig. 1.3). At the same time, the suspensions are blocked with struts to prevent their deformation. The scheme of weighing the vehicle in the inclined state is presented in Figure 1.3.

From the condition of equilibrium with respect to the axis of the front wheels, equality can be written

$$G_a \cos\alpha + G_a \sin\alpha \cdot h_0 - R'_{z2} \cos\alpha \cdot L = 0, \tag{1.3}$$

where h_0 is the height of the center of gravity of the vehicle above the plane passing through the centers of the wheels;

R'_{z2} – the reaction to the rear wheels of the vehicle in an inclined position, which is determined on the scales;

α - the angle of inclination of the vehicle relative to the horizon is calculated.

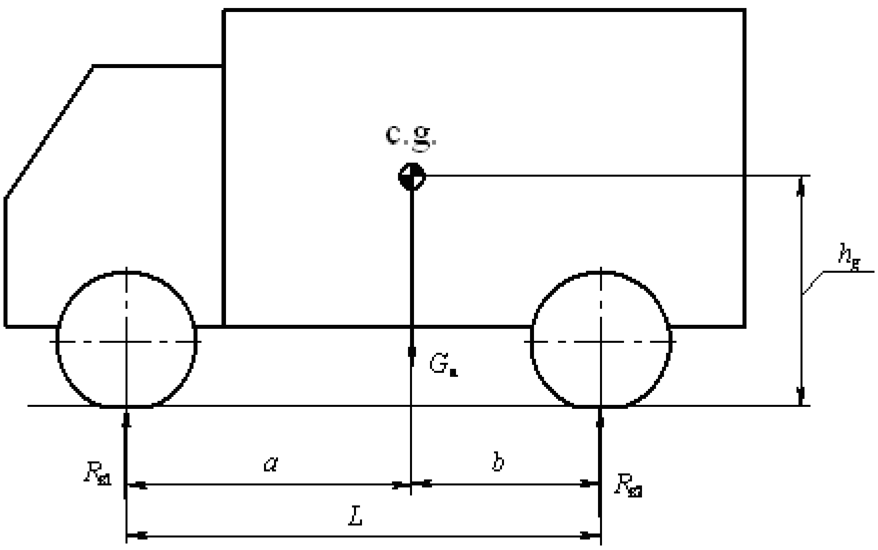


Fig. 1.2. **Scheme for determining the longitudinal coordinates of the center the gravity of the vehicle**

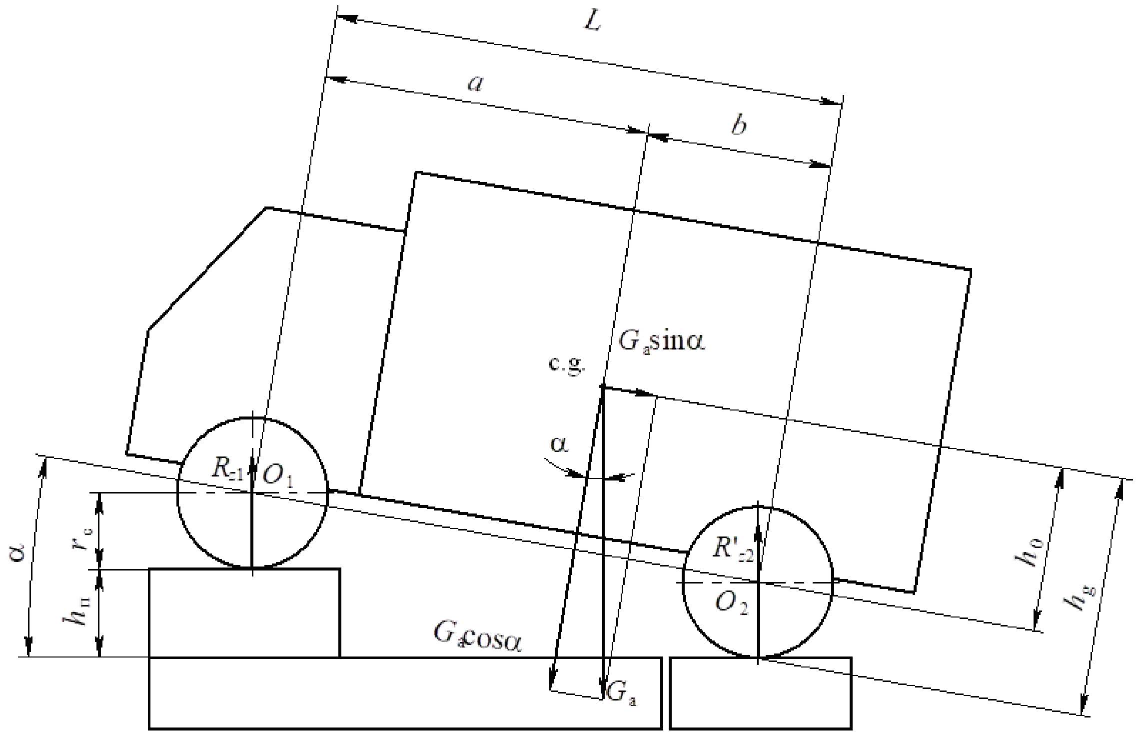


Fig. 1.3. **Scheme for determining the height of the center of gravity of the vehicle**

The height of the center of gravity of the vehicle above the plane passing through the centers of the wheels is determined by the relationship obtained from the balance equation

$$h_0 = \frac{R'_{z2} \cdot \cos \cdot L - G_a \cdot \cos\alpha \cdot a}{G_a \cdot \sin\alpha}. \quad (1.4)$$

In the numerator , we subtract $G_a \cdot \cos\alpha$ from the brackets, and taking into account that $\frac{G_a}{L} \cdot a = R_{z2}$ after transformations we get

$$h_0 = \frac{L}{G_a} \cdot \frac{R'_{z2} - R_{z2}}{\text{tg}\alpha}. \quad (1.5)$$

The height of the center of gravity of the vehicle above the supporting surface is greater by the value of the static radius of the wheel:

$$h_g = h_0 + r_c = \frac{L}{G_a} \cdot \frac{R'_{z2} - R_{z2}}{\text{tg}\alpha} + r_c. \quad (1.6)$$

1.4. Engine characteristics

1.4.1. Speed characteristics of the engine

The speed characteristic of the engine is a dependence of the main parameters:

N_e - effective power;
M_e – effective torque;
g_e – specific fuel consumption

from the engine crankshaft rotation frequency n_e, with constant fuel supply (or throttle position).

The speed characteristic can be external or partial.

External speed characteristic of the engine (ESChE) is called the dependence $N_e = f(n_e)$; $M_e = f(n_e)$; $g_e = f(n_e)$ at *full fuel supply* (which corresponds to a fully open throttle), i.e. at full load.

The partial speed characteristic of the engine is called dependence $N_e = f(n_e); M_e = f(n_e); g_e = f(n_e)$ at *partial engine loads* .

1.4.1.1. The graph of the external speed characteristic internal combustion engine without a speed limiter. The graph of the external speed characteristic usually shows the dependencies $N_e = f(n_e)$; $M_e = f(n_e)$; $g_e = f(n_e)$ (Fig. 1.4). Characteristic points are also indicated: $N_{e\max}$, $M_{e\max}$, M_N, $g_{e\min}$, $n_{e\min}$, n_m, n_N and $n_{e\max}$.

Engines with this characteristic are installed on passenger vehicles.

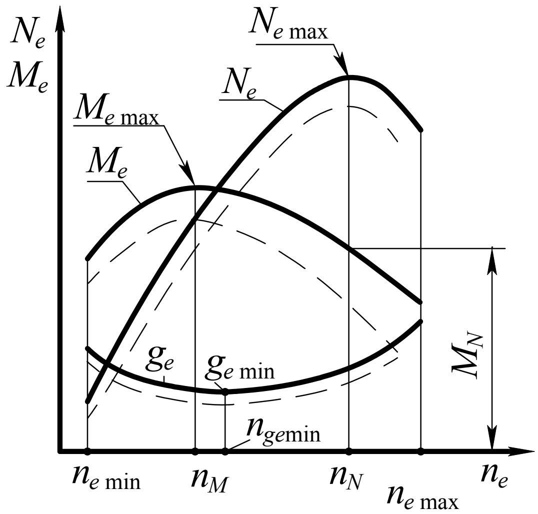


Fig. 1.4. **External speed characteristics of an internal combustion engine (ICE) without a speed limiter** :
$n_{e\min}$ – the minimum crankshaft rotation frequency at which the engine works stably under full load $N_{e\max}$, $M_{e\max}$ – maximum power and torque, respectively; M_N – moment at maximum power; n_m, n_N – crankshaft rotation frequency, respectively, at the maximum moment $M_{e\max}$ and the maximum power $N_{e\max}$; $g_{e\min}$ – minimum specific fuel consumption; n_{ek} – rotation frequency at minimum fuel consumption

Usually, internal combustion engines with a speed limiter have the following shaft speed values:

$n_{e\min}$ = 800 ... 1000 rpm; $n_{e\max}$ = 5000 ... 7000 rpm.

The dashed line on the graph shows the partial characteristics of the engine $N_e^{'}$, dependencies will correspond to it $M_e^{'} = f(n_e)$; $g_e^{'} = f(n_e)$.

1.4.1.2. The graph of the external speed characteristics of the engine with a speed limiter. On trucks, engines with spark ignition usually have a maximum speed limiter (Fig. 1.5). Since the characteristic $N_e = f(n_e)$ at the rotation frequency n_N^* has a gentle extremum, so a decrease in the rotation frequency by 10...20% does not lead to a significant decrease in power. But at the same time, engine wear is significantly reduced. When the rotation frequency n_0 is reached, the limiter of the maximum rotation frequency of the shaft is turned on. Therefore, the maximum frequency of rotation of the motor shaft $n_{e\,max}$ is equal to the frequency n_0. At the maximum rotation frequency $n_{e\,max}$, the maximum power N_{max} of the engine and the maximum speed of the vehicle are achieved , therefore the correct entry $n_{e\,max} = n_0 = n_N = n_v$. _

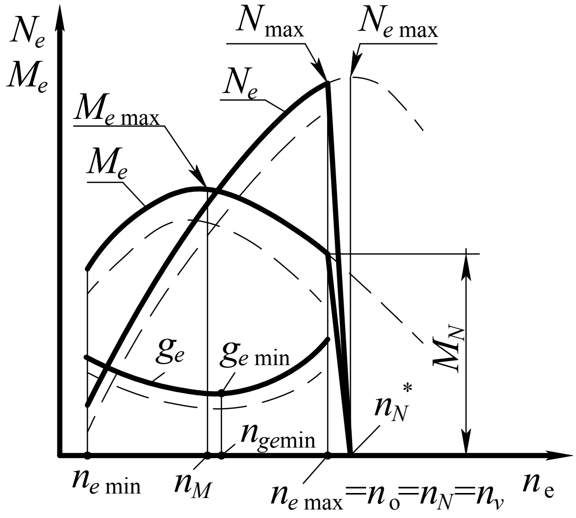


Fig. 1.5. **External speed characteristics of the engine internal combustion with a speed limiter:** $N_{e\,max}$ – the maximum effective power of the engine; N_{max} – maximum engine power; $n_{e\,max}$ – crankshaft rotation frequency at the maximum speed of the vehicle; n_N^* is the theoretical frequency of rotation of the engine shaft at which the maximum effective power $N_{e\,max}$ would be achieved in the absence of a maximum speed limiter

Usually, internal combustion engines with a speed limiter have the following shaft speed values: $n_{e\,max} = n_0 = n_N = n_v = (0{,}8\ \ldots\ 0{,}9)\ n_N^*$; $n_{e\,min} = 600...800$ rpm; $n_{e\,max} = 3000...3200$ rpm.

1.4.1.3. The graph of the external speed characteristics of a diesel engine. Due to the peculiarities of the working process, the dependences $N_e = f(n_e)$ and $M_e = f(n_e)$ have less nonlinearity, a smaller speed range, and a shaft speed regulator with a maximum frequency limiter is mandatory in a diesel engine (Fig. 1.6) .

Usually, diesel engines have the following shaft speed values:

$n_{e\max} = n_0 = n_N = n_v = (1{,}0...0{,}91)\, n_N^*$;

$n_{e\,min} = 900$ rpm;

$n_{e\,max} = 2100\ ...\ 2800$ rpm for truck diesels;

$n_{e\,max} = 3000\ ...\ 4000$ rpm in passenger vehicle diesels.

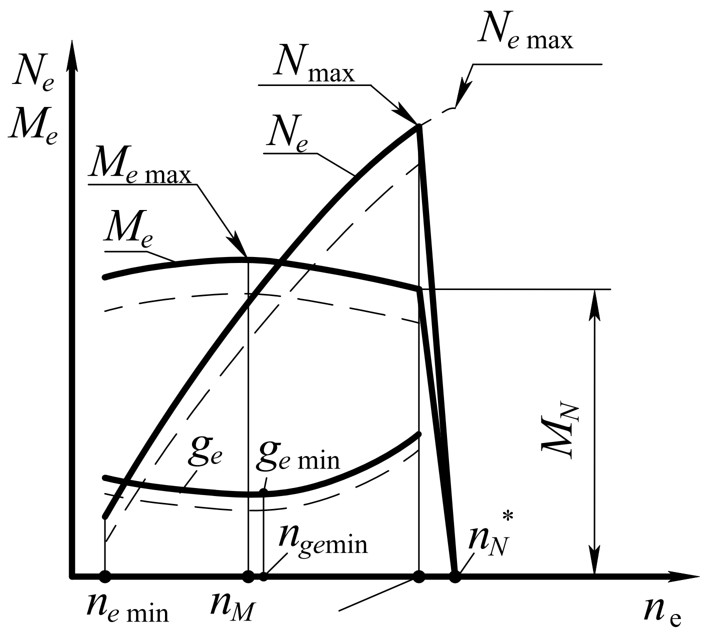


Fig. 1.6. **External speed characteristics of a diesel engine**

1.4.2. Adaptability factor of the engine

Engine loads change during operation depending on vehicle driving conditions. The engine's ability to overcome short-term load changes depends on its type and the organization of the work process. This ability of the engine is characterized by the adaptability coefficients for the moment k_M and the frequency k_ω . These coefficients are defined as the ratio:

$$k_M = \frac{M_{e\max}}{M_N}; \tag{1.7}$$

$$k_\omega = \frac{n_N}{n_M}. \tag{1.8}$$

Figure 1.7 shows the characteristics of two engines with the same value of moments M_N and rotation frequencies n_M and n_N . At the same time, the adaptation coefficients $k_{M1} > k_{M2}$ and $k_{\omega 1} = k_{\omega 2}$. Let us consider the case of the movement of vehicle with such engines at the frequency of rotation of their shafts n_N . In the case of an increase in the movement resistance by the same value ΔM , the engine speed, which has a higher adaptation factor k_{M1} , will decrease by a smaller value $\Delta n_1 < \Delta n_2$. This helps not to switch gears to maintain the speed of the vehicle.

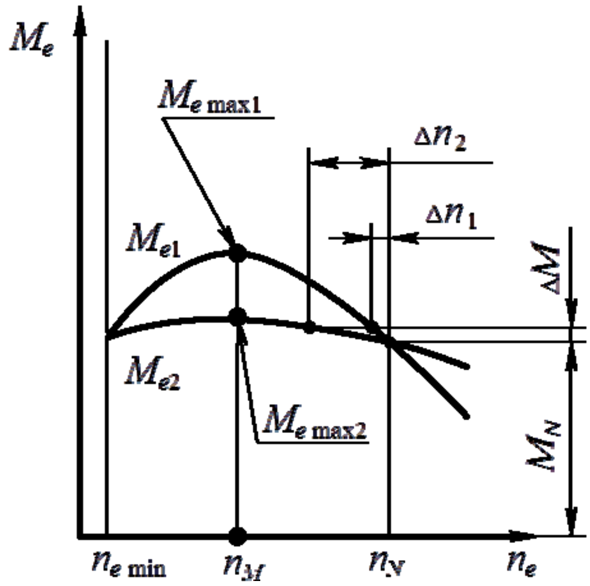


Fig. 1.7. **Characteristics of engines with different k_M and the same k_ω**

The greater k_M and k_ω,, the better the self-adjustment of the engine to changes in the external load. In this case, the engine is able to ensure the movement of the vehicle without switching gears in a wide range of speeds. This ability of the engine is called *elasticity*. Usually, the values of engine adaptability coefficients are in the intervals:

$$k_M = \begin{cases} 1{,}2 - 1{,}4 \text{ for ICE;} \\ 1{,}05 - 1{,}25 \text{ for Diesel.} \end{cases}$$

$$k_\omega = \begin{cases} 1{,}5 - 2{,}5 \text{ for ICE;} \\ 1{,}4 - 2 \text{ for Diesel.} \end{cases}$$

Diesels have a lower ability to adapt to changes in load, so on vehicle with diesel engines, gearboxes must have a greater number of gears.

1.4.3. Analysis of ESChE of different types of engines

Different types of engines are installed on the vehicle. A specific type of engine corresponds to varying degrees to the nature of the load - the start of motion of a stationary vehicle, its acceleration and movement at a constant speed.

The ideal engine characteristic is characteristic 1 (Fig. 1.8a), which provides the possibility of engine operation with maximum power in the entire speed range from zero to maximum rotation frequency. For comparison, let's take the characteristics of three types of engines: internal combustion, gas turbine and electric engine (Fig. 1.8).

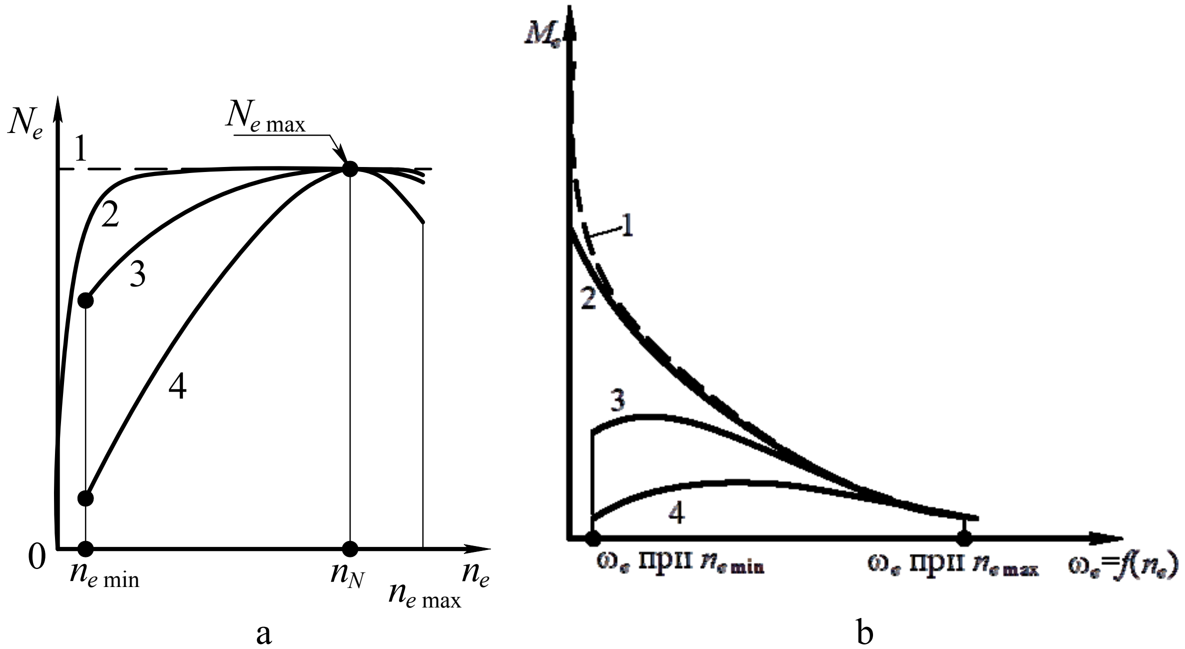


Fig. 1.8. **Characteristics of engines of different types:**
a – power dependence on shaft rotation frequency; b – dependence of the moment on the frequency of rotation of the shaft; 1 – ideal characteristic; 2 – electric motor; 3 – gas turbine; 4 - internal combustion engine

Let's choose engines with the same power $N_{e\,max}$ at the same shaft rotation frequency n_N. The characteristics of the engine depend on its working process. It is obvious that the electric motor is able to develop power close to the maximum power in a wider speed range than the gas turbine and internal combustion engine (Fig. 1.8 a).

It should be noted that the electric motor is able to develop a torque when the shaft is stationary, which means that it provides the possibility of starting the movement of a stationary vehicle without additional devices in the transmission (Fig. 1.8 b). The gas turbine and internal combustion engine have a minimally stable shaft rotation frequency, so the transmissions of vehicle with such engines have a clutch. This is necessary to ensure a smooth connection between the rotating engine shaft and the non-rotating transmission shaft. In addition, the characteristic of the internal combustion engine is such that at a low shaft rotation frequency, the engine is able to develop a torque much smaller in magnitude than with an ideal characteristic. In order to increase the torque and get closer to the ideal characteristics, a gearbox is installed in the transmission in vehicle with a diesel engine.

1.5. Power transmission to the drive wheels. Transmission efficiency

Effective power N_e developed by the engine is transmitted to the driving wheels of the vehicle, part of it is lost in the transmission. Power losses are also estimated by the efficiency factor (efficiency) η_{tr}

$$\eta_{тр} = \frac{N_к}{N_e} = \frac{N_e - N_{тр}}{N_e} = 1 - \frac{N_{тр}}{N_e}, \tag{1.9}$$

where N_k is the power supplied to the drive wheels;

N_e – effective engine power;

N_{tr} is the power lost in the transmission (determined experimentally).

The power lost in the transmission depends on:

- directly proportional to the power transmitted through the transmission (losses due to friction between the teeth of the gears, friction in the bearings of the transmission mechanisms and drive wheels);

– directly proportional to the speed of movement (friction in oil seals, friction related to bearing preload);

– proportional to the square of the engine crankshaft rotation frequency (and, therefore, with a constant transmission in the gearbox);

– proportional to the square of the vehicle speed (hydraulic losses in the gearbox associated with the rotation of its gears in the oil bath).

$$\eta_{тр} = \begin{cases} 0{,}88 - 0{,}92 & \text{– for car;} \\ 0{,}8 - 0{,}9 & \text{– for truck and bus;} \\ 0{,}78 - 0{,}85 & \text{– for off road vehicle.} \end{cases}$$

It is obvious that the efficiency of the transmission depends on the number of toothed connections involved in torque transmission. The more gear connections, the lower the efficiency.

Control questions

1. List the main operational properties and parameters of the vehicle.
2. How are the coordinates of the vehicle center of gravity determined and what do they depend on?
3. Define the external speed characteristic of the engine.
4. What is the difference between the external speed characteristics of the engine without the limiter of the maximum rotation frequency of the crankshaft and with it?
5. What is meant by the coefficient of adaptability?
6. Which engine has better adaptability to changes in external load?
7. What factors affect the efficiency?

TOPIC 2

THE MAIN PARAMETERS AND DYNAMICS OF A VEHICLE WHEEL

2.1. The main parameters of an elastic wheel

Wheel parameters belong to the main parameters of the vehicle.

The main parameters of the wheel from the point of view of the vehicle theory:

– wheel radii;

– tire stiffness;

– permissible load on the wheel;

- permissible speed of the vehicle.

2.1.1. Wheel radii

The elastic (vehicle) wheel is characterized by the following dimensions:

1 – nominal (free) radius r_0;

2 – static radius r_c;

3 – dynamic radius r_d;

4 – rolling radius r_k.

1. Nominal (free) radius r_0 is the radius of the circumference of the running track in the central plane of the unloaded wheel.

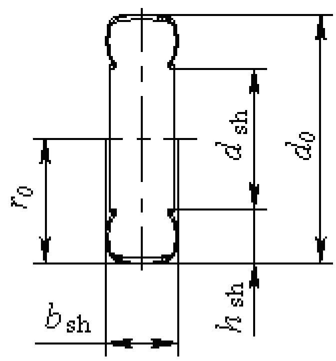


Fig. 2.1. **Dimensions of the wheel in a free (unloaded) state:**
r_0 – free radius; d_0 – diameter of the central running track;
d_{sh} – tire mounting diameter; b_{sh} – tire profile width;
h_{sh} – the height of the tire profile

The free wheel radius can be determined:

- *experimentally* :

Determining the length of the circumference of the treadmill in the central plane and calculating according to the equation:

$$r_0 = \frac{l_0}{2\pi}, \tag{2.1}$$

where l_0 is the circumference length of the tire running track;

– *theoretically* :

Using the parameters of the bus designation, calculating according to the equation

$$r_0 = \frac{d_0}{2} = \frac{d_{sh}}{2} + h_{sh}; \tag{2.2}$$

$$r_0 = \frac{d_0}{2} = \frac{d_{sh}}{2} + b_{sh} \cdot k_h, \tag{2.3}$$

where $k_h = h_{sh}/b_{sh}$ – tire profile coefficient (the ratio of the height hsh of the tire profile to the width of the profile b_{sh}) .

An example . Tire 165/70 *R* 13: 165 – b_w ; 70 – k_h /100% → k_h = 0.7; *R* – radial tire; 13 – d_{dh} (in inches; 1 inch = 25.4 mm);

$$r_0 = \frac{13 \cdot 25,4}{2} + 165 \cdot 0,7 = 280,6 \text{ (mm)}.$$

2. Static radius r_c is the distance from the axis of the stationary wheel, loaded with the load P_z, to the support surface.

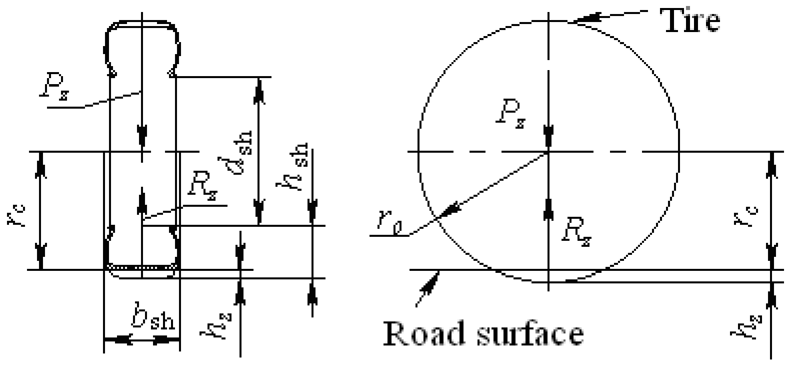


Fig. 2.2. **Scheme for determining the static radius of the wheel**

$$r_c = 0,5 \cdot d_{sh} + k_h \cdot b_{sh} \cdot (1 - \lambda_{sh}), \tag{2.4}$$

where λ_{sh} is the coefficient of radial deformation of the tire:

– for vehicle tires $\lambda_{sh} = 0.14 – 0.16$;

– for truck tires $\lambda_{sh} = 0.1 – 0.13$.

The coefficient of radial deformation of the tire is the ratio of the normal deformation of the tire to the height of the tire profile

$$\lambda_{sh} = \frac{h_z}{h_{sh}}, \tag{2.5}$$

where h_z is the normal deformation of the tire (deflection of the tire) in the vertical plane under the action of P_z.

The normal deformation of the tire is expressed through the parameters of the tire and the coefficient of normal deformation has a value

$$h_z = b_{sh} \cdot k_h \cdot \lambda_{sh}. \tag{2.6}$$

3. *Dynamic radius* r_d is the distance from the wheel axis to the support surface, while the wheel is loaded with a normal load and a torque

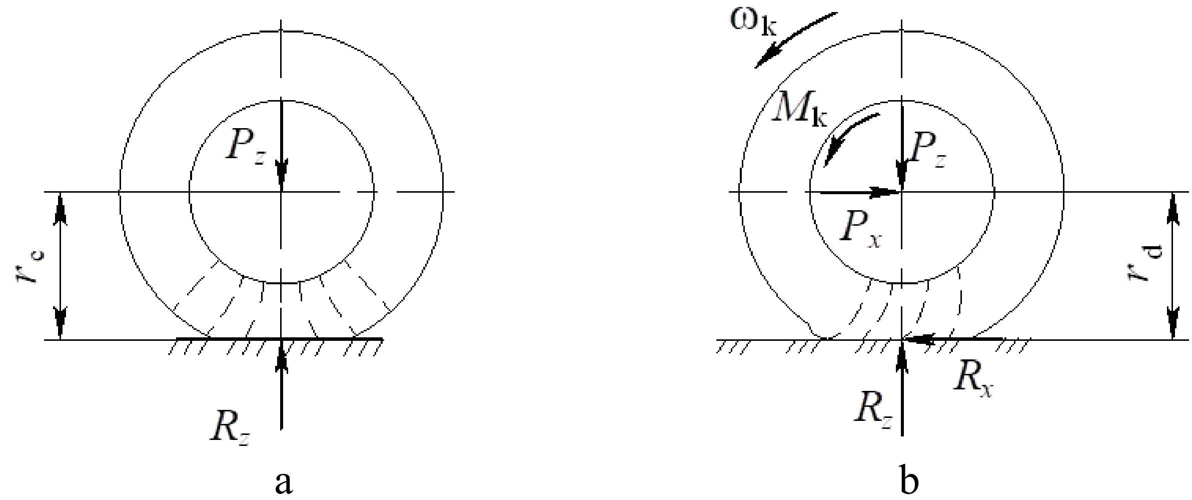


Fig. 2.3. **Schemes of load and deformation of the wheel:**
a – normal load; b – normal load and torque

with an increase in $M_k \rightarrow r_d \downarrow$;

when increasing $\omega_k \rightarrow r_d \uparrow$.

4. Rolling radius r $_k$ is the kinematic characteristic of the wheel, which is defined as the ratio of the longitudinal component of the wheel speed to its angular speed

$$r_k = v_k / \omega_k, \quad (2.7)$$

where v_k is the longitudinal speed of the wheel;

ω_k is the angular speed of the wheel.

That is, the rolling radius of the wheel characterizes the relationship between longitudinal and angular speed and the process of wheel rolling. This characteristic can be defined as the radius of a conventional wheel that rolls on a surface without slipping. The diagram explaining the determination of the rolling radius of the wheel is shown in Figure 2.4.

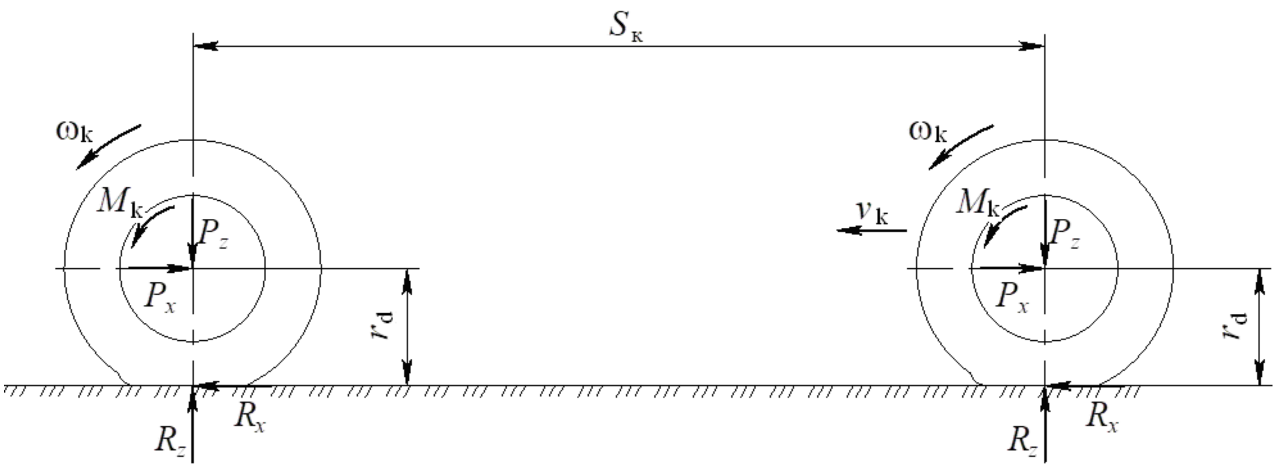


Fig. 2.4. **The scheme for determining the rolling radius elastic wheel**

The path covered by the wheel in the absence of sliding :

$$S_k = 2\pi \cdot r_k \cdot z_k, \quad (2.8)$$

where z_k is the number of wheel revolutions.

Accordingly, in the absence of sliding, the radius of the conditional wheel that has rolled over a distance S is equal to

$$r_k = \frac{S_k}{2\pi \cdot z_k}. \quad (2.9)$$

The rolling radius of the wheel depends on the amount of slip in the contact patch. The rolling radius characterizes the rolling process of the wheel and can vary in the range $0 < r_к < \infty$.

Example:

– the wheel skids (the vehicle is stationary) at the same time $S_к = 0 \rightarrow r_к = 0$;

– the wheel slips (the wheel does not turn, the vehicle moves when braking) at the same time $S_к \neq 0,\ z_к = 0 \rightarrow r_к = \infty$.

2.1.2. Deformation of an elastic vehicle wheel

Under the influence of an external load, the tire undergoes complex deformation. For ease of study, this deformation is divided into four simple ones: normal, lateral, torsional, rotary.

1. Normal (radial) tire deformation is the deformation of the tire under the action of the normal load P_z (Fig. 2.5). At the same time, the air pressure in the tire almost does not change, since the volume of deformation is very small compared to the volume of air in the tire. Therefore, the increase in pressure in the tire as a result of adding a normal load to it is small and amounts to 1–2%.

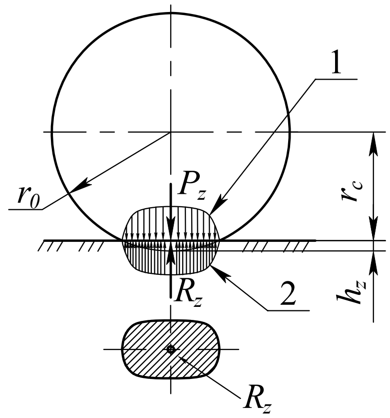


Fig. 2.5. **Wheel load scheme with normal load:** 1, 2 – graphs, respectively, of the pressure of the tire on the road surface and normal reactions; R_z is the normal reaction of the road surface (equivalent of all elementary normal reactions of 2 supporting surfaces)

Characteristics of normal deformation:

– normal deflection of the tire h_z is a linear displacement of the center of the wheel relative to the road surface under the action of a normal load;

- normal tire stiffness.

Coefficient of normal stiffness of the tire

$$c_{\text{shz}} = \frac{P_z}{h_z}. \tag{2.10}$$

The normal deformation of the tire is not the same during loading and unloading (Fig. 2.6). Due to the presence of internal friction in the tire, there is hysteresis in the change of deformation (hysteresis in translation from the Greek language - lag).

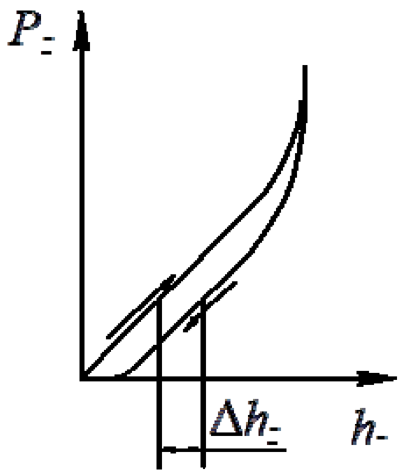


Fig. 2.6. **Dependence of normal strain on load:** Δh_z – bus hysteresis

Normal tire stiffness affects:
– the smoothness of the vehicle;
- damping capacity;
– load in parts of the chassis.
Normal tire stiffness depends on:
– tire designs
– air pressure in the tire;
– tire temperature;
- load.

2. The lateral deformation of the tire is the deformation of the tire under the action of the normal load P_z and the lateral load P_y (Fig. 2.7).

Characteristics of lateral deformation:

– side deflection of the tire h_y is a linear displacement of the center of the wheel relative to the central plane of the wheel under the action of lateral and normal load;

– lateral stiffness of the tire.

Coefficient of lateral stiffness of the tire

$$c_{\text{sh}y} = \frac{P_y}{h_y}. \tag{2.11}$$

The lateral stiffness of the tire affects: its operating conditions, stability and controllability of the vehicle.

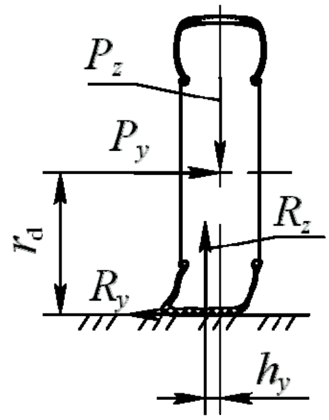


Fig. 2.7. **Diagram of lateral and normal wheel loading load:**
P_y – lateral load; R_y is the lateral reaction of the support surface (equivalent of all elementary lateral reactions of the support surface)

The lateral stiffness of the tire depends on: tire design, air pressure in the tire, adhesion utilized coefficient, tire temperature, load capacity.

<u>*3. The torsional deformation of the tire*</u> is the deformation of the tire under the action of the torque M_k (Fig. 2.8) .

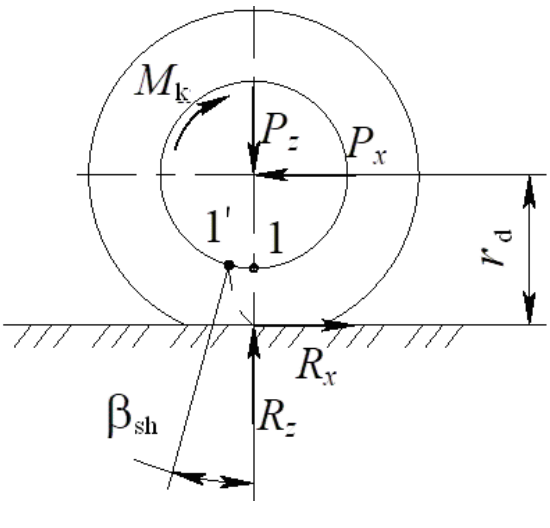


Fig. 2.8. **Wheel torque load diagram**

Torsional deformation characteristics:

- the angle of rotation of the tire β_w is the angular displacement of the point of the wheel rim around the axis of rotation of the wheel relative to the point of the tire stationary in contact as a result of an increase in the torque measured in the plane of rotation of the wheel.

– torsional stiffness of the tire.

Coefficient of torsional rigidity of the tire

$$c_{sh\beta} = \frac{M_k}{\beta_{sh}}. \qquad (2.12)$$

The torsional rigidity of the tire affects the kinematics of the movement of the vehicle wheel.

Torsional stiffness of a tire depends on: tire design, tire air pressure, adhesion utilized coefficient, tire temperature, torque, longitudinal force, load capacity.

4. *The turning deformation of the tire* is the deformation of the tire under the action of the turning moment M_n (Fig. 2.9). The turning moment of the wheel on a stationary vehicle occurs from the side of the steering drive. The rotation of the wheel is resisted by the resulting frictional forces in the contact zone of the tire with the supporting surface. When the tire tread is stationary relative to the support surface, the wheel turns due to its elasticity.

Characteristics of rotational deformation:

– the angular elastic displacement of the wheel Θ_k is the angular displacement of the point of the wheel rim relative to the point of the tire fixed in contact around the normal passing through the center of the wheel, under the action of an increase in the turning moment M_n;

– angular stiffness of the tire.

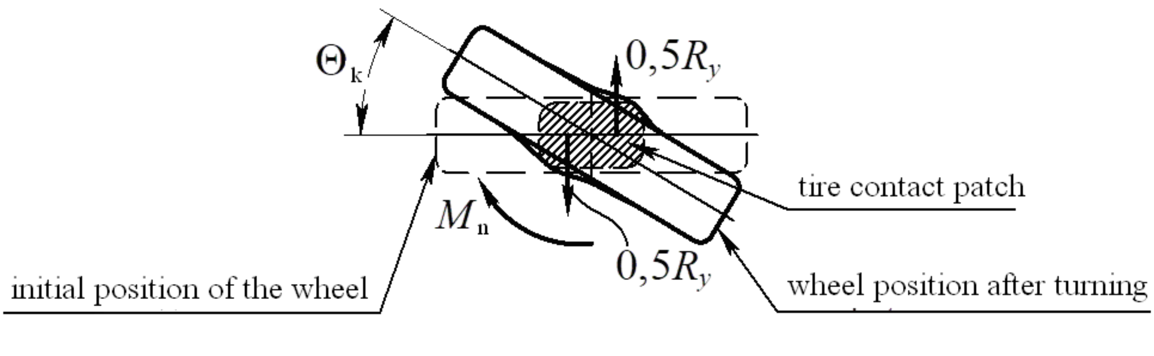


Fig. 2.9. **Scheme of wheel load with turning moment**

Coefficient of angular stiffness of the tire

$$c_{\text{sh}\Theta} = \frac{M_{\text{n}}}{\Theta_{\text{k}}}. \qquad (2.13)$$

The rotational stiffness of the tire affects the controllability and stability of the vehicle.

The turning stiffness of the tire depends on: tire design, air pressure in the tire, adhesion utilized coefficient, tire temperature, turning moment, load capacity.

2.1.3. Permissible wheel loads and vehicle speed

The design of the tire determines the permissible speed of the vehicle and the load on the wheel. The letter indices of the vehicle's permissible speed and digital indices of the carrying capacity are indicated on the tire. These parameters are regulated by the UNECE R30 international rules. Fragments of the regulation are presented in tables 2.1 and 2.2.

Table 2.1 - **Speed indexes of automobile tires**

Speed index	*L*	*M*	*N*	*P*	*Q*	*R*	*S*	*T*	*U*	*H*
Maximum permissible speed v_{a}, km/h	120	130	140	150	160	170	180	190	200	210

Table 2.2 – **Vehicle tire load capacity indices**

Index	70	71	72	73	74	75	76	77	78	79
Load G_{k}, N	3350	3450	3550	3650	3750	3870	4000	4120	4250	4370
Index	80	81	82	83	84	85	86	87	88	89
Load G_{k}, N	4500	4620	4750	4870	5000	5150	5300	5450	5600	5800
Index	90	91	92	93	94	95	96	97	98	99
Load G_{k}, N	6000	6150	6300	6500	6700	6900	7100	7300	7500	7750
Index	100	101	102	103	104	105	106	107	108	109
Load G_{k}, N	8000	8250	8500	8750	9000	9250	9500	9750	10000	10500

Examples:

– tire designation 175/80R16 Q88 – tires allow the vehicle to move at a speed of up to 160 km/h (Q index) and a wheel load of up to 5600 N (index 88);

- tire designation 175 / 80R16 N104 / 102 - tires allow the vehicle to move at a speed of up to 140 km / h (index N) and the load on a single-spoke wheel up to 9000 N (index 104) and on a double-spoke wheel 8500 N. (a tire in a two-spoke wheel has a lower load capacity for a number of reasons, including friction between the slopes).

2.2. Rolling of the wheel on a surface that does not deform

2.2.1. Rolling of an elastic wheel on a surface that does not deform under loads acting in the plane of its rotation

When the vehicle is moving, its wheels can roll in different modes, which are determined by the reasons that caused the rolling.

2.2.1.1. Wheel rolling under the action of a pushing force. If the vehicle is standing on a horizontal surface, then its wheels are stationary. At the same time, a normal load P_z is applied to the wheel, which consists of the part of the vehicle weight that falls on the wheel, plus the weight of the wheel. Since the wheel is elastic, in contact with the rough road surface, its tire receives a normal deflection h_z. In the spot of contact of the tire with the supporting surface, the normal reaction of the supporting surface R_z occurs - the equivalent of all elementary normal reactions of the supporting surface. If a longitudinal load P_x is applied to it from the side of the wheel axis , then due to elementary frictional forces in the contact patch, a total longitudinal reaction R_x is formed (Fig. 2.10).

Longitudinal force P_x and reaction R_x are a pair of forces that on the arm r_d form a torque that causes the wheel to roll. At the same time, the wheel makes a gradual movement with a gradual speed v_k and a rotational movement with an angular speed ω_k.

When the wheel rolls, the tire is constantly deformed in contact with the bearing surface. In the front part of the contact, the tire is compressed, and in the rear it is straightened. The compression of the tire and its straightening are opposed by the forces of internal friction. Since the friction force is always directed against the movement, in the front part of the impression its direction coincides with the load, and in the rear part it is directed against the decreasing load. As a result, in the front part of the tire footprint, the elementary normal reactions are larger in magnitude than in the rear part. As a result of the change in the plot of normal reactions during rolling of the wheel, the total normal reaction R_z acquires wear and tear .

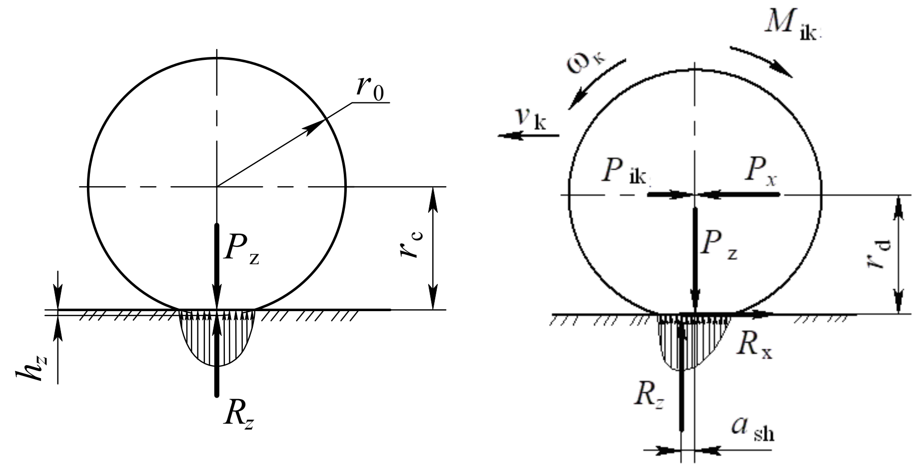


Fig. 2.10. **Scheme of forces and moments acting on a stationary wheel (a), and in managed mode (b):** P_x – longitudinal wheel load; R_x – longitudinal reaction of the road; R_{yk} – wheel inertia force; M_{ik} – moment of inertia of the wheel; a_w – wear of normal road response; r_d – dynamic radius of the wheel; ω_k – angular speed of the wheel; v_k – the progressive speed of the wheel

If the wheel accelerates, then the inertial force of the gradual movement R_{ik} and the moment of inertia of the rotational movement M_{ik} arise . The force of inertia of the wheel is determined by the dependence

$$R_{yk} = m_k j_k , \tag{2.14}$$

where m_k is the mass of the wheel;

j_k – acceleration of the wheel.

The moment of inertia of the wheel depends on its angular acceleration and moment of inertia

$$M_{ik} = J_k \cdot \frac{d\omega_k}{dt}, \tag{2.15}$$

where J_k is the moment of inertia of the wheel;

$\varepsilon = d\,\omega_k/dt$ is the angular acceleration of the wheel.

The force P_x applied to the wheel axis from the side of the vehicle, the force P_z acts from the side of the wheel to the road in the contact zone (in the figure, it has been conventionally moved to the center of the wheel). The forces R_z and R_x are reactions from the side of the road.

Equation of wheel motion for unsteady motion

$$P_x - R_x - P_{ik} = 0. \tag{2.16}$$

For steady motion, $R_{ik} = 0$ and the equation will take the form

$$P_x - R_x = 0 \text{ or } P_x = R_x, \tag{2.17}$$

that is, the longitudinal reaction of the road R_x is equal to the longitudinal force P_x and is applied to the wheel axis and is directed in the direction opposite to the movement. Therefore, the reaction R_x represents the rolling resistance force

$$P_f = P_x = R_x. \tag{2.18}$$

In order to determine the force of resistance to rolling P_f, we will formulate the equation of the moments acting on the wheel relative to its axis:

$$R_x \cdot r_d - R_z \cdot a_{sh} - M_{ik} = 0. \tag{2.19}$$

For a steady mode of motion $M_{ik} = 0$ and the equation will take the form

$$R_x \cdot r_d - R_z \cdot a_{sh} = 0, \tag{2.20}$$

from here

$$R_x = R_z \cdot a_{sh} / r_d. \tag{2.21}$$

introduce the concept *of the rolling resistance coefficient* - the ratio of the longitudinal reaction to the normal reaction on the wheel

$$f = \frac{R_x}{R_z} = \frac{R_z \cdot a_{sh} / r_d}{R_z} = \frac{a_{sh}}{r_d}, \tag{2.22}$$

taking into account the introduced concept, the force of resistance to rolling P_f is determined by the equation

$$P_f = R_x = R_z \cdot a_{\text{sh}} / r_{\text{d}} = R_z \cdot f. \quad (2.23)$$

Accordingly, the moment of rolling resistance is equal to

$$M_f = P_f r_{\text{d}}. \quad (2.24)$$

For unsteady motion, the longitudinal reaction is determined by the equation

$$R_x = R_z \frac{a_{\text{sh}}}{r_{\text{d}}} + \frac{M_{\text{ik}}}{r_{\text{d}}} = R_z \cdot f + \frac{J_{\text{k}}}{r_{\text{d}}} \cdot \frac{d\omega_{\text{k}}}{dt}. \quad (2.25)$$

2.2.1.2. Wheel rolling under the influence of torque (driving mode). When the torque M_{k} is applied to the driving axis, the wheel tends to rotate (Fig. 2.11) . As a result of the frictional forces in the contact zone of the wheel with the support surface, a longitudinal reaction R_x occurs , and as a result of the resistance of the vehicle movement, a longitudinal load P_x occurs on the wheel . This force is the reaction of the axle of the vehicle on the wheel.

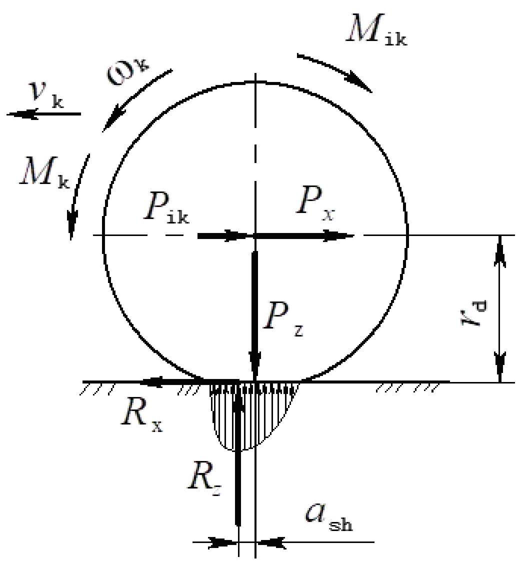


Fig. 2.11. **Scheme of forces and moments acting on the wheel in the driving mode:** M_k is the torque (driving) moment applied from the driving axis to the wheel; P_x – longitudinal wheel load; other designations - see Fig. 2.10

A pair of forces R_x and P_x on the shoulder r_{d} form a reactive moment that is directed against the torque M_{k} . If the torque M_{k} is more reactive, then the wheel acquires translational motion with a speed v_{k} and rotational motion with an angular speed ω_{k} .

The equation of motion of the wheel has the form

$$R_x - P_x - P_{ik} = 0. \tag{2.26}$$

For steady motion, $R_{ik} = 0$ and the equation will take the form

$$P_x - R_x = 0 \text{ or } P_x = R_x, \tag{2.27}$$

that is, the longitudinal reaction of the road R_x is equal to the longitudinal load on the side of the vehicle axis P_x and is directed in the direction opposite to the movement.

The equation of the moments acting on the wheel relative to its axis

$$M_k - R_z \cdot a_{sh} - R_x \cdot r_d - M_{ik} = 0. \tag{2.28}$$

Longitudinal response of the road

$$R_x = \frac{M_k}{r_d} - R_z \frac{a_{sh}}{r_d} - \frac{J_k}{r_d} \cdot \frac{d\omega_k}{dt}. \tag{2.29}$$

For a stable mode of motion, a reaction operates in the contact patch

$$R_x = \frac{M_k}{r_d} - R_z \frac{a_{sh}}{r_d} = \frac{M_k}{r_d} - R_z \cdot f = \frac{M_k}{r_d} - P_f, \tag{2.30}$$

introduce the concept of *total traction force* - it is the ratio of torque to dynamic radius

$$P_k = \frac{M_k}{r_d}. \tag{2.31}$$

This conditional force is equal to the real longitudinal force *that would occur when the wheel rolls at a constant speed* v_{const} *and in the absence of rolling losses.*

Taking into account the adopted provisions, the longitudinal reaction of the road and its equal pushing force is determined by the equation

$$R_x = P_x = P_k - P_f. \tag{2.32}$$

2.2.1.3. Rolling of the wheel under the action of the braking moment (braking mode). If, in the controlled rolling mode of the wheel, artificial resistance to its rotation is created with the help of a braking mechanism, then the rolling mode of the wheel will turn into the braking mode (Fig. 2.12) .

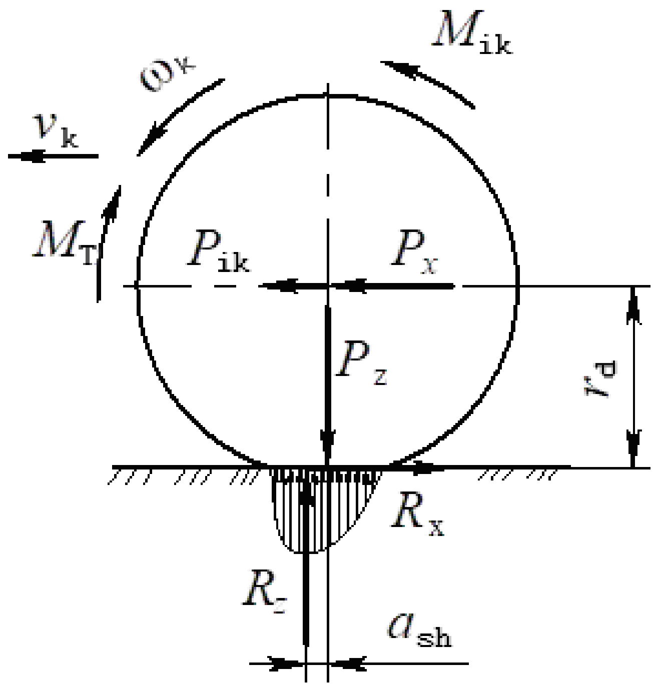


Fig. 2.12. **Scheme of forces and moments acting on the wheel in braking mode:** M_t – braking moment ; other designations - see Fig. 2.10

At the same time, the braking moment M_T has a direction against the rotation of the wheel. As a result, the wheel begins to slow down and there is a force of inertia directed in the direction of movement and a moment of inertia M_{ik} directed in the direction of rotation of the wheel. The force P_x tends to move the wheel at a speed v_k , and the reaction R_x is directed against the movement and tends to stop it. The moment created by the pair of forces P_x and R_x causes the wheel to rotate with an angular velocity ω_k. Therefore, in braking mode, the wheel moves in the longitudinal plane and rotates around its axis.

The equation of the moments acting on the wheel relative to the axis:

$$M_T + R_z \cdot a_{sh} - R_x \cdot r_d - M_{ik} = 0. \qquad (2.33)$$

The braking moment created by the braking mechanism:

$$M_T = R_x \cdot r_d - R_z \cdot a_{sh} + M_{ik} = 0. \qquad (2.34)$$

Braking power

$$P_{\text{T}} = \frac{M_{\text{T}}}{r_{\text{d}}} = R_x - R_z \frac{a_{\text{sh}}}{r_{\text{d}}} + \frac{J_{\text{k}}}{r_{\text{d}}} \cdot \frac{d\omega_{\text{k}}}{dt}. \tag{2.35}$$

Longitudinal response of the road

$$\begin{aligned} R_x &= \frac{M_{\text{T}}}{r_{\text{d}}} + R_z \frac{a_{\text{sh}}}{r_{\text{d}}} - \frac{J_{\text{k}}}{r_{\text{d}}} \cdot \frac{d\omega_{\text{k}}}{dt} = \\ &= \frac{M_{\text{T}}}{r_{\text{d}}} + R_z \cdot f - \frac{J_{\text{k}}}{r_{\text{d}}} \cdot \frac{d\omega_{\text{k}}}{dt} = P_{\text{т}} + P_f - \frac{J_{\text{k}}}{r_{\text{d}}} \cdot \frac{d\omega_{\text{k}}}{dt} \end{aligned}. \tag{2.36}$$

For steady motion, the longitudinal reaction of the road is equal to

$$R_x = P_{\text{т}} + P_f. \tag{2.37}$$

2.2.1.4. Free wheel rolling mode. By the torque M_{k}, and the longitudinal force P_x is zero (Fig. 2.13).

This mode of rolling of the wheel occurs on the driving axis during the transition from the driven mode to the driving mode (and vice versa) at the moment when the torque is equal in magnitude to the rolling resistance moment M_f. In this case, the longitudinal reaction R_x in the contact zone of the wheel with the support surface is zero and, accordingly, the longitudinal force $P_x = 0$.

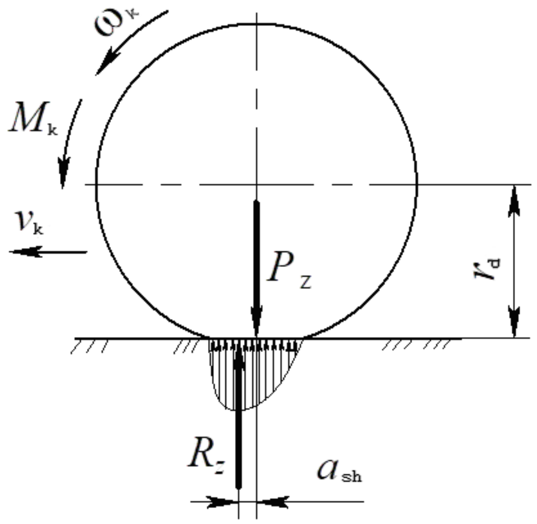


Fig. 2.13. **Scheme of forces and moments acting on the wheel in free mode:**
M_{k} – torque (driving) moment applied from the driving axle to the wheel;
other designations - see Fig. 2.10

Equation of moments acting on the wheel

$$M_{\text{k}} - R_{\text{z}} \cdot a_{\text{sh}} = 0 \Rightarrow M_{\text{k}} - M_f = 0. \tag{2.38}$$

2.2.1.5. Neutral wheel rolling mode. *The neutral rolling mode* is a mode in which the wheel is driven into rotation simultaneously by the torque M_k and the *pushing* force P_{x} . Such a rolling mode of the wheel occurs on the driving axis during the transition from the driven mode to the free mode (and vice versa) at the moment when the torque is less than the rolling resistance moment $M_f = R_z \cdot a_{\text{sh}}$.

The neutral mode can be both after the driven mode when the torque increases from zero to M_f , and after the free mode when the torque decreases from $M_{\text{k}} = M_f$ to zero (Fig. 2.14).

If, in the driven mode, a torque begins to be applied to the wheel, the longitudinal reaction R_x in the contact zone of the wheel with the support surface begins to decrease and, accordingly, the longitudinal force P_x decreases (Fig. 2.14a) . If the torque increases to $M_{\text{k}} = R_z \cdot a_{\text{sh}}$, the wheel will go into free rolling mode (Fig. 2.13). If the torque continues to increase, the leading wheel rolling mode will occur.

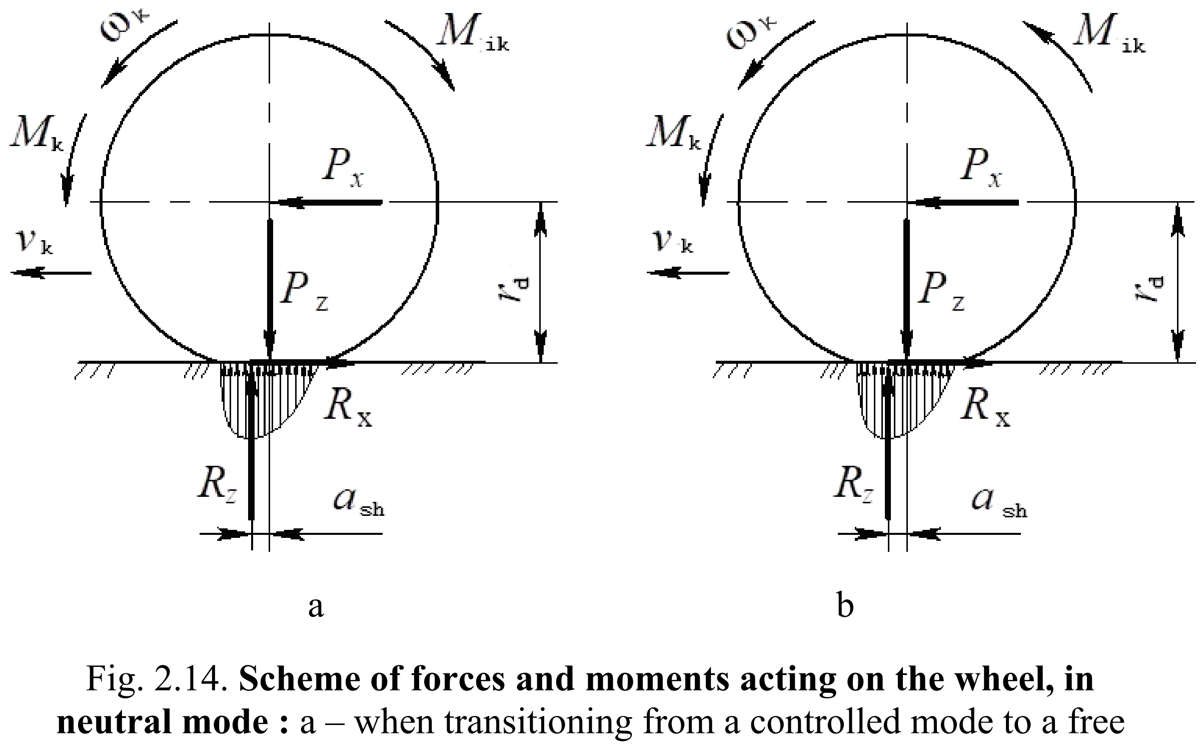


Fig. 2.14. **Scheme of forces and moments acting on the wheel, in neutral mode :** a – when transitioning from a controlled mode to a free one; b – when transitioning from a free mode to a controlled one; other designations - see Fig. 2.10

If the torque M_{k} on the wheel decreases in the free mode, the longitudinal reaction R_x appears in the contact zone of the wheel with the support surface and begins to increase, and the longitudinal force P_x increases accordingly (Fig. 2.14b) . If the torque decreases to zero, the wheel will enter the controlled rolling mode (Fig. 2.10).

Equation of moments acting on the wheel in neutral mode:

$$M_k - R_z \cdot a_{\text{sh}} + R_x \cdot r_{\text{d}} \pm J_{\text{k}} \cdot \frac{d\omega_{\text{k}}}{dt} = 0. \qquad (2.39)$$

2.2.1.6. Dependence of the rolling radius on the rolling mode of the wheel. The mode of movement of the wheel is determined by the nature and magnitude of the loads applied to it. As a result, the kinematic radius of the wheel changes according to the change in the loads acting on the wheel in all possible rolling modes. The dependence of the rolling radius on its rolling mode is presented in Figure 2.15.

For the convenience of analyzing the dependence of the rolling radius of the wheel on its rolling mode, Figure 2.15 below shows the dependence of the longitudinal reactions on the moment acting on the wheel.

1 – braking mode;

– the moment on the wheel is equal to M_{t}, the rolling radius of the wheel changes depending on the value of the moment in the range $r_{\text{ko}} < r_{\text{k}} < \infty$, if the wheel goes into full sliding mode (Fig. 2.15, limit 1'), then $r_{\text{k}} = \infty$;

2 – guided mode

– the moment on the wheel $M_{\text{k}} = 0$, the rolling radius of the wheel $r_{\text{k}} = r_{\text{ko}}$.

3 – neutral mode

– the moment on the wheel $0 < M_{\text{k}} < M_f$, the rolling radius of the wheel $r_{\text{ko}} < r_{\text{k}} < r_{kc}$.

4 – free mode

– wheel moment $M_{\text{k}} = M_f$, wheel rolling radius $r_{\text{k}} = r_{\text{kc}} = \text{const}$.

5 - leading mode

– moment on the wheel $M_{\text{k}} > M_f$, wheel rolling radius $0 < r_{\text{k}} < r_{\text{kc}}$, (if complete skidding, limit 5'), then $r_{\text{k}} = 0$.

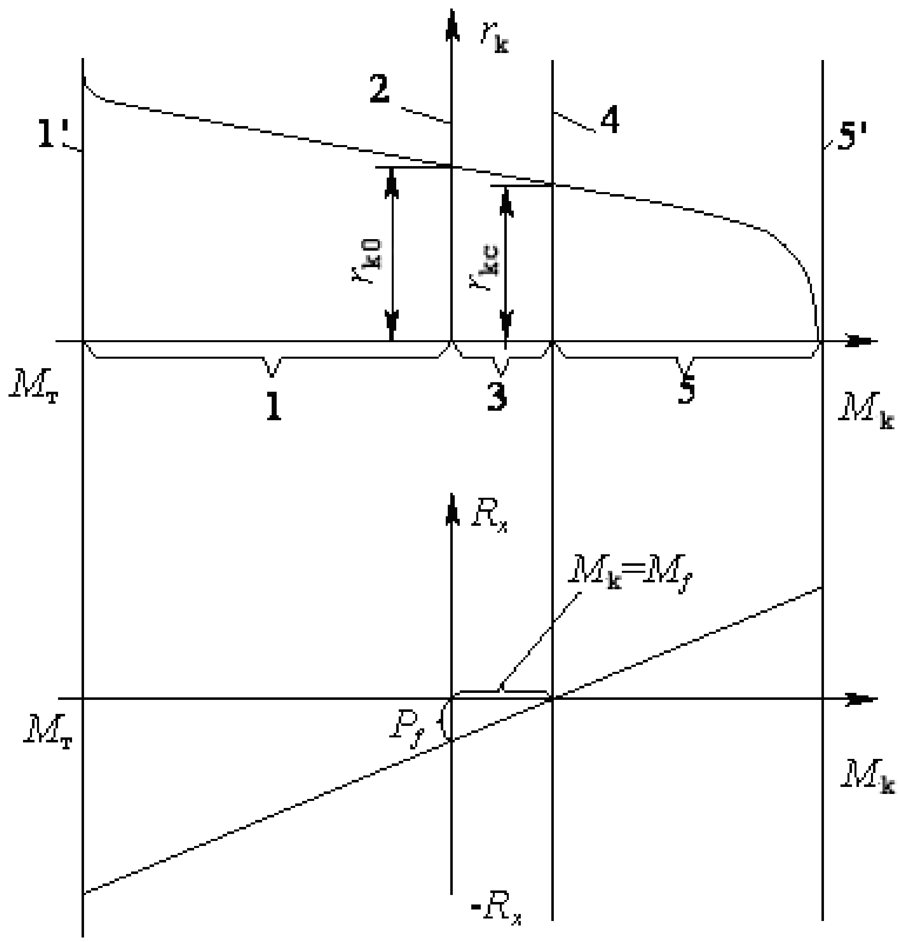


Fig. 2.15. **Dependence of the rolling radius of the wheel from rolling mode:** 1 – braking mode; 2 – controlled mode; 3 – neutral mode; 4 – free mode; 5 - leading mode

2.2.2. Rolling of a wheel on a surface that does not deform under loads acting in the longitudinal and transverse planes

Consider the process of wheel rolling. The rectilinear rolling of the wheel on a surface that does not deform can be disturbed if the forces and moments acting on it cause the velocity vector of its movement to deviate from the plane of rotation. In this case, it is considered that its rolling stability is broken. The reasons for breaking the rolling stability of rigid and elastic wheels are different.

2.2.2.1. Rolling of a rigid wheel on a non-deformable surface under the action of a lateral force. The rolling of a rigid wheel is considered stable if it does not slide relative to the supporting surface. The stability of the motion of a rigid vehicle wheel is explained by the diagram shown in Figure 2.16.

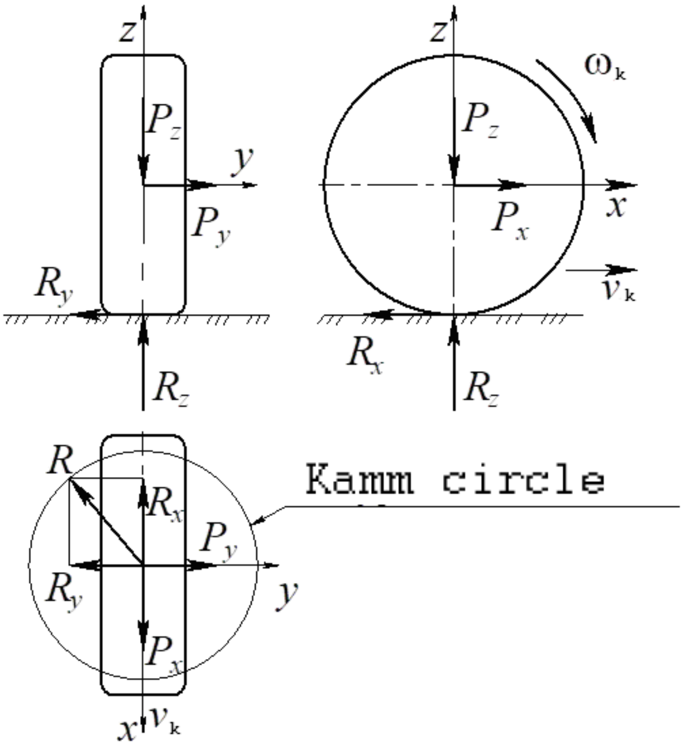


Fig. 2.16. **The scheme of interaction of a rigid wheel with a support surface under the action of a lateral force:** P_z – normal load on the wheel; R_z is the equivalent of the normals elementary reactions; P_y is the lateral force acting on the wheel; R_y is the equivalent of tangential elementary reactions in the transverse plane; R_x is the equivalent of tangential elementary reactions in the longitudinal plane; R is the equivalent of two reactions R_x and R_y

The diagram shows a wheel that is loaded with a normal load P_z and rolls in the driven mode due to the effect of a longitudinal load P_x. In addition, it is acted upon by a lateral force P_y. Corresponding reactions occur in the contact patch of the wheel. In the controlled wheel rolling mode under the action of a lateral force, the velocity vector v_k may deviate from the rolling direction x if the net effect of the two reactions R_x and R_y – R exceeds the wheel adhesion utilized force $P_\varphi = R_z \cdot \varphi$.

Equivalent reaction R of the elementary reactions of the road surface on the wheel is determined according to a known relationship $R = \sqrt{R_x^2 + R_y^2}$. The limit value of R is determined by the condition of the wheel's adhesion to the support on top of it $R \le P_\varphi$.

$$R = \sqrt{R_x^2 + R_y^2} \le P_z \cdot \varphi.$$

The geometric locus of the location points of the end of the vector of the limiting value of the equivalent reaction R , when the direction of action of the forces on the wheel changes, forms a circle (Fig. 2.16). This circle is called the *Kamm circle* in honor of Professor V. Kamm, who proposed such a graphical presentation of the reaction vector when the direction of the total force acting on the wheel changes. The rolling stability of a cruel wheel depends on the mode of its movement and is ensured if the lateral force

$$P_y = R_y \le \sqrt{R^2 - R_x^2}, \text{ or } P_y = R_y \le \sqrt{\left(P_z \cdot \varphi\right)^2 - R_x^2} \ . \quad (2.40)$$

The maximum lateral force $P_{y\,max}$, at which the wheel rolls without lateral slip at $R_x = 0 \Rightarrow P_{y\max} = R_{y\max} \le P_z \cdot \varphi$.

The minimum lateral force $P_{y\,min}$ at which the wheel rolls without lateral slip at $R_x = P_\varphi = R_z \cdot \varphi \Rightarrow R_{y\min} = 0$.

2.2.2.2. Rolling of an elastic wheel on a non-deformable surface under the action of a lateral force. The rolling of an elastic wheel is considered stable if there is no deviation of the velocity vector v_k from the rolling direction x (Fig. 2.17).

In contact with the bearing surface, the tire will undergo normal deformation and have contact in the form of a spot, the leading edge of which is marked by point 1 on the central tread tread. As a result of the lateral force P *, the* tire acquires lateral deformation. At the same time, the central plane of the wheel is shifted from the center of the footprint by a_y , and the central running track of the tread is curved. Points 2 and 3 on the central running track of the tread when the wheel is rolling touch the supporting surface with a lateral lead, as shown in Figure 2.17. If we connect points 1, 2 and 3, we will get the trajectory of the wheel, which means the direction of the velocity vector v_k .

When a lateral force P_y is applied to a wheel with an elastic tire, it rolls along the road with a lateral deflection. The angle between the vector of the speed of the wheel v_k and the plane of its rotation is called the *angle of lateral displacement* δ . Unlike a rigid wheel, a tire with a flexible tire deviates from the straight-line direction of travel under any lateral impact, even if there is no lateral slip in the footprint. The tire's ability to resist lateral deflection under the action of a lateral force is characterized by *the coefficient of resistance to lateral deflection of the elastic wheel* k_δ .

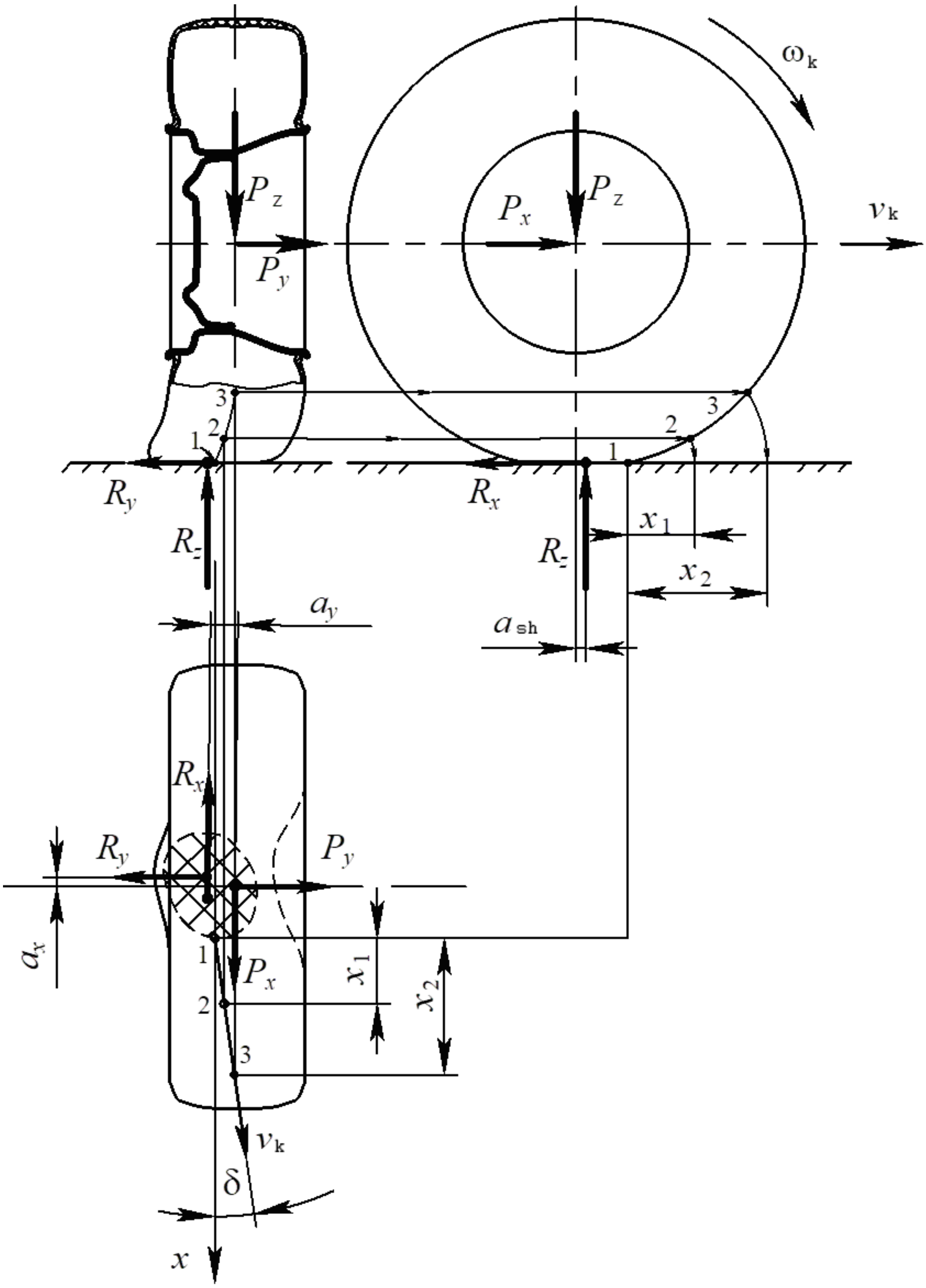


Fig. 2.17. **The rolling pattern of an elastic wheel when a lateral force acts on it**

The dependence of the lateral deflection angle on the lateral force is determined experimentally. Figure 2.18 shows the dependences between the lateral force and the angle of lateral deflection at different air pressure p_{in} the tire. The dependence $P_y = f(\delta)_1$ was obtained at a higher air pressure in the tire than $P_y = f(\delta)_2$.

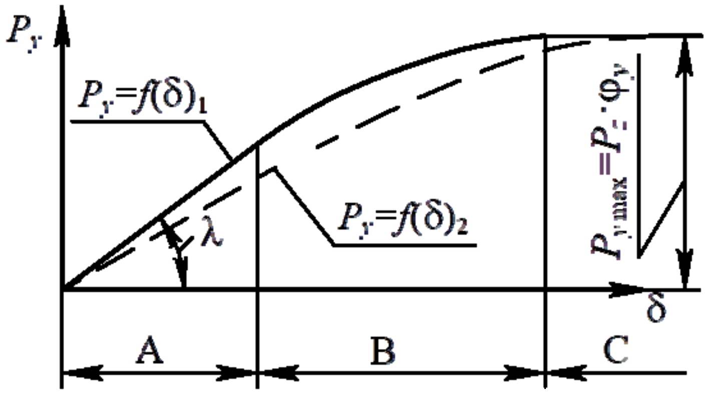


Fig. 2.18. **The relationship between the lateral force and the angle of the lateral deflection when rolling the elastic wheel**

The dependence $P_y = f(\delta)_1$ (see Fig. 2.18) has three sections A, B, C. In section A, the dependence between the lateral force and the angle of lateral deflection is proportional.

$$P_y = k_\delta \cdot \delta , \tag{2.41}$$

where k_δ is the coefficient of resistance to lateral displacement of the elastic wheel, N/degree.

The coefficient of resistance to lateral drift is a proportionality coefficient, which is defined as the tangent of the angle of inclination λ оф the dependence $P_y = f(\delta)$ to the abscissa.

$$k_\delta = \text{tg}\lambda = \frac{P_y}{\delta} . \tag{2.42}$$

The coefficient of resistance to lateral deviation depends on the design of the tire, the size of the tire, and the air pressure.

The larger the size of the tire and the air pressure in it, the greater the coefficient of resistance to lateral deviation. In diagonal tires, the coefficient of resistance to lateral deflection is greater than in radial tires. The more layers of the frame in the tire, the higher the value of k_δ.

In section A, the wheel rolls under the action of a lateral force with lateral deviation in the absence of lateral tire slippage in the contact patch. The maximum value of the side deflection angle of this tire depends on the value of the lateral force. On a dry hard surface with a nominal load on the wheel, section A is usually 3 ° ... 4 ° .

In the section B, in addition to the lateral deviation of the tire, there is an elastic lateral slippage of the wheels relative to the support surface. Conditionally lateral movement in sections A and B is considered a lateral diversion.

Section C corresponds to the value of the lateral force at which complete sliding of the tire impression relative to the support surface is observed. The ratio $\varphi_y = P_{y\max} / R_z$ is called *the coefficient of lateral adhesion* .

2.3. Rolling resistance coefficient of an elastic wheel

The coefficient of rolling resistance of an elastic wheel has a complex dependence on many external and internal factors. The dependence of the rolling resistance coefficient on some factors is given below.

1. *Type and condition of the road surface.* The coefficient of rolling resistance is determined by the experimental-calculation method when the wheel is rolling at a low speed ($v < 10$ m/s). Some results are shown in Table 2.3.

Table 2.3 – **Typical values of the rolling resistance coefficient**

Type and condition of the road surface		f_0
Asphalt-concrete and cement-concrete road	In order	0.007 - 0.015
	In bad condition	0.015 - 0.02
The gravel road is in good condition		0.02 - 0.025
Paving is in good condition		0.025 - 0.03
Dirt road	Dry stiff	0.025 - 0.03
	After the rain	0.05 - 0.15
	During the off-road period	0.1 - 0.25
Sand	Dry	0.1 - 0.3
	Raw	0.06 - 0.15
Frozen road, ice		0.015 - 0.03
Compacted snow		0.03 - 0.05
Slush		0.1 - 0.3

2. *Speed of movement.* The rolling resistance coefficient of an elastic wheel has a non-linear dependence on the speed of movement (Fig. 2.19).

This dependence is determined not only by the speed of the wheel, but also by a number of other parameters.

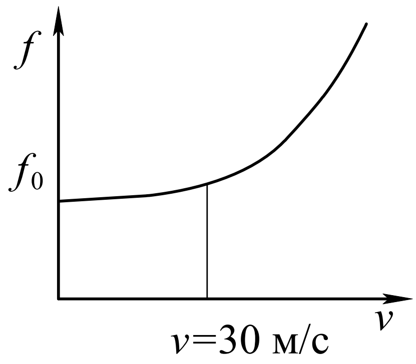


Fig. 2.19. **Dependence of the rolling resistance coefficient from the speed of the wheel**

Up to a speed of less than 30 m/s, the coefficient of rolling resistance changes slightly and in some tasks it can be considered constant. At a speed of more than 30 m/s, the rolling resistance coefficient increases significantly and its value can be determined empirically

$$f = f_0 \cdot \left(1 + \frac{v^2}{1500}\right), \tag{2.43}$$

where f_0 – the coefficient of movement resistance at low speed;

1500 – an empirical coefficient.

3. Tire temperature. As the temperature of the tire increases, its rolling resistance coefficient decreases (Fig. 2.20). This is due to a decrease in deformation due to an increase in air pressure in the tire and a decrease in hysteresis losses in the rubber (a decrease in frictional forces). In the reference literature, the value of f_0 for a heated tire is usually given.

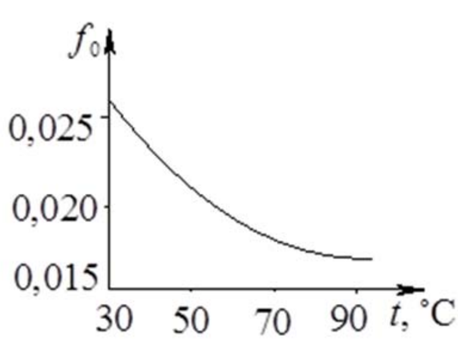


Fig. 2.20. **Dependence of the rolling resistance coefficient on tire temperature**

4. Air pressure in the tire. The internal pressure in the tire on different support surfaces affects the coefficient in different ways (Fig. 2.21). On hard, smooth surfaces, the value of the coefficient decreases. On a soft surface for each tire, there is an optimal pressure value at which the total deformation of the tire and the support surface, and therefore the coefficient, has a minimum value.

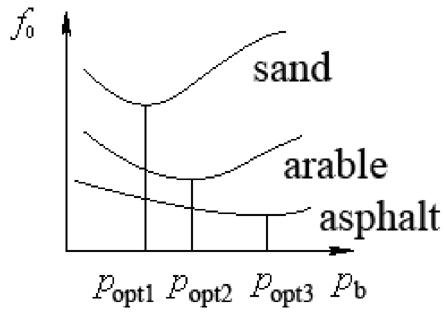


Fig. 2.21. **Dependence of the rolling resistance coefficient on pressure air in the tire on different support surfaces**

An excessive increase in pressure in the tire leads to an increase in the depth of the rut, with a decrease in pressure, the deformation of the tire increases, which leads to an increase in the coefficient.

5. Load on the tire. An increase in the load on the tire causes an increase in its deformation and, as a result, an increase in the rolling resistance coefficient (Fig. 2.22).

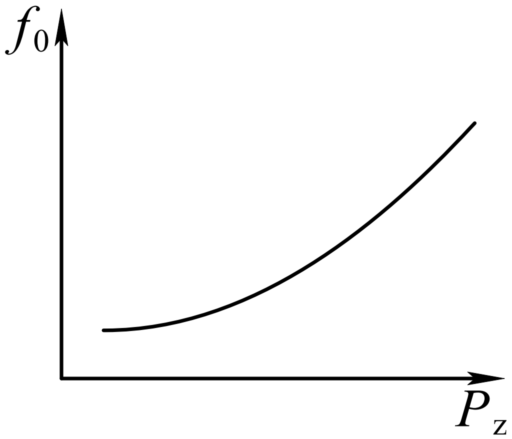


Fig. 2.22. **Dependence of the rolling resistance coefficient on the normal load on the tire**

6. *Design parameters of the tire.* The value of the rolling resistance coefficient depends on a large number of its design parameters:

– increase in tread thickness $f\uparrow$;
(with tire wear $f\downarrow$);
– increasing the width of the rim to the width of the tire profile $f\downarrow$;
– reduction of the tire profile coefficient $f\downarrow$;
(at the same time $\downarrow$ the dependence of f on v);
– frame structure:
at $v < 30 \dots 35$ m/s $f_{\text{rad}} < f_{\text{diagonal}}$;
– increase in wheel diameter $\rightarrow f\downarrow$;
(the worse the road, the greater the impact);
– increasing the width of the wheel:
on hard roads $f\uparrow$ slightly;
on soft $f\downarrow$ significantly.

7. The torque applied to the wheel. An increase in the torque applied to the wheel leads to an increase in the wear of the normal reaction and the work of friction in the contact of the tire with the supporting surface, which leads to an increase in the value of the rolling resistance coefficient (Fig. 2.23). The rolling resistance coefficient of the wheel to which the torque is applied can be approximately determined by the equation

$$f = \frac{a_{\text{sh}}}{r_{\text{d}}} + \frac{M_{\text{k}} \cdot (r_{\text{ko}} - r_{\text{k}})}{R_z \cdot r_{\text{ko}} \cdot r_{\text{d}}}.$$

The first term characterizes the friction losses in the tire during its deformation, the second term characterizes the friction losses in the contact between the tire and the supporting surface.

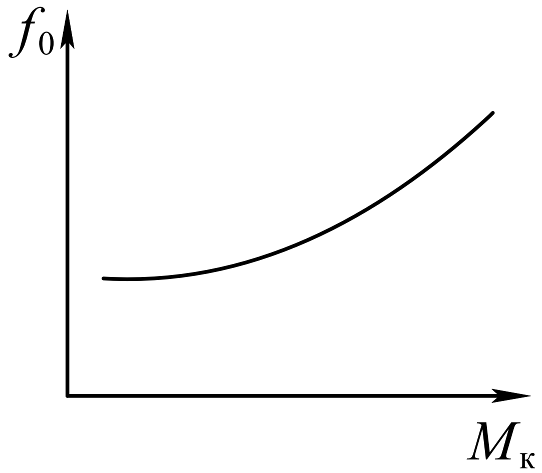


Fig. 2.23. **Dependence of the rolling resistance coefficient on the moment, connected to the wheel**

2.4. Coefficient of adhesion of an elastic wheel to the supporting surface of the road

2.4.1. Limit cases of wheel rolling

It was established above that there is a direct relationship $R_x = f(r_k, M_k)$ between the longitudinal reaction on the wheel and the moment applied to it. It should be noted that this relationship has a non-linear nature, which depends on the size of the wheel, the load on the wheel and sliding in contact with the support surface. This nonlinearity has a significant effect on the relationship between the longitudinal reaction on the wheel and the moment applied to it in extreme cases of rolling of the wheel. The extreme case of wheel rolling in the driving mode is characterized by its skidding, and in the braking mode by wheel wear.

To analyze the rolling mode of tires of different sizes under a changing load and rolling wheels with slippage, it is convenient to use the dependence of the relative value R_x / R_z on some dimensionless value associated with the rolling radius.

This dimensionless value associated with the rolling radius is called:

– for the leading mode: by *the slip coefficient* s_b;

– for the braking mode: by *the coefficient of longitudinal slip* s.

Slip coefficient

$$s_b = \frac{v_T - v_k}{v_T} = \frac{r_{kc} \cdot \omega_k - r_k \cdot \omega_k}{r_{kc} \cdot \omega_k} = 1 - \frac{r_k}{r_{kc}}, \qquad (2.44)$$

where v_T – the theoretical incremental speed of the wheel ($v_T = r_{kc} \cdot \omega_k$, r_{kc} – rolling radius at $R_x = 0$ – free rolling mode);

ω_k – angular speed of the wheel;

v_k – wheel speed ($v_k = r_k \cdot \omega_k$, r_k – rolling radius corresponding to the moment that is actually transmitted).

Coefficient of longitudinal slip

$$s = \frac{v_k - v_T}{v_k} = \frac{r_k \cdot \omega_k - r_{kc} \cdot \omega_k}{r_k \cdot \omega_k} = 1 - \frac{r_{kc}}{r_k}. \qquad (2.45)$$

The slip coefficient s_b (slip s), multiplied by 100%, reflects how many percent of the tire area in the contact patch slides relative to the support surface.

Figure 2.24 shows the change in the ratio of the sliding area (shaded) in the tire contact zone with an increase in the applied torque .

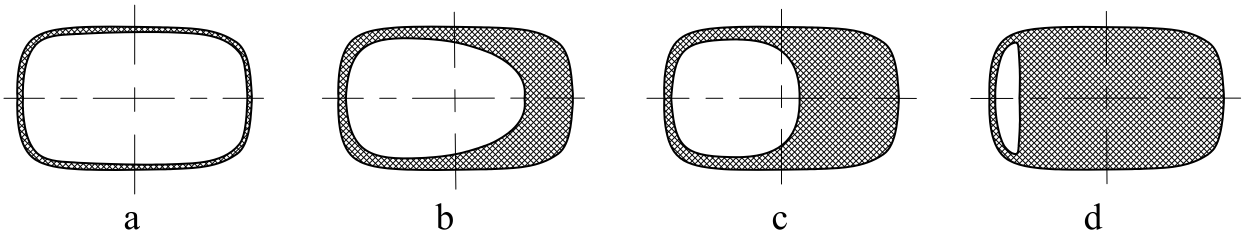


Fig. 2.24. **The appearance of areas of rest and sliding in the zone of contact of the tire with the supporting surface**

The dependence of the value R_x/R_z on sliding (and skidding), obtained experimentally, is shown in Figure 2.25. The magnitude of the critical longitudinal slip s_{kp} (critical slip s_{bkp}), at which R_x/R_z reaches its maximum value, as well as the intensity of the decrease in R_x/R_z with a further increase in s or s_b depend on both the characteristics of the tire tread material and the bearing surface, as well as from the speed of movement.

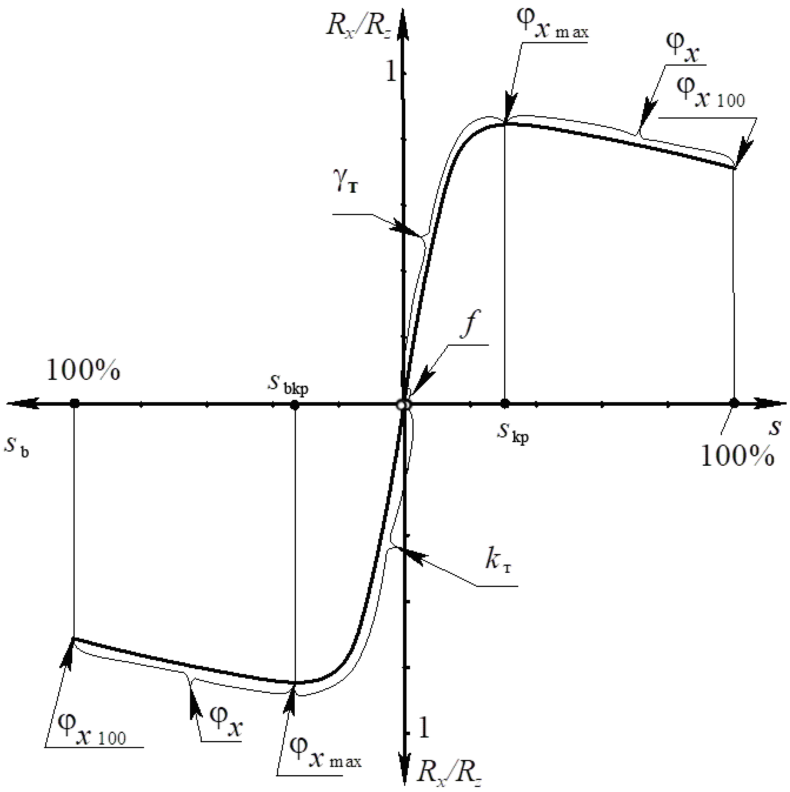


Fig. 2.25. **Dependence R_x/R_z from sliding s (slipping s_b)**

The ratio of reactions R_x / R_z determines the share of the longitudinal reaction of the wheel from the normal reaction of the wheel, therefore, in the general case, we will call this ratio *the coefficient of longitudinal force* k_n. The longitudinal force factor multiplied by 100% represents how much percent of the normal wheel reaction is the longitudinal force.

If $s < s_{kp}$ or $s_b < s_{bkp}$, then the reaction R_x is determined only by the value of the moment that is applied to the wheel. An increase in the moment leads to an increase in both the reaction R_x and the coefficient s or s_b. In this case, the longitudinal force coefficient (that is, the ratio of the longitudinal reaction to the normal reaction) is called:

– in traction mode: by *the traction coefficient* k_t;

– in driven and neutral modes: rolling *resistance coefficient* f;

– in the braking mode: by *the coefficient of the specific braking force* γ_t.

If $s \geq s_{cr}$ or $s_b \geq s_{bkr}$, then the reaction R_x is limited by the conditions of interaction (adhesion utilized) of the wheel with the supporting surface. In this case, the coefficient of longitudinal force is called *the coefficient of longitudinal adhesion utilized of the wheel* φ_x *with the supporting surface* .

If $s = s_{kp}$ or $s_b = s_{bkp}$, the reaction R_x acquires the maximum possible value $R_{x\,max}$ in terms of adhesion and the coefficient of longitudinal adhesion of the wheel to the support surface has a maximum value:

$$\varphi_{x\max} = \frac{R_{x\max}}{R_z}. \tag{2.46}$$

That is, it is the coefficient of longitudinal reaction, which receives the maximum value for a given wheel in given rolling conditions.

When $s > s_{kp}$ or $s_b > s_{bkp}$, the reaction R_x is also limited by the conditions of interaction (adhesion utilized) of the wheel with the support surface. Bringing a greater moment to the wheel will lead to an increase in the angular speed of the wheel ω_k and a decrease in the longitudinal reaction R_x and an increase in longitudinal sliding (slipping). In this case, the coefficient of longitudinal force is also called *the coefficient of longitudinal adhesion of the wheel to the supporting surface* . If $s = 1$

(100%) or $s_b = 1$ (100%), the coefficient of longitudinal adhesion of the wheel decreases to $\varphi_{x\,100}$.

The traction coefficient k_t and the coefficient of specific braking force γ_T reflect the share of the coefficient of longitudinal adhesion utilized of the wheel realized in the given rolling conditions. Therefore, they are also called *coefficients of realized adhesion utilized* in the corresponding modes. In this regard, specialists call the entire dependence $R_x / R_z = f(s)$ *"φ - s diagram"*, which for braking mode is depicted in the form presented in Figure 2.26.

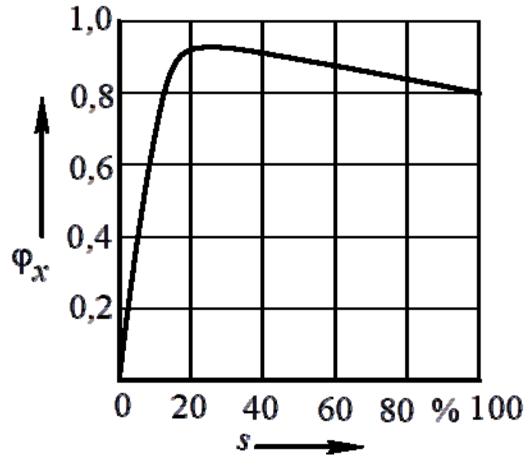


Fig. 2.26. **Image φ_x - *s* diagram**

2.4.2. Factors affecting the coefficient of adhesion

Type and condition of the coating has a significant effect on the coefficient of longitudinal adhesion utilized φ_x (see Table 2.4).

Table 2.4 – **Values of longitudinal adhesion utilized coefficients on different types of coating**

The type and condition of the road surface	$\varphi_{x\,max}$	$\varphi_{x\,100}$
Dry asphalt and concrete	0.8-0.9	0.7-0.8
Wet asphalt	0.5-0.7	0.45-0.6
Wet concrete	0.75-0.8	0.65-0.7
Gravel	0.55-0.65	0.5-0.55
Dirt road: dry wet	 0.65-0.7 0.5-0.55	 0.6-0.65 0.4-0.5
Compacted snow	0.15-0.2	0.15
Ice	0.1	0.07

Coefficient of sliding (slipping) of the wheel significantly affects the adhesion utilized coefficient. When the sliding coefficient increases, the adhesion utilized coefficient first increases and then decreases (Fig. 2.27). At the same time, the amount of change in the adhesion coefficient depends on the type and condition of the surface, speed of movement.

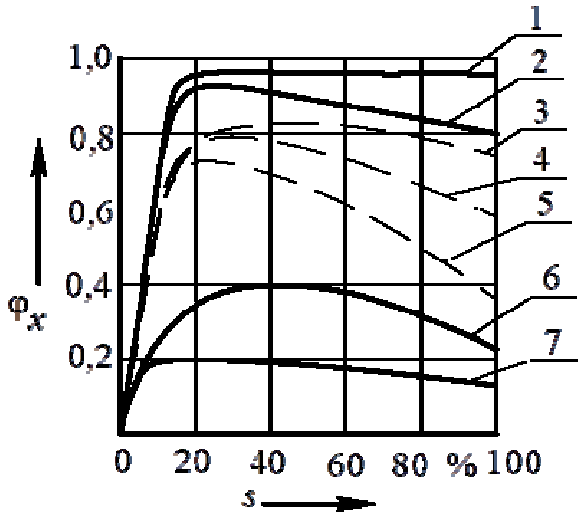


Fig. 2.27. Chart **image** $\varphi_x - s$ **for various rolling conditions wheels:**
1 – dry road (speed 10 km/h)); 2 – dry road (speed 50 km/h);
3 – wet road (speed 10 km/h); 4 – wet road (speed 50 km/h);
5 – wet road (speed 90 km/h); 6 – hardened snow; 7 – wet ice

The speed of movement and the presence of moisture, dust, dirt on the supporting surface. When the speed of movement increases, the adhesion utilized coefficient first increases, and then decreases (Fig. 2.28).

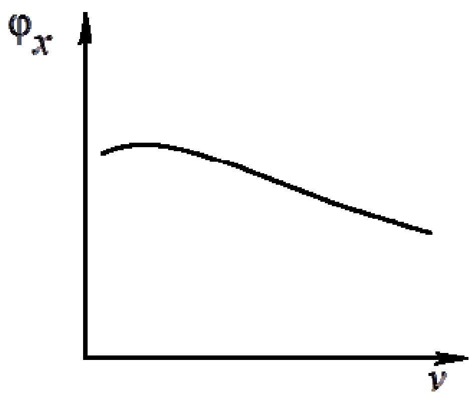


Fig. 2.28. **Dependence of the adhesion utilized coefficient on the speed of movement**

The presence of dust, dirt and moisture on the supporting surface causes deterioration of the wheel's adhesion to it and, as a consequence. the adhesion utilized coefficient decreases. If there is a layer of liquid (water) on the solid support surface, the *effect of aquaplaning of the wheel may occur* (Fig. 2.29). When the wheel rolls at a certain speed, the liquid does not have time to squeeze out of the contact zone and the contact area of the tire with a solid surface decreases.

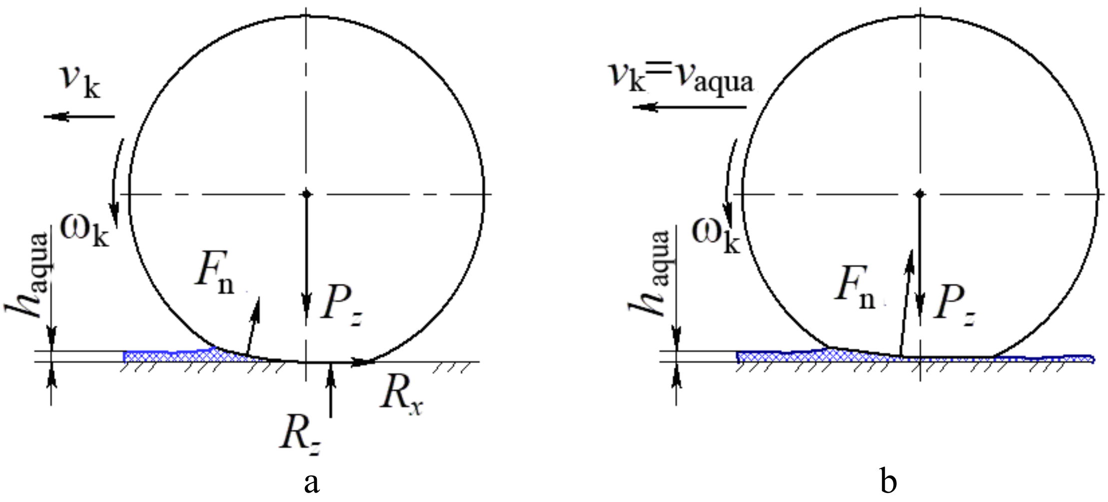


Fig. 2.29. **A scheme that explains the occurrence of the effect aquaplaning wheels:** a – reducing the contact of the tire with the surface; b – complete loss of contact; h_{aqua} is the height of the liquid film; F_n is the lifting force of the liquid film

At the same time, the wheel may lose contact with the hard surface, which dramatically reduces the grip conditions. The coefficient of adhesion of the wheel to the supporting surface in the presence of a liquid film determines the dependence

$$\varphi_{x\ \text{aqua}} = \varphi_x \left(1 - \frac{\kappa_m}{l_k \cdot p_B} \cdot v \right), \tag{2.47}$$

where k_m – an empirical coefficient;

v – movement speed, m/s;

l_k – the length of contact of the tire with a solid surface, m;

p_B – tire pressure, MPa.

The speed at which there is a complete loss of contact of the tire with a solid support surface is called *the aquaplaning speed* and is determined by Horn's empirical equation

$$v_{\text{aqua}} = 6{,}34\sqrt{p_B}\ . \qquad (2.48)$$

Air pressure in the tire affects the adhesion coefficient differently depending on the type and condition of the bearing surface. On a solid flat support surface, with an increase in air pressure in the tire, the adhesion coefficient decreases due to a decrease in the area of the contact imprint. On hard surfaces with a layer of mud in the wet state or on dirt roads with a wet surface, the increase in pressure helps push this mud into contact with the solid base. Therefore, on such surfaces, this leads to an increase in the coefficient of adhesion (Fig. 2.30).

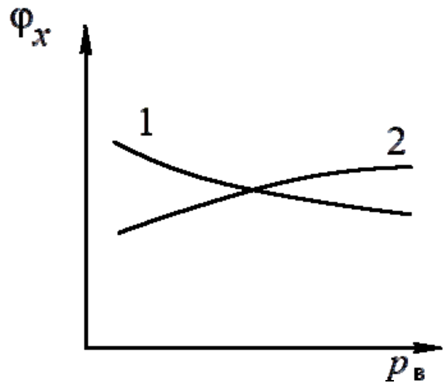


Fig. 2.30. **Dependence of the adhesion utilized coefficient on tire pressure** : 1 – on a clean solid support surface; 2 - on wet, dirt roads with a hard surface

Normal load on the wheel. When the load on the wheel increases, the adhesion coefficient with the support surface slightly decreases (Fig. 2.31).

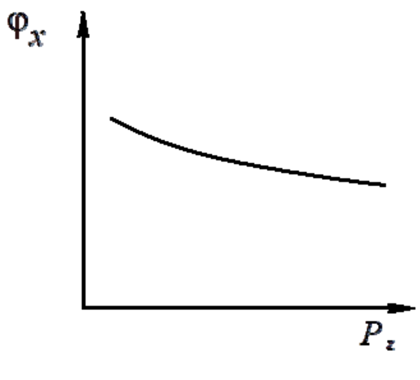


Fig. 2.31. **Dependence of the adhesion utilized coefficient on the load on the wheel**

But at the same time, the force of wheel adhesion $P_{\varphi} = \varphi_x \cdot R_z$ increases, because the reduction of the coefficient of adhesion φ_x occurs much more slowly than the growth of the normal reaction R_z (equal to P_z).

The design features of the wheel affect the clutch coefficient in different ways:

– when the wheel diameter increases, the adhesion utilized coefficient increases, but slightly;

– the impact of the tread pattern is assessed by the saturation factor of the tread pattern, which is defined as the ratio of the area of the tread protrusions in contact to the area of the entire contact of the tire with the supporting surface. With an increase in the saturation coefficient of the tire tread pattern, the coefficient of adhesion on dry roads with a hard surface increases, and on wet roads and on soft surfaces, it decreases;

- the shape of the pattern and the height of the tread significantly affect the traction coefficient when driving on wet hard roads. The better the removal of liquid and liquid dirt from the contact zone, the greater the coefficient of adhesion. When reducing the height of the tire tread, the traction coefficient is significantly reduced.

Control questions

1. Name the main parameters of an elastic wheel.
2. List the wheel radii and give their definition.
3. Name the types of deformation of an elastic wheel.
4. List the modes of motion of the rolling wheel.
5. Draw a diagram of the forces acting on the wheel in the driven, driving and braking modes and determine the longitudinal reactions.
6. How does the rolling radius depend on the rolling mode of the wheel?
7. What is the coefficient of resistance to lateral drift and what factors affect its value?
8. What is the coefficient of rolling resistance and what factors affect its value?
9. What is the coefficient of adhesion and what factors affect its value?
10. What are slip and slip coefficients?
11. What is φ_x - s chart?
12. What are the critical slip and slip coefficients?

TOPIC 3

FORCES AND REACTIONS ACTING ON THE VEHICLE

3.1. Dynamic interaction of the vehicle with the environment

During movement, a number of forces act on the vehicle, which are called *external* :

$G_a = m_a \cdot g$ - gravitational force;

R_{z1}, R_{z2}, R_{x1}, R_{x2} – forces of interaction between the wheels and the road (road reactions);

P_{is} the force of interaction of the vehicle with the air environment (reaction of the air environment).

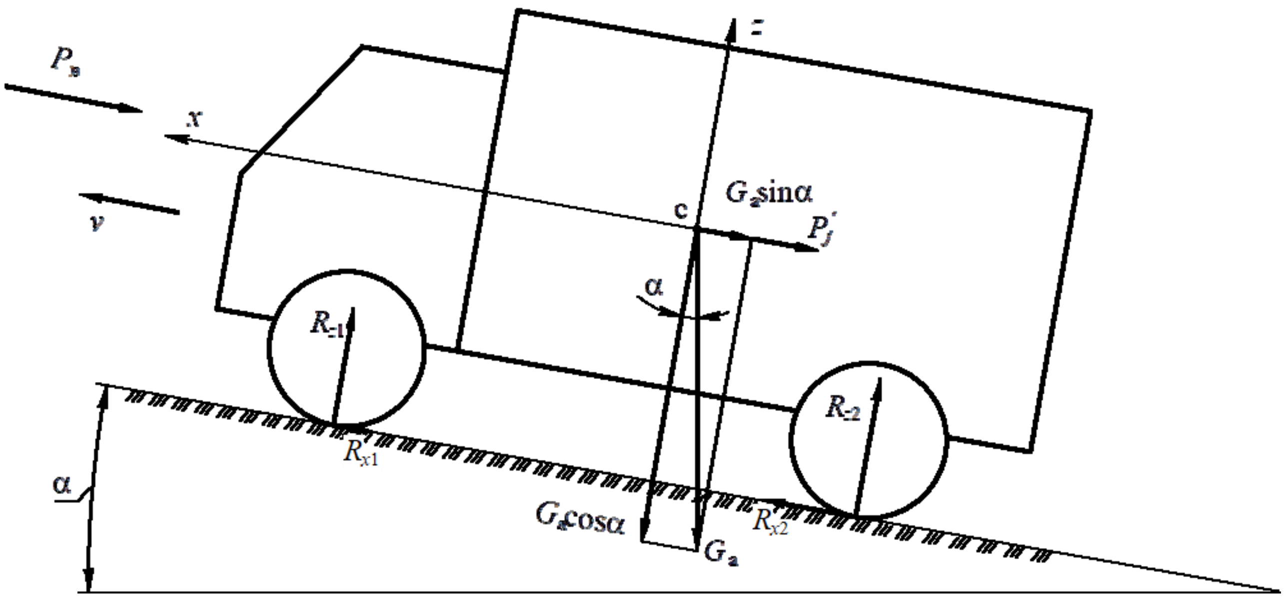


Fig. 3.1. **Scheme of external forces acting on the vehicle while moving:**
c – center of gravity of the vehicle; *x , z* – longitudinal and vertical axes;
α – elevation angle; *v* – movement speed; G_a - gravitational force;
P_B – air resistance force; R_{z1}, R_{x1}, R_{z2}, R_{x2} – road reactions on the wheels of the front and rear axles

The force of gravity G_a is applied at the center of gravity of the vehicle. When moving along a surface with a longitudinal slope, this force is divided into two vectors, as shown in Figure 3.1. The action of these vectors causes reactions on the wheels along the *z* and *x* axes : R_{z1}, R_{z2},

R_{x1}, R_{x2} from the side of the road surface. When the vehicle moves, there is a reaction of the air environment, which is called the force of *air resistance* $R_{B.}$ The force of air resistance is the resulting force of interaction of the vehicle with the air environment and is applied in the center of windage (metacenter). In the case of a change in the dynamic state of the vehicle, that is, when it accelerates or decelerates, the inertial force of the vehicle mass moving forward is directed against the acceleration vector. This force is applied in the center of gravity of the vehicle. The positions of the vehicle center of buoyancy and its center of gravity are not much different, so it can be assumed that the center of gravity of the vehicle coincides with the center of gravity.

3.2. Analysis of forces acting on a vehicle moving in a longitudinal plane

To analyze the traction and braking dynamics of the vehicle, we will assume that the external forces acting on it coincide with the longitudinal plane of the vehicle. The forces acting in the longitudinal plane are divided into *driving forces* , the direction of which coincides with the direction of the velocity vector of the center of gravity, and *resistance forces* , the direction of which is opposite to this vector.

Conventionally, driving forces include the full force of traction on the driving wheels. All other forces acting on the vehicle are considered resistance forces. If any of these forces in specific conditions turn out to be directed towards movement, then they are considered negative resistance forces.

3.2.1. Longitudinal reactions of the road on the wheels of the vehicle

In the general case, consider the process of driving a vehicle with rear driving wheels and front driven wheels (see Fig. 3.2). According to the wheel rolling theory, the longitudinal reaction on the wheels of the front axle with driven wheels is determined by the equation

$$R_{x1} = R_{z1} \cdot f_1 + \frac{J_{\text{k1}}}{r_{\text{d}}} \cdot \frac{d\omega_{\text{k1}}}{dt}, \tag{3.1}$$

where R_{z1} is the normal response of the road to the front axle of the vehicle;

f_1 – coefficient of resistance to rolling of the wheels of the front axle;

J_{k1} – the total moment of inertia of the wheels of the front axle;

r_d – dynamic wheel radius;

ω_{k1} – angular speed of the wheels of the front axle.

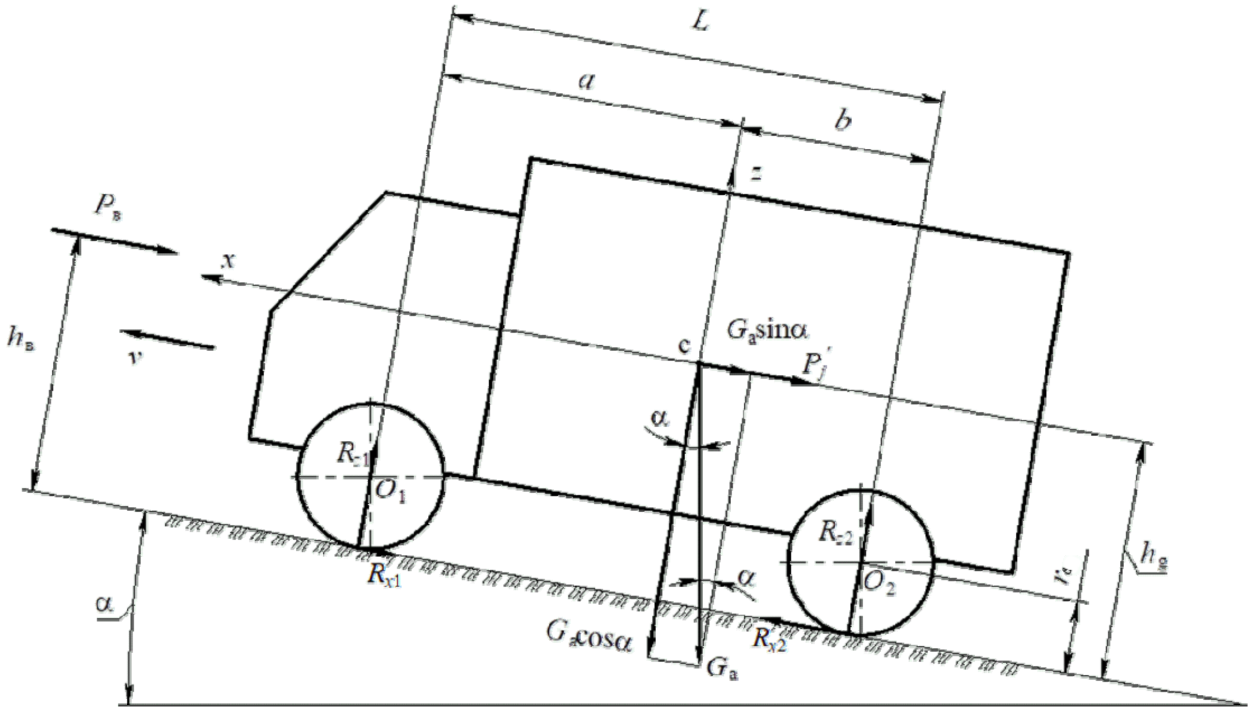


Fig. 3.2. **Scheme of forces acting on the vehicle when moving uphill** :
c – center of gravity of the vehicle; x, z – longitudinal and vertical axes;
L – vehicle base; a, b, h_g – coordinates of the center of gravity;
$h_в$ – the height of the center of windage; α – elevation angle;
v- speed of movement

According to the wheel rolling theory, the longitudinal reaction on the wheels of the rear axle with the driving wheels connected to the transmission is determined by the equation

$$R_{x2} = \frac{M_k}{r_d} - R_{z2} \cdot f_2 - \frac{J_{k2}}{r_d} \cdot \frac{d\omega_{k2}}{dt} - \frac{J_e \cdot \eta_{tr} \cdot u_{tr}}{r_d} \cdot \frac{d\omega_e}{dt}, \quad (3.2)$$

where $\frac{M_k}{r_d} = P_k$ – the total traction force of the driving wheels;

R_{z2} – normal reaction of the road to the rear axle of the vehicle;
f_2 – coefficient of rolling resistance of the wheels of the rear axle;
J_{k2} – the total moment of inertia of the wheels of the rear axle;
ω_{k2} – angular velocity of the wheel of the rear axle;
J_e – the moment of inertia of the rotating parts of the engine and transmission;
ω_e is the angular speed of the engine crankshaft.

3.2.2. Dependence of total traction force P k and vehicle speed on engine parameters

The main driving force is the total traction force P_k on the driving wheels of the vehicle. It arises as a result of the fact that the torque of the engine M_e is brought to them .

Torque on the drive wheels

$$M_k = M_e \cdot \eta_{tr} \cdot u_{tr} . \quad (3.3)$$

Full traction on the driving wheels

$$P_k = \frac{M_k}{r_d} = \frac{M_e \cdot \eta_{tr} \cdot u_{tr}}{r_d} . \quad (3.4)$$

The speed of the vehicle in the absence of skidding of the driving wheels is strictly related to the frequency of rotation of the crankshaft of the engine

$$v = v_k = r_k \cdot \omega_k = r_k \frac{\omega_e}{u_{tr}} = r_k \frac{2 \cdot \pi \cdot n_e}{60 \cdot u_{tr}} , \quad (3.5)$$

where v is the speed of the vehicle, m/s.

The speed of the vehicle in the dimension "km/h" will be denoted by v and will be determined from dependence (3.5)

$$v_a = r_k \frac{2 \cdot \pi \cdot n_e}{60 \cdot u_{tr}} \cdot \frac{3600}{1000} = 0{,}377 \frac{r_k \cdot n_e}{u_{tr}} . \quad (3.6)$$

Since it is assumed that there is no skidding of the driving wheels, the equality $r_k = r_d = r_c$ is valid .

3.2.3. The rolling resistance force P f of the vehicle

The force of rolling resistance occurs when the vehicle is moving and is due to friction in the tire, friction between the tire and the road, and losses due to the formation of ruts

$$P_f = P_{f1} + P_{f2} = R_{z1} \cdot f_1 + R_{z2} \cdot f_2, \qquad (3.7)$$

where P_{f1}, P_{f2} are the rolling resistance force of the wheels of the front and rear axles, respectively.

We can assume that $f_1 \approx f_2 \approx f$, then

$$P_f = (R_{z1} + R_{z2}) \cdot f = G_a \cdot \cos\alpha \cdot f. \rightarrow P_f = G_a \cdot f \cdot \cos\alpha. \qquad (3.8)$$

If the vehicle moves uphill (downhill), then $P_f \downarrow$, and the stronger the angle α ισ .

The rolling resistance of a vehicle on a horizontal road

$$P_f = G_a \cdot f. \qquad (3.9)$$

3.2.4. Lifting resistance force P_α

Lifting resistance force is the component of the vehicle gravity parallel to the lift plane. Then

$$P_\alpha = G_a \cdot \sin\alpha. \qquad (3.10)$$

In road construction, the tangent of the angle of inclination of the road to the horizon is called the *longitudinal uphill (i)* (Fig. 3.3).

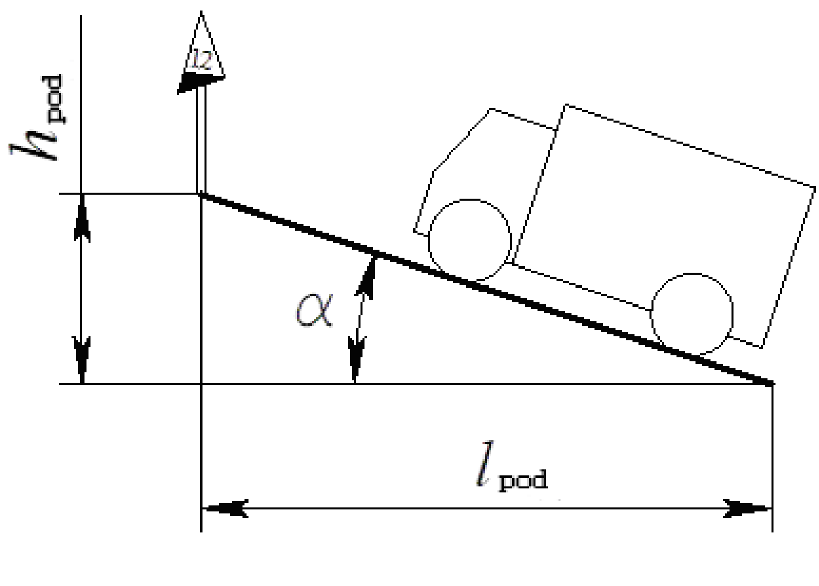


Fig. 3.3. **Scheme of the uphill of the road**

$$tg\alpha = \frac{h_{\text{pod}}}{l_{\text{pod}}} = i,$$

where l_{pod}, h_{pod} - the length and height of the uphill, respectively.

The longitudinal uphill i is measured in fractions or percentages.

At small angles ($\alpha \leq 5°$) tg $\alpha \approx$ sinα, therefore, sin$\alpha \approx i$.

Accordingly, resistance force of lifting

$$P_{\alpha} = G_{\text{a}} \cdot i. \tag{3.11}$$

The resistance force of lifting can be both positive and negative. The sign of P_{α} is determined by the sign of the angle α, which is considered positive on the rise.

3.2.5. Road resistance force P_{ψ}

This force is determined by the sum of the forces of rolling resistance and lifting resistance. Using the values of these forces, we determine

$$P_{\psi} = P_f + P_{\alpha} = G_{\text{a}} \cdot f \cdot \cos\alpha + G_{\text{a}} \cdot \sin\alpha = G_{\text{a}} \cdot (f \cdot \cos\alpha + \sin\alpha), \tag{3.12}$$

for small α (up to 5°) cos $\alpha \approx 1$, therefore, sin$\alpha \approx i$, and we can write

$$P_{\psi} = G_{\text{a}} \cdot (f + i). \tag{3.13}$$

The sum ($f \cdot$cosα + sinα) $\approx f + i \approx \psi$ is called *total road resistance coefficient.* Taking into account this equation (3.13) will take the form

$$P_{\psi} = G_{\text{a}} \cdot \psi. \tag{3.14}$$

3.2.6. The force of air resistance R_w

The air acts on the surface of the vehicle with many elementary forces. The sum of all the elementary forces acting on the surface of the vehicle is called *the total aerodynamic force*

$$P_w = c_w \cdot F \cdot q, \tag{3.15}$$

where c_w is the dimensionless coefficient of the total aerodynamic force;

F – the area of the vehicle cross-section, m ^{2};

q is the velocity head, kg/(m·s^2), which is equal to the kinematic energy of a cubic meter of air moving at a speed equal to the speed of the vehicle relative to the air environment v_w.

Midsection area of the vehicle is the largest cross-sectional area of the vehicle in a plane perpendicular to its longitudinal axis.

The velocity head is equal to the kinematic energy of a cubic meter of air moving at a speed equal to the speed of the vehicle relative to the air environment v_w

$$q = \frac{\rho_\text{v} \cdot v_w^2}{2}, \tag{3.16}$$

where v_w is the speed of the vehicle relative to the air environment, m/s;

ρ_v – air density, kg/m ^{3}.

Dimensionality of high-speed pressure: $\frac{\text{kg}}{\text{m}^3} \cdot \frac{\text{m}^2}{\text{s}^2} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \cdot \frac{\text{m}}{\text{m}^3} = \frac{\text{J}}{\text{m}^3}$.

The density of air at normal atmospheric pressure p_v = 101 325 Pa depends on the temperature:

at t = 0 ° C – ρ_v = 1.293 kg/m ^{3};

at t = 20 ° C – ρ_v = 1.204 kg/m ^{3}.

The projection of the full aerodynamic force on the longitudinal axis of the vehicle is called *the force of air resistance* P_v (or the force of frontal resistance). This force is applied in the center of the sail of the vehicle

$$P_\text{v} = c_x \frac{\rho_\text{v} \cdot v^2}{2} F_\text{a} = 0,5 \cdot c_x \cdot \rho_v \cdot F_\text{a} \cdot v^2, \tag{3.17}$$

where c_x is the coefficient of aerodynamic drag (dimensionless);

F_a is the frontal resistance area of the vehicle , m ^{2}.

Introduce the notation k_v = 0.5·s_x·ρ_v. Taking into account the accepted notation, equation (3.17) will take the form

$$P_\text{v} = \text{k}_\text{v} \cdot F_\text{a} \cdot v^2, \tag{3.18}$$

or

$$P_\text{v} = \frac{\text{k}_\text{v} \cdot F_\text{a} \cdot v_\text{a}^2}{3.6^2}, \tag{3.19}$$

where k_v is the air resistance coefficient, $\left[\frac{kg}{m^3}\right]$ or $\left[\frac{N \cdot s^2}{m^4}\right]$.

Let's introduce another notation $W_v = k_v \cdot F_a$ - the vehicle smoothness factor, then

$$P_v = W_v \cdot v^2 \text{ or } P_v = \frac{W_v \cdot v_a^2}{3.6^2}. \quad (3.20)$$

Approximately the frontal area of the vehicle F_a, m^2, can be determined by the equation:

$$F_a = m_F \cdot B_1 \cdot H_a, \quad (3.21)$$

where m_F is the filling factor of the frontal area of the vehicle;

B_1 – track of the front wheels of the vehicle, m;

H is the overall height of the vehicle, m.

The values of the coefficient m_F *should* be taken as follows:

– for trucks with an on-board platform, dump trucks, tanks 0.9;
– passenger vehicle 1.0;
– vans ; 1.05
– buses (coach). 1.1

The value of the coefficient of air resistance in :

– passenger vehicle .... 0.2 ... 0.35 $N{\cdot}s^2/m^4$;
– buses .... 0.35 ... 0.55 $N{\cdot}s^2/m^4$;
– trucks .... 0.5...0.8 $N{\cdot}s^2/m^4$.

The force of air resistance can be represented in the form of components.

Form resistance (50%...60% P_v) – due to the difference between the increased frontal pressure that occurs in front of the vehicle and the reduced pressure caused by the vortices behind it. The shape of such parts of the body as the hood, wings, windshield, roof, side glass, trunk is of decisive importance.

Internal resistance (10%...15% P_v), which is created by air flows that pass inside the vehicle for ventilation or heating of the body, as well as engine cooling.

Surface friction resistance (5%...10% P_v), which is caused by the viscous forces of the boundary layer of air moving near the surface of the vehicle, and depends on the size and roughness of this surface.

Induced resistance (5 %...10 % P_v), which is caused by the interaction of forces acting in the direction of the vertical axis of the vehicle and across the vehicle.

Additional resistance (15% P_v), which is created by various protruding parts: headlights, turn signals, handles, number plates.

3.2.7. Acceleration resistance force P_j

The force of the acceleration resistance is determined by the inertial force of the forwardly moving masses of the vehicle and the inertial forces of the rotating masses

$$P_j = P_j' + P_{j\text{dv}} + P_{j\text{k}}, \qquad (3.22)$$

where P_j' is the force of inertia of translationally moving masses;

$P_{j\,\text{dv}}$ – force of inertia of the rotating masses of the engine and transmission, reduced to the driving wheels;

$R_{j\,\text{k}}$ is the force of inertia of the rotating masses of the wheels of the vehicle.

Figure 3.4 shows the diagram of forces and moments of inertia to determine the resistance to acceleration of the vehicle.

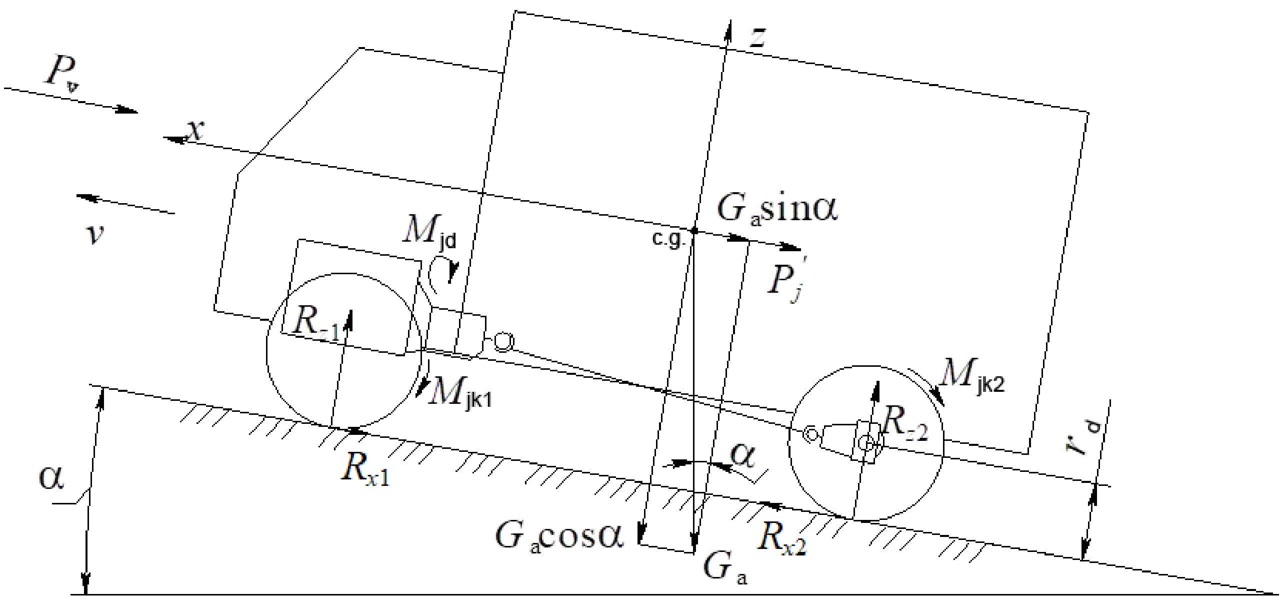


Fig. 3.4. **Scheme for determining the resistance force of acceleration of the vehicle:** M_{jd} - moment of inertia of the rotating parts of the engine; M_{jk1}, M_{jk2} - moments of inertia of the wheels of the front and rear axles vehicle

a) the force of inertia of the forwardly moving masses of the vehicle arises when the vehicle accelerates

$$P'_j = \frac{G_\text{a}}{g} \cdot \frac{dv}{dt}, \quad (3.23)$$

where $\frac{dv}{dt} = j_\text{a}$ – acceleration of the vehicle , m/s ^{2}.

This force is applied in the center of gravity of the vehicle and is directed in the opposite direction of acceleration.

b) inertia forces of rotating masses

– rotating masses of the engine, reduced to the drive wheel:

$$P_{j\text{dv}} = \frac{M_\text{jd} \cdot u_\text{tr} \cdot \eta_\text{tr}}{r_\text{d}} = \frac{J_e \cdot u_\text{tr} \cdot \eta_\text{tr}}{r_\text{d}} \cdot \frac{d\omega_e}{dt} = \frac{J_e \cdot u_\text{tr}^2 \cdot \eta_\text{tr}}{r_\text{d} \cdot r_\text{k}} \cdot \frac{dv}{dt}. \quad (3.24)$$

In the transformation of the equation (3.24) the relation is used

$$\omega_e = \frac{v}{r_\text{k}} \cdot u_\text{tr}.$$

– a rotating wheel

$$P_{j\text{к}} = \frac{M_\text{jk}}{r_\text{d}} = \frac{J_\text{k}}{r_\text{d}} \cdot \frac{d\omega_\text{k}}{dt} = \frac{J_\text{k}}{r_\text{d} \cdot r_\text{k}} \cdot \frac{dv}{dt}. \quad (3.25)$$

In the transformation of the equation (3.25) the relation is used

$$\omega_\text{k} = \frac{v}{r_\text{k}}.$$

After adding the inertial forces of translationally moving masses and the inertial forces of its rotating masses, we get the acceleration resistance force

$$P_j = P'_j + P_{j\text{dv}} + \Sigma P_{j\text{k}} =$$

$$= \frac{G_\text{a}}{g} \cdot \frac{dv}{dt} + \frac{J_e \cdot u_\text{tr}^2 \cdot \eta_\text{tr}}{r_\text{d} \cdot r_\text{k}} \cdot \frac{dv}{dt} + \frac{\Sigma J_\text{k}}{r_\text{d} \cdot r_k} \cdot \frac{dv}{dt} = \left(\frac{G_\text{a}}{g} + \frac{J_e \cdot u_\text{tr}^2 \cdot \eta_\text{tr} + \Sigma J_\text{k}}{r_\text{d} \cdot r_\text{k}} \right) j_\text{a}. \quad (3.26)$$

Let's enter the notation

$$\text{M}_{\text{pr}} = \frac{G_{\text{a}}}{\text{g}} + \frac{J_e \cdot u_{\text{tr}}^2 \cdot \eta_{\text{tr}} + \Sigma J_{\text{k}}}{r_{\text{d}} \cdot r_{\text{k}}}, \tag{3.27}$$

where M_{pr} is the reduced mass of the vehicle.

Reduced mass M_{pr} is a conditional mass used for the traction calculation of the vehicle.

Taking into account the adopted notation (3.27), equation (3.26) will take the form

$$P_j = \text{M}_{\text{pr}} \cdot j_{\text{a}}. \tag{3.28}$$

Let's introduce the concept - the coefficient of consideration of rotating masses

$$\delta_{\text{vr}} = \frac{\text{M}_{\text{pr}}}{m_{\text{a}}} = 1 + \frac{J_e \cdot u_{\text{tr}}^2 \cdot \eta_{\text{tr}} + \Sigma J_{\text{k}}}{G_{\text{a}} \cdot r_{\text{k}} \cdot r_{\text{d}}} g. \tag{3.29}$$

Taking into account the introduced concept, the force of resistance to acceleration of the vehicle is determined by the equation

$$P_j = \frac{G_{\text{a}}}{g} \cdot \delta_{\text{vr}} \cdot j_{\text{a}}. \tag{3.30}$$

The coefficient δ_{vr} – shows how many times the force required for acceleration with a given acceleration of the progressively moving and rotating masses of the vehicle is greater than the force required for accelerating only its progressively moving masses.

Often, in practice, the coefficient of consideration of rotating masses δ_{vr} is determined by an empirical equation:

– for overclocking $\delta_{\text{vr}} = 1 + 0{,}04\left(u_{\text{k}}^2\right)\frac{G_{\text{a}}}{G}$; $\delta_{\text{vr}} = 1{,}04 + 0{,}04 u_{\text{k}}^2$;

– for rolling $\delta_{\text{vr}} = 1 + 0{,}04\frac{G_{\text{a}}}{G}$,

where u_{k} is the transmission ratio of the gear box;

G_{a} is the force of gravity of the full mass vehicle;

G is the weight of the vehicle.

3.3. A change in normal reactions during the movement of the vehicle

The normal reactions of the road acting on the vehicle wheels do not remain constant. They change depending on the forces and moments acting on the vehicle. To determine the dynamic values of the normal reactions on the axles of the vehicle, we will use the diagram shown in Figure 3.5.

After projecting all the forces on the supporting surface of the road, we get

$$R_{x2} - R_{x1} - P_{\text{v}} - P_{\alpha} - P_j^{'} = 0\,. \tag{3.31}$$

To determine the normal reaction on the front axle, let's make the equation of moments relative to the contact zone of the wheels of the rear axle

$$\Sigma M = 0; \quad G_{\text{a}} \cdot \cos\alpha \cdot b - \left(P_j' + P_{\alpha}\right) \cdot h_g - P_{\text{v}} \cdot h_{\text{v}} - R_{z1} \cdot L = 0\,. \tag{3.32}$$

Let us assume that $h_g \approx h_{\text{in}}$, then we will rewrite the dependence (3.32) in the form

$$G_{\text{a}} \cdot \cos\alpha \cdot b - \left(P_j^{'} + P_{\alpha} + P_{\text{v}}\right) \cdot h_g - R_{z1} \cdot L = 0\,. \tag{3.33}$$

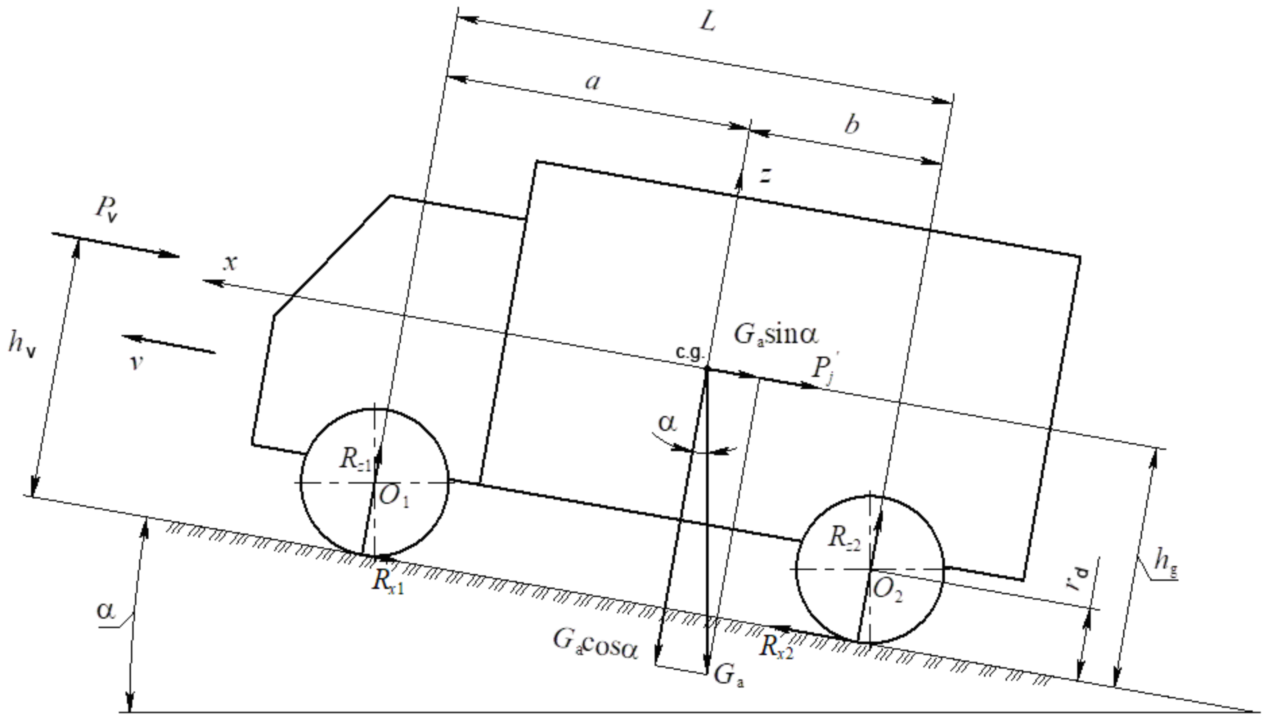


Fig. 3.5. **Scheme for determining normal reactions on vehicle axles**

From equation (3.33) we get

$$R_{z1} = G_{\text{a}} \cdot \cos\alpha \frac{b}{L} - \left(P_j^{'} + P_{\alpha} + P_{\text{v}}\right) \cdot \frac{h_g}{L}. \quad (3.34)$$

Taking into account equation (3.31), let's transform (3.34) into the form

$$R_{z1} = G_{\text{a}} \cdot \cos\alpha \frac{b}{L} - \left(R_{x2} - R_{x1}\right) \cdot \frac{h_g}{L}. \quad (3.35)$$

write down the values of R_{x2} and R_{x1}, while assuming that the coefficients of rolling resistance on both axes are the same and equal to f

$$R_{x1} = R_{z1} \cdot f + \frac{J_{\text{k1}}}{r_{\text{d}} \cdot r_{\text{k}}} \cdot \frac{dv}{dt}; \quad (3.36)$$

$$R_{x2} = \frac{M_{\text{k}}}{r_{\text{d}}} - R_{z2} \cdot f - \frac{J_{\text{k2}}}{r_{\text{d}} \cdot r_{\text{k}}} \cdot \frac{dv}{dt} - \frac{J_{\text{e}} \cdot u_{\text{tr}}^2 \cdot \eta_{\text{tr}}}{r_{\text{d}} \cdot r_{\text{k}}} \cdot \frac{dv}{dt}. \quad (3.37)$$

Substitute the values of R_{x2} and R_{x1} into the equation (3.35) and, taking into account that $R_{z1} + R_{z2} = G_{\text{a}} \cdot \cos\alpha$, convert it to the form

$$R_{z1} = G_{\text{a}} \cdot \cos\alpha \frac{b}{L} - \begin{pmatrix} \frac{M_{\text{k}}}{r_{\text{d}}} - G_{\text{a}} \cdot f - \frac{J_{\text{k2}}}{r_{\text{d}} \cdot r_{\text{k}}} \cdot \frac{dv}{dt} - \\ - \frac{J_{\text{k1}}}{r_{\text{d}} \cdot r_{\text{k}}} \cdot \frac{dv}{dt} - \frac{J_{\text{e}} \cdot u_{\text{tr}}^2 \cdot \eta_{\text{tr}}}{r_{\text{d}} \cdot r_{\text{k}}} \cdot \frac{dv}{dt} \end{pmatrix} \cdot \frac{h_g}{L} =$$

$$= G_{\text{a}} \cdot \cos\alpha \frac{b}{L} + G_{\text{a}} \cdot \cos\alpha \cdot f \frac{h_g}{L} - \frac{M_{\text{k}}}{r_{\text{d}}} \cdot \frac{h_g}{L} + \frac{\Sigma J_{\text{k}} + J_{\text{e}} \cdot u_{\text{tr}}^2 \cdot \eta_{\text{tr}}}{r_{\text{d}} \cdot r_{\text{k}}} \cdot \frac{h_g}{L} \cdot j_{\text{a}},$$

we will finally get

$$R_{z1} = G_{\text{a}} \cdot \cos\alpha \frac{b + f \cdot h_g}{L} - \frac{M_{\text{k}}}{r_{\text{d}}} \cdot \frac{h_g}{L} + \frac{\Sigma J_{\text{k}} + J_{\text{e}} \cdot u_{\text{tr}}^2 \cdot \eta_{\text{tr}}}{r_{\text{d}} \cdot r_{\text{k}}} \cdot \frac{h_g}{L} \cdot j_{\text{a}}. \quad (3.38)$$

To determine the normal reaction on the rear axle, let's make the moment equation relative to the contact zone of the wheels of the front axle

$$\Sigma M = 0; \quad G_{\text{a}} \cdot \cos\alpha \cdot a + \left(P'_j + P_\alpha\right) \cdot h_g + P_{\text{v}} \cdot h_{\text{v}} - R_{z1} \cdot L = 0. \quad (3.39)$$

Using similar transformations outlined in determining the normal reaction on the front axle, we obtain the equation that determines the normal reaction on the rear axle:

$$R_{z2} = G_{\text{a}} \cdot \cos\alpha \frac{a - f \cdot h_g}{L} + \frac{M_{\text{k}}}{r_{\text{d}}} \cdot \frac{h_g}{L} - \frac{\Sigma J_{\text{k}} + J_{\text{e}} \cdot u_{\text{tr}}^2 \cdot \eta_{\text{tr}}}{r_{\text{d}} \cdot r_{\text{k}}} \cdot \frac{h_g}{L} \cdot j_{\text{a}}. \quad (3.40)$$

With uniform movement on a horizontal road, the normal reactions on the axles of the vehicle are correspondingly equal:

$$R_{z1} = G_{\text{a}} \cdot \frac{b + f \cdot h_g}{L} - \frac{M_{\text{k}}}{r_{\text{d}}} \cdot \frac{h_g}{L}; \quad (3.41)$$

$$R_{z2} = G_{\text{a}} \cdot \frac{a - f \cdot h_g}{L} + \frac{M_{\text{k}}}{r_{\text{d}}} \cdot \frac{h_g}{L}. \quad (3.42)$$

With a stationary vehicle on a horizontal road, the normal reactions on the axles of the vehicle are correspondingly equal:

$$R_{z1} = G_{\text{a}} \frac{b}{L} = G_1; \quad (3.43)$$

$$R_{z2} = G_{\text{a}} \frac{a}{L} = G_2, \quad (3.44)$$

where G_1 and G_2 are the weight that falls on the front and rear axles in a static state.

As the torque increases, the normal reaction on the front axle decreases, and on the rear axle it increases by the same amount.

The change in reactions R_{z1} and R_{z2} during movement compared to the load falling on the axle in a static state G_1 and G_2 are estimated by the coefficient of change of reactions.

Coefficient of reaction change (load redistribution) is the ratio of the normal reaction when moving to the reaction acting on the same axle when the vehicle is stationary on a horizontal road.

Accordingly, for the front and rear axles:

$$m_{z1} = \frac{R_{z1}}{G_1};\ m_{z2} = \frac{R_{z2}}{G_2}. \tag{3.45}$$

For completed vehicle designs, the coefficient of change of reactions has the following values:

– during acceleration m_{z1} = 0.65...0.8; m_{z2} = 1.1...1.3;
– during braking m_{z1} = 1.25...1.4; m_{z2} = 0.6...0.75.

Control questions

1. What forces act on the vehicle during its movement?
2. How does the total traction and speed of the vehicle depend on the engine parameters?
3. Determine the forces acting on the vehicle as it moves.
4. What is the coefficient of consideration of the rotating masses of the vehicle?
5. What is the curb weight of a vehicle?
6. Determine the normal reactions on the axles of the vehicle during its movement.
7. What is the coefficient of variation of the normal reactions on the axles of the vehicle?

TOPIC 4

TRACTION DYNAMICS OF THE VEHICLE AND TRACTION AND SPEED PROPERTIES OF THE VEHICLE

4.1. Gauges and indicators of traction and speed properties of the vehicle

Traction-speed properties (TSP) are the properties of the vehicle that determine the ranges of changes in driving speeds and maximum accelerations during acceleration in traction mode.

Traction mode is the vehicle's driving mode, in which the power and torque required for movement are supplied from the engine to the drive wheels through the transmission.

The methods of evaluating traction and speed properties can be used when solving problems:

– *analysis:* determination of speeds, accelerations and extreme road conditions in which the vehicle can move with given design parameters *(verified traction calculation of the vehicle);*

- *synthesis:* determination of design parameters that can ensure the given values of speeds and accelerations in given road conditions, as well as finding the limit road conditions *(design traction calculation of the vehicle).*

A unit of measurement that characterizes this property qualitatively (for example, the speed of a vehicle) is called a meter of an operational property.

The indicator of the operational property is the number that determines the value of the meter of this property, its quantity (for example, the value of the maximum speed).

The following meters are the most widely used and sufficient for the comparative evaluation of traction and speed properties of vehicle:

1) maximum speed;*

2) conditional maximum speed;

3) acceleration time on the 400 and 1000 m course;

4) acceleration time to the set speed;*

5) speed characteristic of acceleration - coasting;*

6) speed characteristics of acceleration in a higher gear;

7) speed characteristics on a road with a variable longitudinal profile;

8) minimum stable speed;*

9) the maximum climb that can be overcome;*

10) constant speed on long climbs;

11) acceleration during acceleration in traction mode;*

12) traction force on the hook;

13) the length of the dynamically overcome ascent.

*) The most used gauges for evaluating the traction and speed properties of a vehicle in the training process.

4.2. The equation of motion of a vehicle

The equation of motion of the vehicle is fundamental in traction dynamics. It connects the forces that move the vehicle with the forces of resistance and allows you to determine the nature of the vehicle movement at each moment of time. When studying the dynamics of the vehicle, it is believed that its capabilities are limited only by the power of the engine and the traction of the tires of the driving wheels with the road.

Consider the diagram of the forces acting on the vehicle during its uneven movement on the uphill (Fig. 4.1).

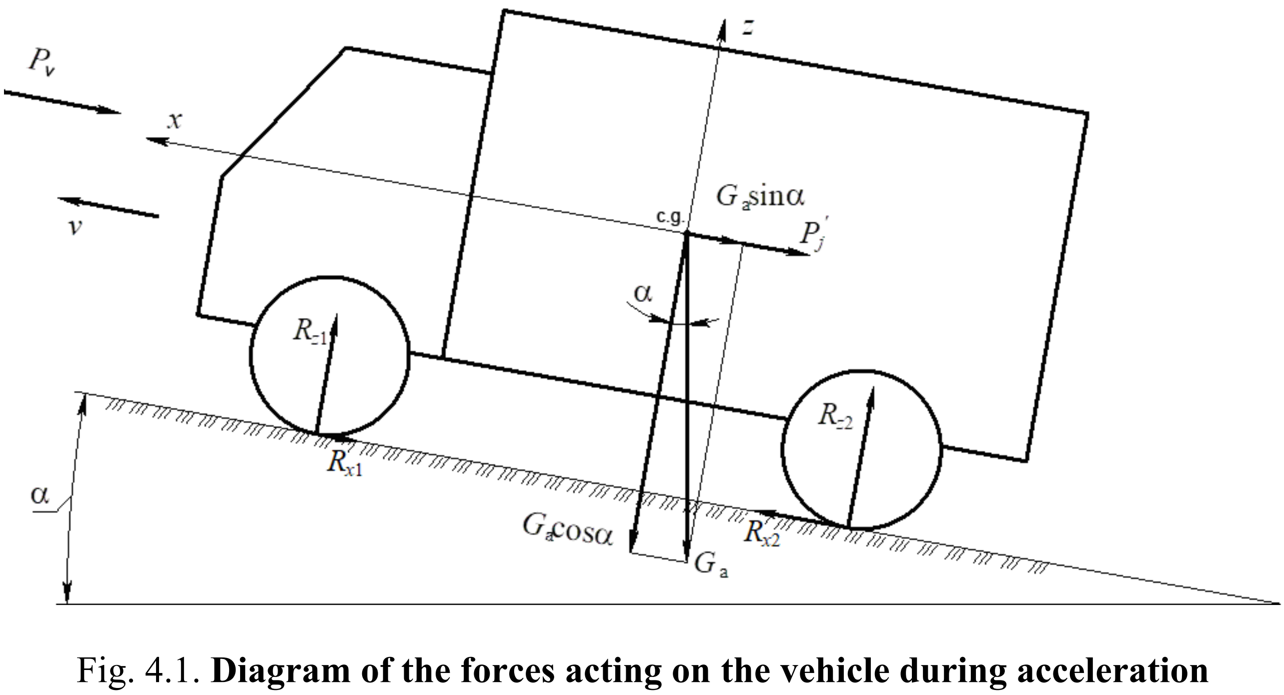


Fig. 4.1. **Diagram of the forces acting on the vehicle during acceleration on the uphill of the road**

At the center of gravity of the vehicle, the force of gravity G_a, the force of inertia of the forward moving masses $P_j^{'}$, and the force of lifting resistance P_α are applied.

We project all forces onto the road plane

$$R_{x2} - R_{x1} - P_\alpha - P_v - P_j^{'} = 0, \tag{4.1}$$

where $R_{x2} = \frac{M_k}{r_d} - R_{z2} \cdot f - \frac{J_{k2}}{r_d r_k} j_a - \frac{J_e \eta_{tr} u_{tr}^2}{r_d r_k} j_a$;

$R_{x1} = R_{z1} f + \frac{J_{k1}}{r_d r_k} j_a$;

$P_j^{'} = \frac{G_a}{g} j_a$.

Let's open equation (4.1)

$$\frac{M_k}{r_d} - R_{z2} f - \frac{J_{k2}}{r_d r_k} j_a - \frac{J_e \eta_{tr} u_{tr}^2}{r_d r_k} j_a - R_{z1} f - \frac{J_{k1}}{r_d r_k} j_a - P_\alpha - P_v - \frac{G_a}{g} j_a = 0,$$

taking into account (3.4), (3.12), (3.18), (3.26) and (3.30) we obtain

$$P_k - P_\psi - P_v - P_j = 0 \tag{4.2}$$

It should be noted two conventions adopted when deriving the equation of motion. ***First,*** the force P_k refers to the steady motion of the vehicle, and the equation is valid for the general case of motion. It would be more correct when analyzing acceleration to reduce the value of P_k in accordance with the power consumption for changing the energy of the rotating parts, and not to conditionally change the mass of the vehicle by introducing the coefficient δ_{vr}. Such a method is admissible only because in most cases it does not affect the results of calculations, and experimental dependences of the force P_k on speed are known only for stable modes of motion. ***Secondly***, the rolling resistance force P_f is included in the road resistance force P_ψ, as an external movement resistance force. When moving on roads with a hard surface, the main part of the energy spent on rolling is caused by internal friction in the tires, and the force P_f can be called external only conditionally.

The equation of motion allows you to determine the possibility and nature of the vehicle movement in given road conditions.

Continuous movement is possible under the condition

$$P_k \geq P_\psi + P_v. \tag{4.3}$$

This inequality connects the design factors of the vehicle with the factors that cause resistance to movement. Its execution is necessary, but not sufficient for the continuous movement of the vehicle, since the latter is possible only in the absence of skidding of the driving wheels. Given this, continuous movement is possible if the condition is met

$$R_{z2}\varphi_x = P_\varphi \geq P_k \geq P_\psi + P_v. \tag{4.4}$$

The condition of uniform movement of the vehicle

$$P_\varphi \geq P_k = P_\psi + P_v. \tag{4.5}$$

If you increase the force P_k compared to the sum $P_\psi + P_v$, then this will lead to acceleration of the vehicle. However, this will continue until the traction force is equal to the adhesion utilized force P_φ. A further increase in P_k will cause only accelerated wheel slip without changing the vehicle movement parameters.

4.3. Traction (power) balance of the vehicle

If the equation of motion of the vehicle (4.5) is written in the form of equation (4.6), then we get the equation of the force balance of the vehicle

$$P_k = P_v + P_\psi + P_j. \tag{4.6}$$

This dependence is used in the design of new and evaluation of traction and speed properties of existing vehicle.

When designing new vehicle in accordance with the requirements, which traction and speed properties of the vehicle determine:

– required engine parameters M_e, N_e, n_e;

– gear ratios of the transmission u_k, u_0.

When evaluating the traction and speed properties of existing vehicle, the following are determined:

– maximum speed v_{max};

– acceleration j_a;

– overcoming uphill i;

- other traction and speed indicators.

In this case, you need to know the road conditions, the main structural parameters of the vehicle and its traction-speed characteristics.

The traction-speed characteristic of the vehicle is the dependence of the total traction force on the driving wheels on the speed of the vehicle in all gears of the transmission $P_k = f(v)$. This characteristic represents the ultimate traction-speed characteristics of the vehicle and is calculated according to the parameters of the external speed characteristic of the engine.

The analysis of traction-speed properties is conveniently performed on the basis of the graphical solution of equation (4.6) of the vehicle power balance. For this equation of the power balance, we will present it in the form of a graph of the traction-speed characteristic of the vehicle, on which we plot the speed of the vehicle v on the abscissa axis (Fig. 4.2) , and on the ordinate axis the total traction force on the drive wheels R_k on all gears of the transmission. Acceleration of the vehicle from a standstill to speed $v_{a\,min}$ occurs with clutch slippage. This section is small and is not taken into account in the analysis of traction and speed properties.

P_ψ is also plotted on the graph . The value of the air resistance force P_v the graph is plotted from the curve P_ψ, obtaining the curve of the total values of the forces $P_v + P_\psi$.

Ordinate of any point of the curves P_{k1}, P_{k2}, P_{k3} characterizes the magnitude of the traction force on the driving wheels at a given speed of movement in the corresponding gears. For example, the traction force R_k (see Fig. 4.2) when moving in third gear at a speed v_1. At the same time, the ordinate P_ψ characterizes the force of road resistance at this speed, and the ordinate $P_v + P_\psi$ is the force of the total resistance of the road and air. The segment between the ordinates P_{k3} and $P_v + P_\psi$ represents the remainder of the traction force on the drive wheels at the speed v_1. This balance is called *the traction power reserve R_z.*

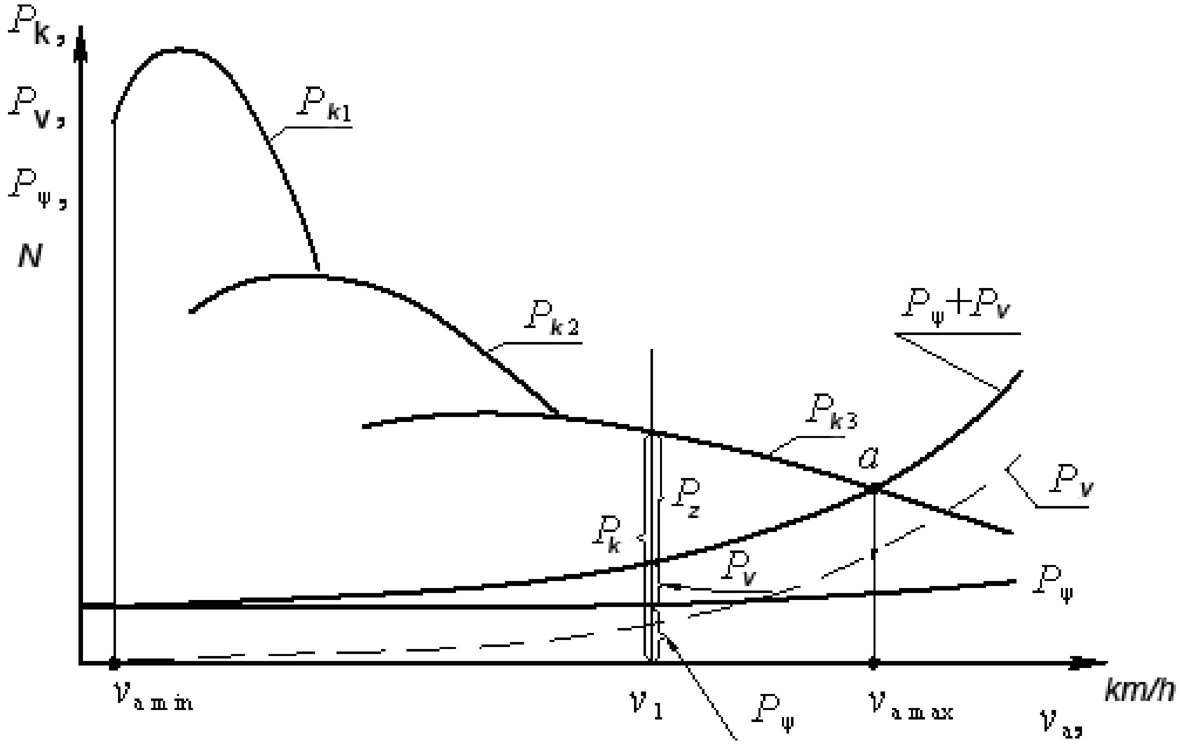


Fig. 4.2. **Graph of traction (power) balance of the vehicle**

It can be spent on accelerating the vehicle, towing a trailer, or overcoming additional resistance to movement at a speed of v_1. As can be seen from the graph, the margin of traction in lower gears is always greater than in higher ones. That is why movement in difficult road conditions is carried out in lower gears.

With the help of the power balance graph, you can solve various tasks related to the study of the traction and speed properties of the vehicle. Example:

– *determination of the maximum speed* v_{max}. The maximum speed v_{max} of the vehicle is determined by the intersection point *a* of the curve of the total traction force P_k in the highest gear and the curve of the sum of the resistance forces $P_v + P_\psi$ (Fig. 4.2) . At this point, the traction reserve and vehicle acceleration j_a are equal to zero. The speed of its movement is maximum, since its further increase is impossible;

- *determining the traction force on the driving wheels of the vehicle when moving at a uniform speed* . To drive the vehicle at a uniform speed, lower than the maximum speed, it is necessary to reduce the engine power. For this, the fuel supply level is reduced and the engine will operate at a partial speed characteristic. On the graph of the traction balance (Fig. 4.3), the traction characteristics of a vehicle with partial fuel supply are depicted by dashed lines. At the same time, the traction force on the drive wheels when moving in the corresponding gears is marked P^{α}_{k1}, P^{α}_{k2}, P^{α}_{k3}. It should be noted that uniform movement of the vehicle at a speed of v_1 is possible both in the second and third gears. At the same time, the traction force on the driving wheels in both gears will have the same value, equal to the sum of the road resistance and air resistance

$$P^{\alpha}_{k2} = P^{\alpha}_{k3} = P_k = P_{\psi} + P_v .$$

The movement of the vehicle at a speed of v_1 (Fig. 4.3) in the first gear is impossible because even at the maximum frequency of rotation of the engine shaft, the indicated speed of the vehicle is not reached;

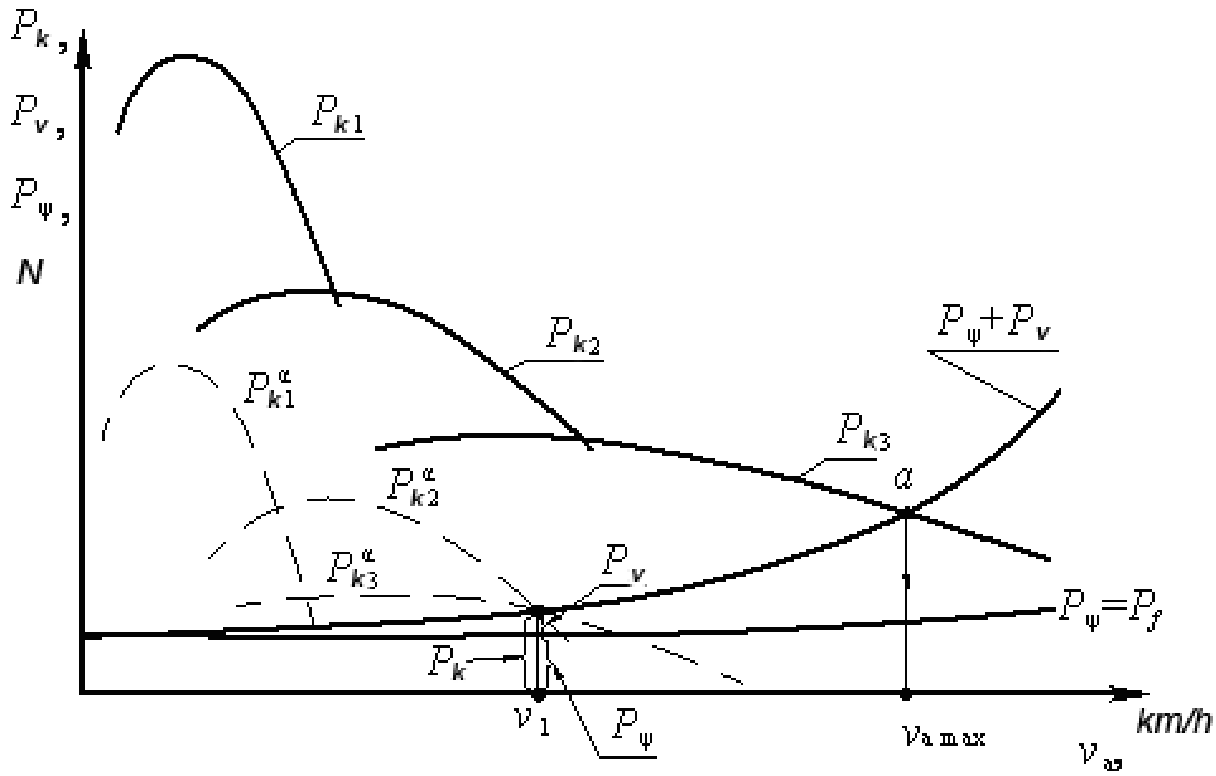


Fig. 4.3. **Determination of traction force on the driving wheels of a vehicle when moving at a uniform speed**

– *determination of the maximum resistance force of the road* $P_{\psi\,max}$ (see Fig. 4.4) . Usually, overcoming the maximum resistance force of the road occurs in a reduced gear at a low speed of the vehicle. At the same time, the force of air resistance is too small and therefore it can be assumed to be zero (see Fig. 4.4).

$$P_{\psi\max} = P_{k\max} - P_{v} \approx P_{k\max}; \quad (4.7)$$

– *definition of the maximum overcome uphill i* $_{\max}$. The value of the maximum rise that the vehicle will overcome is calculated according to the value of the maximum resistance force of the road (Fig. 4.4).

$$P_{\psi\max} = P_{\alpha\max} + P_f = P_{k\max} - P_v \rightarrow P_{\alpha\max} = P_{k\max} - \left(P_f + P_v\right),$$

revealing the lift resistance force $P_{\alpha\max}$, after transformations we get the value of the maximum overcome uphill $i_{\max}$

$$G_a \cdot i_{\max} = P_{k\max} - \left(P_f + P_v\right) \rightarrow i_{\max} = \frac{P_{k\max} - \left(P_f + P_v\right)}{G_a}. \quad (4.8)$$

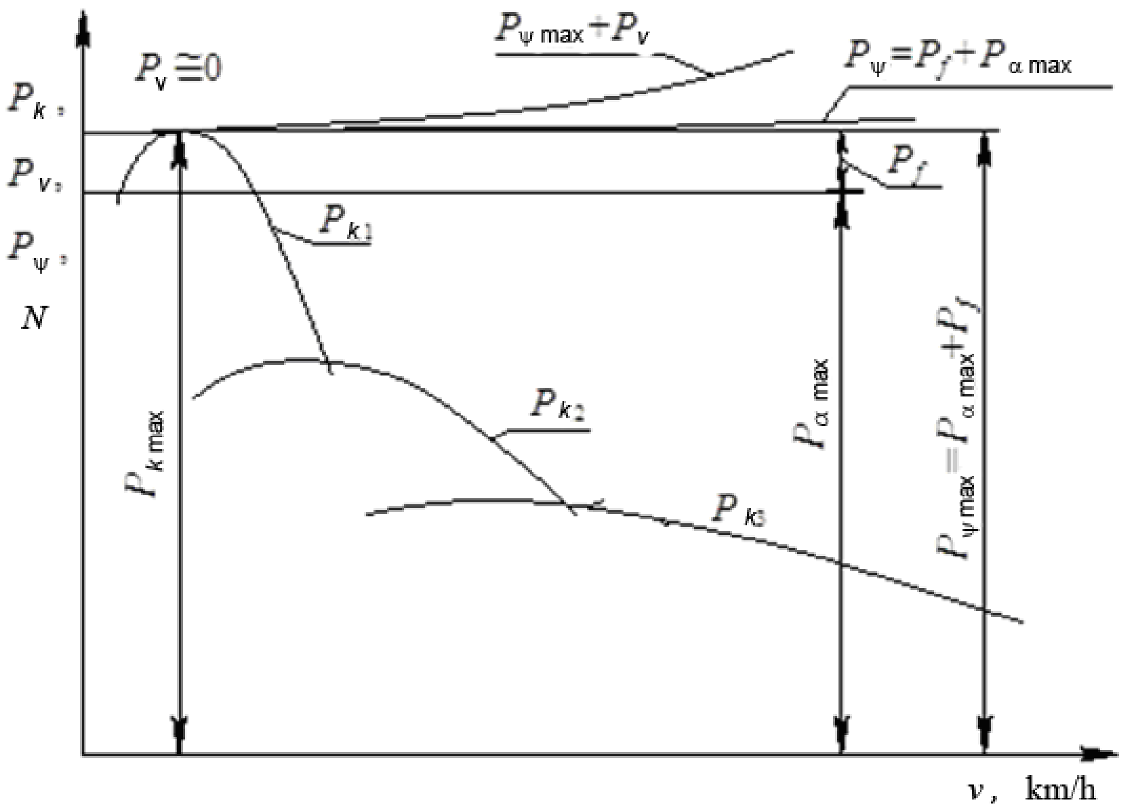


Fig. 4.4. **Determination of the maximum resistance force of the road, which the vehicle will overcome**

The maximum road resistance force overcome by a vehicle moving uniformly at a given speed v_1 is defined as the difference between the total traction force P_k and the air resistance force P_v at this speed (see Fig. 4.5).

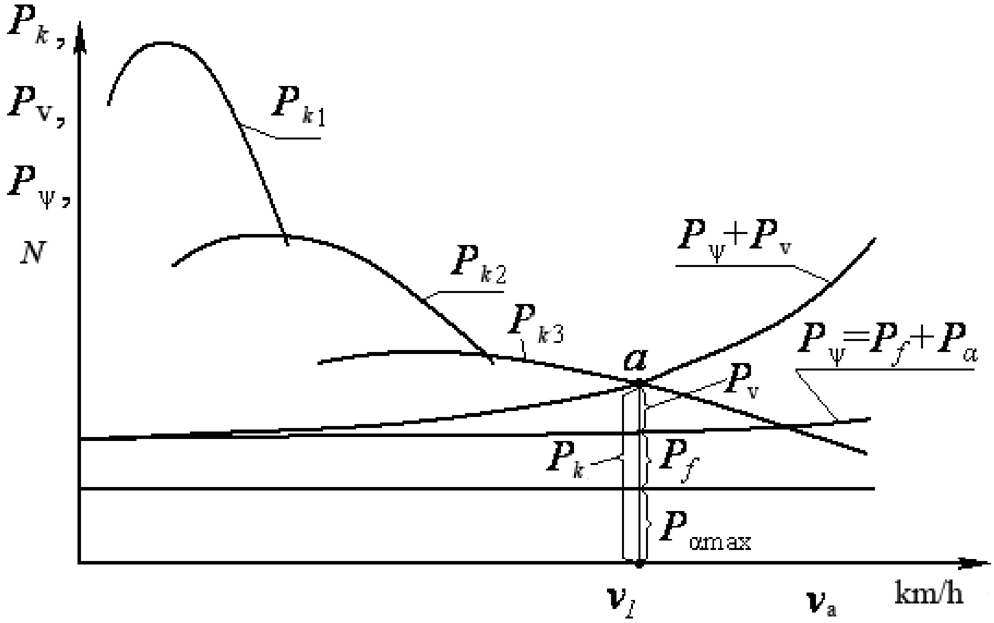


Fig. 4.5. **Determination of the maximum resistance force of the road when moving at a speed of** v_1

The value of the maximum uphill i_{max}, which the vehicle will be able to overcome, moving at a speed of v_1, is calculated according to the equation (4.8), replacing $P_{k\,max}$ with P_k.

- *determination of the maximum possible acceleration of the* j_{amax} *vehicle at a given speed*. The value of the maximum possible acceleration of the vehicle at a speed of v_2 is calculated from the value of the traction power reserve P_z, which is determined according to the graph (Fig. 4.6).

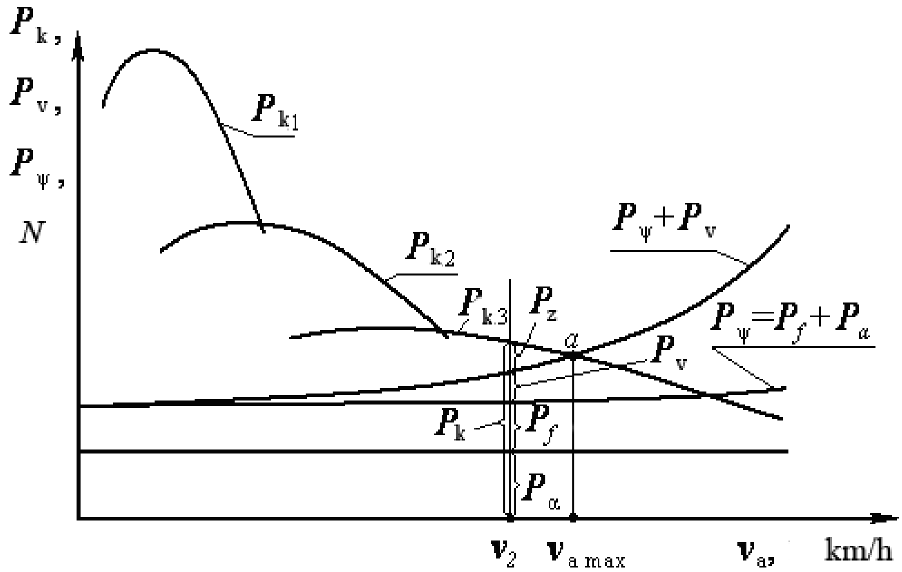


Fig. 4.6 . **Determination of the maximum possible acceleration of a vehicle moving at a speed of** v_2

At the same time, it is considered that the entire reserve of traction force is used to accelerate the vehicle, i.e. $P_z = P_j$.

$$P_j = P_k - \left(P_\psi + P_v\right) = P_z;$$

$$\frac{G_a}{g}\delta_{vr} \cdot j_a = P_z \rightarrow j_{a\,max} = \frac{P_z}{G_a \cdot \delta_{vr}} g. \quad (4.9)$$

- determination of the possibility of skidding of the driving wheels . If it is necessary to determine the possibility of skidding of the driving wheels during steady motion of the vehicle, the dependence of the adhesion utilized force P_φ of the driving wheels on the speed is plotted on the graph of the traction balance (Fig. 4.7).

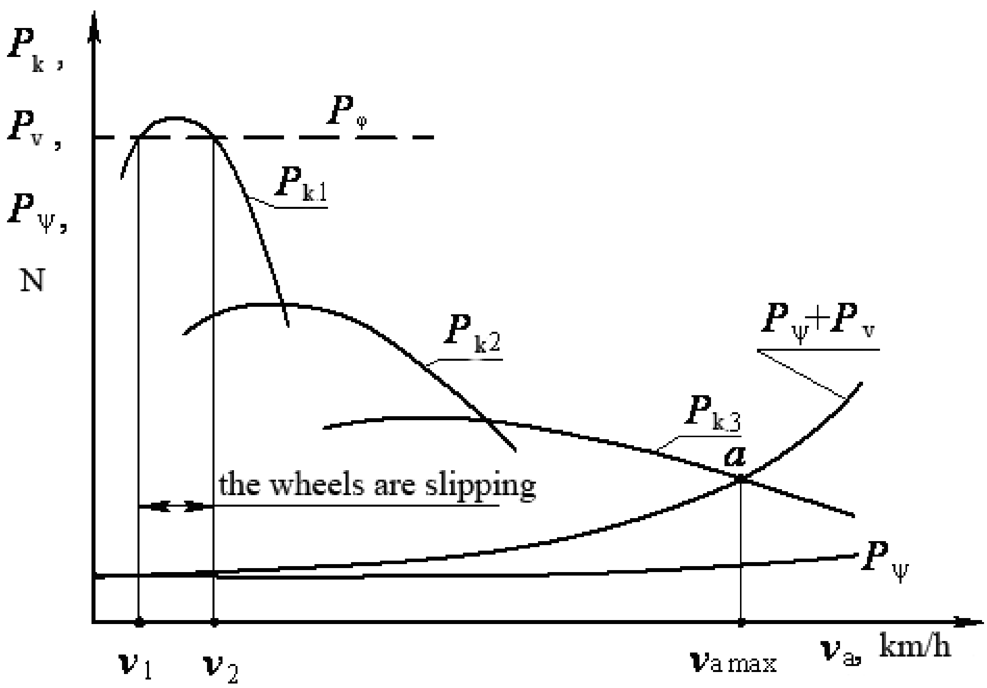


Fig. 4.7. **Determination of skidding of the driving wheels of the vehicle**

Determination of the possibility of skidding of the driving wheels is carried out according to the conditions

$$P_\varphi < P_k \text{ – wheels skid;} \quad (4.10)$$

$$P_\varphi \geq P_k \text{ – wheels do not skid.} \quad (4.11)$$

As can be seen from Figure 4.7, the vehicle moves without skidding of the driving wheels at a speed less than v_1. But in the section from v_1 to v_2, the force P_k exceeds the force P_φ, so the wheels slip, which determines the limitation of the vehicle passability. In this case, it is rational to reduce the fuel supply so as to reduce the traction force P_k to the level of the clutch force P_φ. If constant movement occurs at a speed of v_2 or more, there is no wheel slip even with full fuel supply.

The vehicle power balance method is convenient for analyzing the dynamics of a particular vehicle, but it is inconvenient to compare the dynamics of different vehicle (for example, truck and passenger car), since their weight and total traction force are significantly different. To analyze the dynamics of different vehicle, it is convenient to use the dynamic factor and the dynamic characteristic of the vehicle.

4.4. Dynamic factor and dynamic characteristics of the vehicle

In the force balance equation, we move the air resistance force P_v to the left part, and we write the road resistance force P_ψ and acceleration P_j in expanded form

$$P_k = P_v + P_\psi + P_j; \tag{4.12}$$

$$P_k - P_v = P_\psi + P_j; \tag{4.13}$$

$$P_k - P_v = G_a \cdot \psi + \frac{G_a}{g} \cdot \delta_{vr} \cdot j_a; \tag{4.14}$$

$$\frac{P_k - P_v}{G_a} = \psi + \frac{\delta_{vr}}{g} \cdot j_a. \tag{4.15}$$

In the theory of the vehicle, the notation is used D – dynamic factor of the vehicle.

$$D = \frac{P_k - P_v}{G_a}.$$

The *dynamic factor of the vehicle* is the ratio of the total traction force on the driving wheels minus the force of air resistance to the force of gravity of the vehicle and is a dimensionless quantity that depends only on the design parameters of the vehicle and determines the quality of its construction from the point of view of its dynamics.

The dynamic characteristic of the vehicle is a graphic representation of the dynamic factor depending on the speed of the vehicle at different gears in the box (Fig. 4.8).

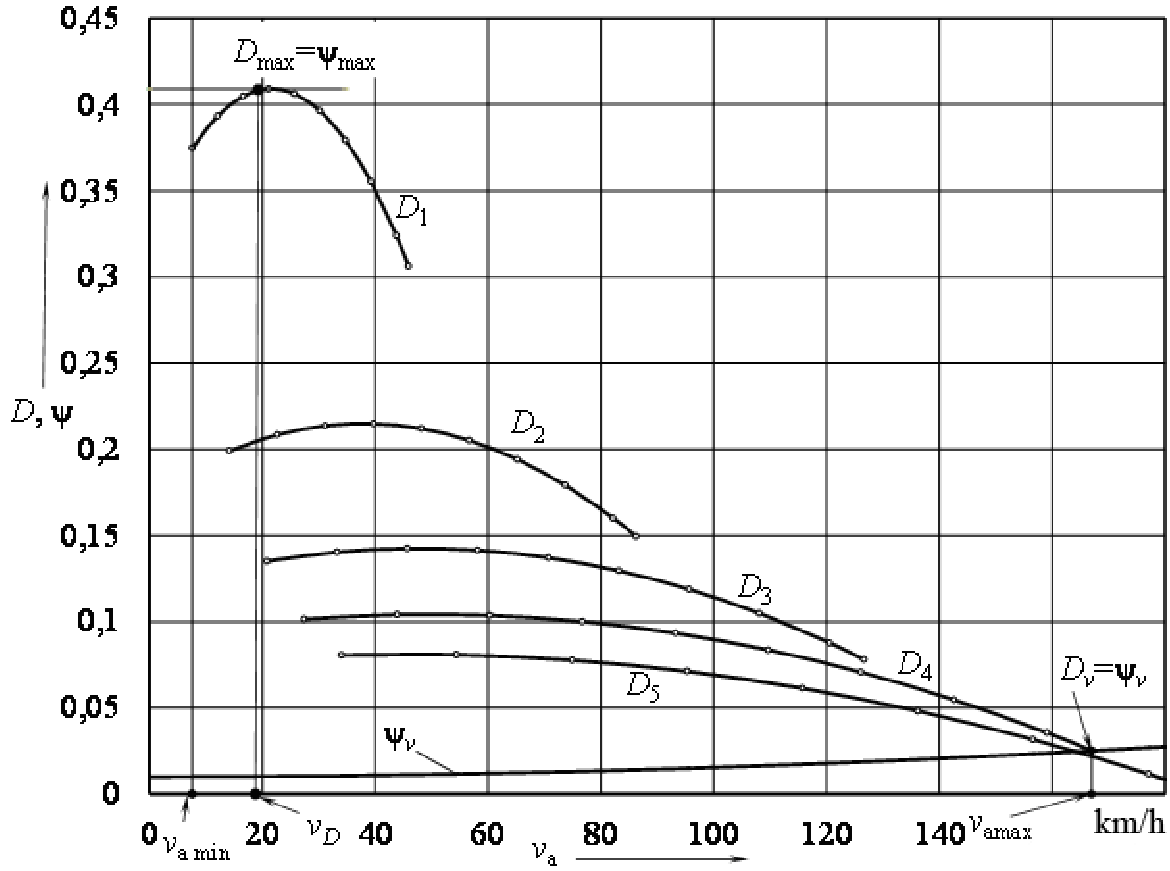


Fig. 4.8. **Dynamic characteristics of the vehicle**

Let's write down the dynamic factor of the vehicle in the form

$$D = \psi + \frac{\delta_{vr}}{g} \cdot j_a . \qquad (4.16)$$

At a constant speed v_a = const $\rightarrow j_a = 0$ and, accordingly, $D = \psi$. Therefore, any point of the *D* curves on the graph corresponds to the total road resistance coefficient during uniform movement.

It should be noted that Figure 4.8 shows:

– at $v_{max} \to D_v = \psi_v$;

– at $v_D \to D_{max} = \psi_{max}$,

where D_v is the dynamic factor of the vehicle at v_{max};

ψ_v is the coefficient of road resistance that can be overcome by a vehicle at v_{max};

D_{max} – the maximum dynamic factor of the vehicle;

ψ_{max} – the maximum coefficient of road resistance that the vehicle can overcome;

v_D is the critical speed of the vehicle under traction conditions.

The critical speed of the vehicle under the traction condition v_D exists in each gear, respectively, at the maximum dynamic factor on it. If the speed of the vehicle in a given gear is greater than the corresponding critical speed, the engine has the opportunity to adapt to the increase in load. Ago:

– when $v_a > v_D$ – the vehicle movement is stable;

– when $v_a < v_D$ – vehicle movement is possible, but unstable.

Dynamic factor of adhesion utilized. The dynamic adhesion utilized factor is determined similarly to the dynamic traction factor, replacing the full traction force on the driving wheels with the traction force of the driving wheels

$$D_\varphi = \frac{P_\varphi - P_v}{G_a}. \quad (4.17)$$

Considering that when towing the vehicle P_v is insignificant, then

$$D_\varphi \simeq \frac{P_\varphi}{G_a} = \frac{G_\varphi \cdot \varphi_x}{G_a}, \quad (4.18)$$

where P_φ is the adhesion utilized force of the driving wheels;

G_φ is the weight falling on the drive wheels.

For a four-wheel drive vehicle

$$D_\varphi = \frac{G_a \cdot \varphi_x}{G_a} = \varphi_x. \quad (4.19)$$

The condition of continuous movement

$$D \geq \psi . \tag{4.20}$$

The condition of continuous movement of the vehicle without wheel slippage

$$D_{\varphi} \geq D \geq \psi . \tag{4.21}$$

With the help of dynamic characteristics, you can solve real tasks related to the traction properties of the vehicle:

1. Determination of v_{max} at a given ψ and the reverse problem: determination of ψ_{max} at a given speed v (see Fig. 4.8).

2. Determination of the maximum uphill i_{max}, which can be overcome by a vehicle in any gear with a given rolling resistance f.

The approximate value of the maximum uphill $i_{max} = \text{tg}\,\alpha_{max}$ can be determined if it is possible to accept the assumption that $\cos\alpha_{max} \approx 1$ and $\sin\alpha_{max} \approx \text{tg}\,\alpha_{max}$, that is, when $i_{max} < 0{,}2$ ($\alpha_{max} < 12°$)

$$D_{max} = \psi_{max} = f \cdot \cos\alpha_{max} + \sin\alpha_{max} \Rightarrow D_{max} \approx f + \sin\alpha_{max} . \tag{4.22}$$

From here we get

$$\sin\alpha_{max} \simeq tg\alpha_{max} \simeq i_{max} = D_{max} - f . \tag{4.23}$$

elevation angle α_{max} is obtained after transformations of condition (4.22)

$$\alpha_{max} = \arcsin\frac{D_{max}}{\sqrt{f^2+1}} - \text{arctg}\, f . \tag{4.24}$$

The transformation of condition (4.22) is given in paragraph 10.3.1.

3. Determination of the maximum acceleration j_{max} that the vehicle can develop on a given section of the road in any gear

$$D_{max} = \psi + \frac{\delta_{vr}}{g} j_{max} \Rightarrow j_{max} = \frac{D_{max} - \psi}{\delta_{vr}} \cdot g . \tag{4.25}$$

4. Determination of the area of steady motion of the vehicle without wheel slippage under the condition (4.26). This is described in more detail in subsection 4.6.

$$D_{\varphi} \geq D. \tag{4.26}$$

4.5. Influence of weight (load) on traction and speed properties

The analysis is carried out with the help of a graph of dynamic characteristics, supplemented by a nomogram of the vehicle load. Let's determine the ratio of dynamic factors of loaded and unloaded vehicle

$$D = \frac{P_k - P_v}{G} \left| \begin{array}{l} D_{100} = \dfrac{P_k - P_v}{G_{100}} \\ D_0 = \dfrac{P_k - P_v}{G_0} \end{array} \right| \begin{array}{l} P_k - P_v = D_{100} \cdot G_{100} \\ P_k - P_v = D_0 \cdot G_0 \end{array} \left| \begin{array}{l} D_{100} \cdot G_{100} = D_0 \cdot G_0 \\ D_0 = \dfrac{D_{100} \cdot G_{100}}{G_0} \end{array} \right., \tag{4.27}$$

where D_0, D_{100} is the dynamic factor of the vehicle without load and with full load, respectively (load 0% and 100%);

G_0, G_{100} – the weight of the vehicle, respectively, without load and with full load (load 0% and 100%).

So, for the analysis on the dynamic characteristic graph, you need to plot another ordinate D_0 on the appropriate scale

$$m_{D_0} \cdot G_0 = m_{D_{100}} \cdot G_{100} \Rightarrow m_{D_0} = m_{D_{100}} \frac{G_{100}}{G_0}. \tag{4.28}$$

Figure 4.9 shows an example of the dynamic characteristics of a vehicle taking into account the load. The dyne ordinate of the power factor of the vehicle without load D_0 is built at some distance to the left of the ordinate D_{100}. The value of the dynamic factor of the vehicle without load D_0 is postponed, taking into account the scale m_{D0}.

For example, if the ratio $D_{100} / D_0 = 1.43$, and the scale $m_{D100} = 0.007$ mm^{-1}, then $m_{D0} = 1.43 \cdot m_{D100} = 0.01$, that is, the segment 0–0.1 on the ordinate D_{100} is 1.43 times more than on the ordinate D_0.

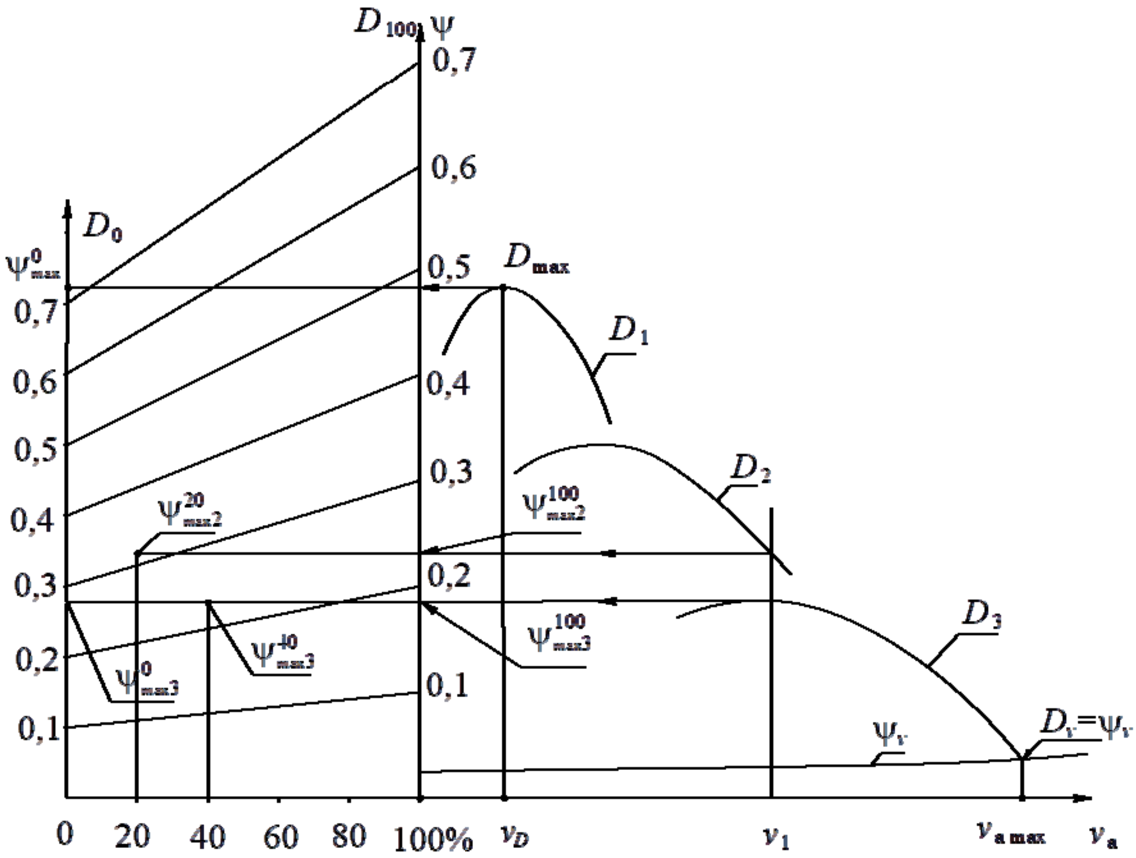


Fig. 4.9. **Influence of vehicle loading on dynamic characteristics**

The segment of the abscissa of the nomogram of loads between the coordinates D_{100} and D_0 is divided into equal parts corresponding to the load of the vehicle, expressed as a percentage.

Equal values of the dynamic factor of the vehicle on the ordinates at full load and without load are connected by segments. Each point of these segments corresponds to the value of the dynamic factor of the vehicle, the values of which they connect, but with the corresponding load.

When determining the value of the dynamic factor at any speed for any load of the vehicle, the value of D_{100} is first determined . Then draw a horizontal line to the intersection with the ordinate D_0 . The obtained value of the dynamic factor characterizes the dynamics of the vehicle without load. If a vertical segment is drawn from the abscissa of the load nomogram for a given load value, then the point of intersection with the horizontal line between D_0 and D_{100} corresponds to the value of the vehicle dynamic factor for the given load. In case of uniform motion, the equality $D = \psi$.

Figure 4.9 shows an example of determining D and ψ for a vehicle without load, with a load of 40% and 100% when moving at a constant speed v_1 in third gear and in second gear with a load of 20% and 100%. Vehicle movement at speed v_1 is possible in second and third gears. At the same time, with a full load in the second gear, it will overcome the resistance $\psi_{\max 2}^{100} = 0{,}235$, on the third $\psi_{\max 3}^{100} = 0{,}18$. In the third gear without load, the vehicle will be able to overcome the resistance $\psi_{\max 3}^{0} = 0{,}28$, and with a load of 40% it is possible overcome resistance $\psi_{\max 3}^{40} = 0{,}23$. In second gear with a load of 20% the vehicle will overcome the resistance $\psi_{\max 2}^{20} = 0{,}32$.

The vehicle will be able to overcome the determined values of the road resistance, provided that there is no skidding of the driving wheels.

4.6. The influence of the adhesion utilized coefficient on traction-speed properties

The condition for the possibility of driving a vehicle

$$P_\varphi \geq P_\text{k} \geq P_\psi + P_\text{v}, \tag{4.29}$$

where $P_\varphi = G_\varphi \cdot \varphi_x$ is the adhesion utilized force of the driving wheels with the supporting surface (G_φ is the towed weight of the vehicle).

Vehicle curb weight is the weight falling on the drive wheels:

– in the case of rear driving wheels, the towed weight is defined as

$$G_{\varphi 2} = m_{z2} \cdot G_2, \tag{4.30}$$

where m_{z2} is the load redistribution coefficient on the rear axle;

– in the case of front driving wheels, the drawbar weight is defined as

$$G_{\varphi 1} = m_{z1} \cdot G_1, \tag{4.31}$$

where m_{z1} is the load redistribution coefficient on the front axle.

Dynamic factor of adhesion utilized

$$D_\varphi = \frac{P_\varphi - P_\text{v}}{G_\text{a}} = \frac{G_\varphi \cdot \varphi_x - P_\text{v}}{G_\text{a}}. \tag{4.32}$$

At a low speed, $P_{\text{v}} \approx 0$, so it can be assumed that the dynamic factor depends on the adhesion utilized

$$D_{\varphi} = \frac{G_{\varphi} \cdot \varphi_x - P_{\text{v}}(\approx 0)}{G_{\text{a}}} = \frac{G_{\varphi}}{G_{\text{a}}} \cdot \varphi_x . \qquad (4.33)$$

For a vehicle with all driving wheels

$$D_{\varphi} = \frac{G_{\text{a}} \cdot \varphi_x}{G_{\text{a}}} = \varphi_x . \qquad (4.34)$$

The condition for the vehicle to be able to move without the wheels slipping

$$D_{\varphi} \geq D \geq \psi . \qquad (4.35)$$

Figure 4.10 shows an example of evaluating the influence of the adhesion utilized coefficient on the traction-speed properties of the vehicle.

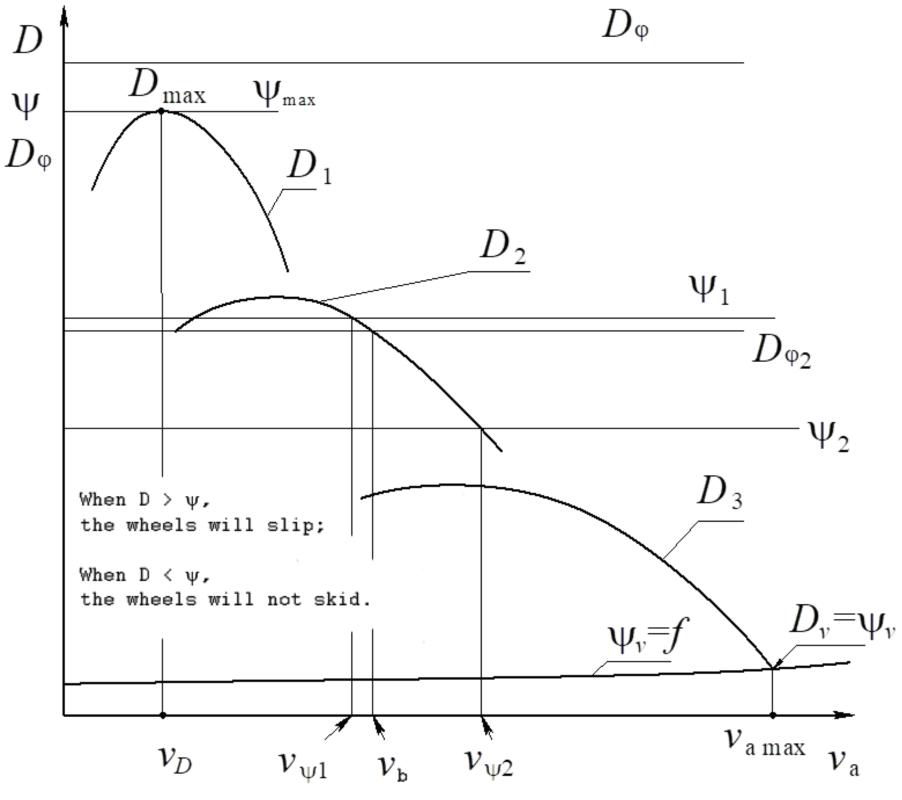


Fig. 4.10. **Analysis of the possibility and nature of the vehicle movement**

At a certain value of the adhesion utilized coefficient φ_x, the drive of the rear driving wheels of the vehicle provides a dynamic adhesion utilized factor $D_{\varphi 2}$. In the case of steady movement of a vehicle with full fuel supply on a road with a resistance coefficient ψ_2, its wheels will slip in the range of speeds from v_{min} to v_b . Further steady movement of the vehicle is possible at speeds less than $v_{\psi 2}$, and the driving wheels will not skid.

If the road resistance coefficient increases to the value ψ_1 , for example, caused by an increase in the slope, the movement of the vehicle becomes impossible due to the skidding of the rear driving wheels, since the condition $\psi_1 > D_{\varphi 2}$ is observed . If you turn on the four-wheel drive, the dynamic factor behind the clutch increases to D_φ and the vehicle will be able to move along the road with a resistance coefficient ψ_1 at the maximum possible speed $v_{\psi 1}$. At the same time, the vehicle will be able to overcome the maximum possible road resistance ψ_{max} without skidding of the driving wheels.

4.7. Dynamic vehicle passport

The dynamic passport represents the dynamic characteristics of the vehicle from the load nomogram and the skid control schedule. Thus, in order to obtain a dynamic vehicle passport, the schedule presented in Figure 4.9 must be supplemented with a skid control schedule. The slip control graph is the dependence of the dynamic factor behind the slip on the degree of load on the driving wheels of the vehicle. The dynamic factor of a vehicle under skidding depends on the coefficient of adhesion of the driving wheels to the supporting surface, the number of driving axles and their location, as well as the type, weight of the vehicle and its distribution between the axles. So, for a vehicle with a rear drive axle, the dynamic factor of the adhesion utilized is equal

$$D_{\varphi_{100}} = \frac{G_2 \cdot \varphi_x}{G_a}; \tag{4.36}$$

$$D_{\varphi_0} = \frac{G_{02} \cdot \varphi_x}{G_0}, \tag{4.37}$$

where $D_{\varphi_{100}}, D_{\varphi_0}$ is the dynamic factor with full load and without load;

G_a , G_0 , G_2 , G_{02} – respectively, the weight of the vehicle with a full load, without a load and the weight falling on the rear driving wheels with a full load and without a load.

As an example, consider a vehicle with a classic layout with a rear drive axle. In this case, we can accept $G_2 = 0.55 \cdot G_a$ and $G_{02} = 0.45 \cdot G_0$. The adhesion utilized coefficient of the driving wheels of the vehicle varies in the range $\varphi_x = 0.1...0.8$. To construct the slip control graph, we will take the values 0.2, 0.4, 0.6, 0.8, 1.0. Calculations of the dynamic factor based on the vehicle adhezion with full load and without load are presented in Table 4.1.

Table 4.1 – **Results of calculation of the dynamic factor by adhesion utilized**

	full load	without load
φ_x	$D_{\varphi_{100}} = \frac{G_2 \cdot \varphi_x}{G_a} = 0,55 \cdot \varphi_x$	$D_{\varphi_0} = \frac{G_{02} \cdot \varphi_x}{G_0} = 0,45 \cdot \varphi_x$
0.2	$D_{\varphi_{100}} = 0,55 \cdot 0,2 = 0,11$	$D_{\varphi_0} = 0,45 \cdot 0,2 = 0,09$
0.4	$D_{\varphi_{100}} = 0,55 \cdot 0,4 = 0,22$	$D_{\varphi_0} = 0,45 \cdot 0,4 = 0,18$
0.6	$D_{\varphi_{100}} = 0,55 \cdot 0,6 = 0,33$	$D_{\varphi_0} = 0,45 \cdot 0,6 = 0,27$
0.8	$D_{\varphi_{100}} = 0,55 \cdot 0,8 = 0,44$	$D_{\varphi_0} = 0,45 \cdot 0,8 = 0,36$
1.0	$D_{\varphi_{100}} = 0,55 \cdot 1,0 = 0,55$	$D_{\varphi_0} = 0,45 \cdot 1,0 = 0,45$

To plot the slip control graph, we plot the value of the dynamic factor by adhesion of the vehicle without load on the left ordinate, and on the right of the vehicle with a full load (Fig. 4.11).

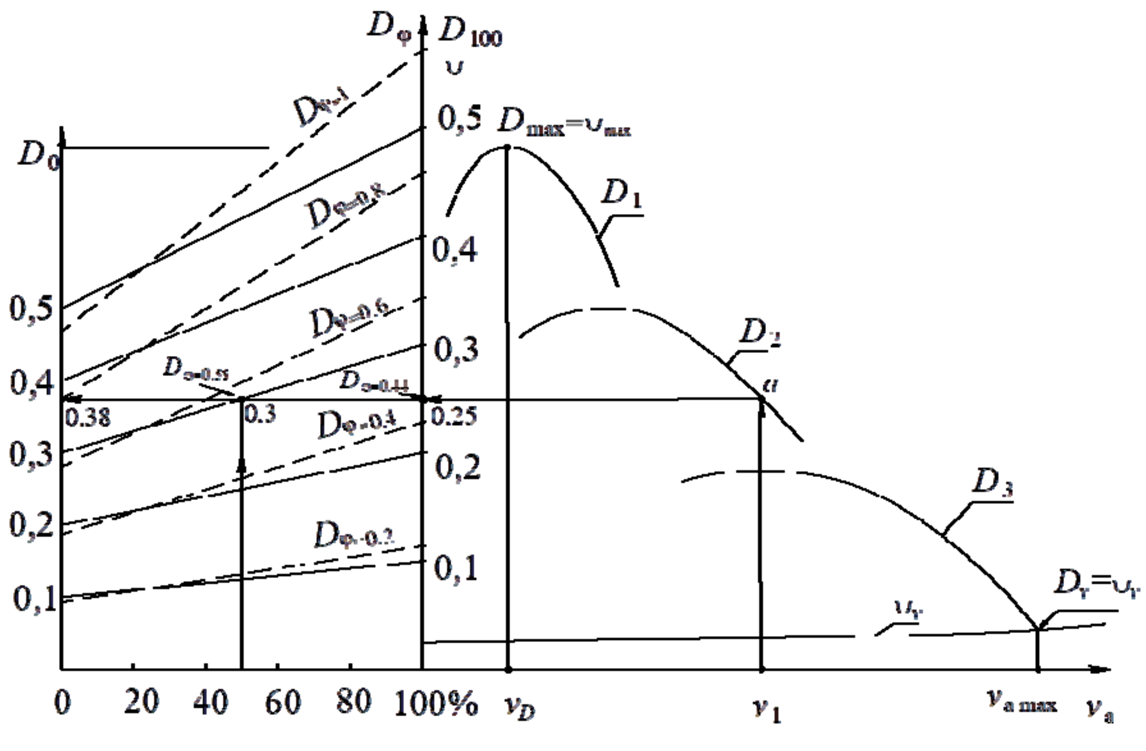


Fig. 4.11. **Dynamic vehicle passport**

The value of the dynamic factor of the adhesion utilized for the same coefficient of adhesion utilized on the left and right ordinates is connected by segments. Above each segment, we will write the value of the adhesion utilized coefficient to which this dynamic adhesion utilized factor would correspond.

Figure 4.11 shows an example of determining the traction-speed properties for a vehicle with different loads when moving at a constant speed v_1 in second gear. With a full load, the vehicle will be able to move on the road with a drag coefficient $\psi = 0.25$, with a load of 50% - $\psi = 0.3$, and without a load - $\psi = 0.38$. But for this it is necessary that the adhesion utilized coefficient φ_x of the driving wheels for the corresponding vehicle load is equal to or greater than 0.44, 0.56, 0.8.

4.8. Power balance of the vehicle

Indicators of vehicle dynamics, along with the use of power balance, can be obtained using the power balance. The vehicle power balance equation from the power balance equation. Because, by definition, power $N = P \cdot v$, then, multiplying the power balance equation by the speed v of the vehicle, we get the power balance equation

$$P_{\text{k}} = P_{\psi} + P_{\text{v}} + P_j \Big| \times v; \tag{4.38}$$

$$P_{\text{k}} \cdot v = P_{\psi} \cdot v + P_{\text{v}} \cdot v + P_j \cdot v. \tag{4.39}$$

The equation of the vehicle power balance

$$N_{\text{k}} = N_{\psi} + N_{\text{v}} + N_j, \tag{4.40}$$

where N_{k} is the power supplied to the driving wheels of the vehicle (supplied from the engine through the transmission);

N_{ψ} – power used to overcome the total road resistance;

N_{v} – power spent to overcome air resistance;

N_j – power spent on acceleration of the vehicle.

The power N_{k}, kW, is connected to the driving wheels of the vehicle

$$N_{\text{k}} = P_{\text{k}} \cdot v = N_e - N_{\text{tr}} = N_e \cdot \eta_{\text{tr}}, \tag{4.41}$$

where N_{tr} is the power lost in the transmission;

η_{tr} – transmission efficiency.

The power spent on overcoming the total road resistance

$$N_\psi = P_\psi \cdot v = G_a \cdot \psi \cdot v, \text{ [W], v [m/s];} \tag{4.42}$$

$$N_\psi = \frac{G_a \cdot \psi \cdot v_a}{3600}, \text{ [kW], } v_a \text{ [km/h].} \tag{4.43}$$

Power expended to overcome air resistance

$$N_v = P_v \cdot v = \kappa_v \cdot F_a \cdot v^2 \cdot v = \kappa_v \cdot F_v \cdot v^3, \text{ [W], v [m/s];} \tag{4.44}$$

$$N_v = \frac{\kappa_v \cdot F_a \cdot v_a^3}{46656}, \text{ [kW], v a [km/h].} \tag{4.45}$$

The power spent on accelerating the vehicle

$$N_j = P_j \cdot v = \frac{G_a}{g} \cdot \delta_{vr} \cdot j_a \cdot v, \text{ [W], v [m/s];} \tag{4.46}$$

$$N_j = \frac{G_a}{3600g} \cdot \delta_{vr} \cdot j_a \cdot v_a, \text{ [kW], v a [km/h].} \tag{4.47}$$

4.8.1. Vehicle power balance graph

The solution of the power balance equation can be performed by the grapho-analytical method. For this, it is necessary to build a graph of the vehicle power balance (see Fig. 4.12), which shows the dependences of N_e, N_k, N_ψ, N_v on the speed of the vehicle.

At the same time, the dependence $N_e = f(v_a)$ is depicted for the speed of movement, on that gear in the transmission, at which the maximum speed of the vehicle is reached. For the same transmission, the total dependence $(N_\psi + N_v) = f(v_a)$ is depicted. The dependence of power on the driving wheels of the vehicle $N_k = f(v_a)$ is depicted for each gear in the transmission.

Figure 4.12 shows the vehicle power balance at a speed of 80 km/h. Each segment of the vertical bounded by curly brackets characterizes the value of the corresponding power. It is shown that in the power balance there is a component N_z, which characterizes the power reserve on the drive wheels when moving at a given speed. This reserve of N power can be used to accelerate the vehicle, overcome additional road resistance, tow a trailer.

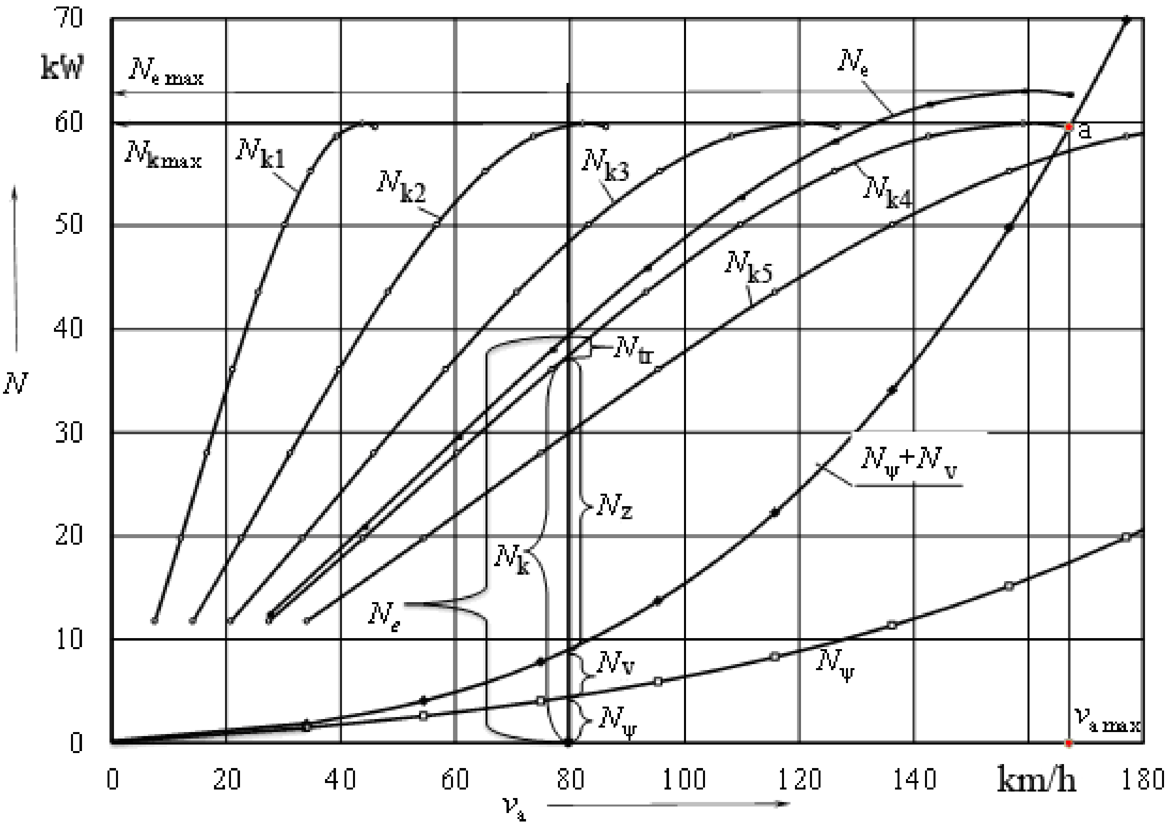


Fig. 4.12. **Power balance chart of a passenger vehicle**

At the maximum driving speed, there is no power reserve, so the vehicle can no longer accelerate.

4.8.2. Analysis of traction and speed properties of a vehicle using a power balance graph

1 . *Determination of the maximum speed of the vehicle.* The maximum speed of the vehicle is a constant speed because there is no traction reserve and the vehicle is unable to accelerate. That is, acceleration $j_a = 0$ and, accordingly, the power spent on acceleration $N_j = 0$. In this case, the power balance equation has the form $N_k = N_\psi + N_v$.

The graphic solution of this equation (Fig. 4.13) is the point of intersection of the graphs $N_к = f(v_a)$ and $N_\psi + N_v = f(v_a)$. There are two such points "a" and "b" in the figure. At the same time, as a solution, it is necessary to determine the point to which the highest speed corresponds, that is, in Figure 4.13, it is point "a".

2 . *Determination of the maximum road resistance that a vehicle can overcome at a given speed v_1.* Overcoming the maximum road resistance by the vehicle occurs at a constant speed, since the entire reserve of power is used for this.

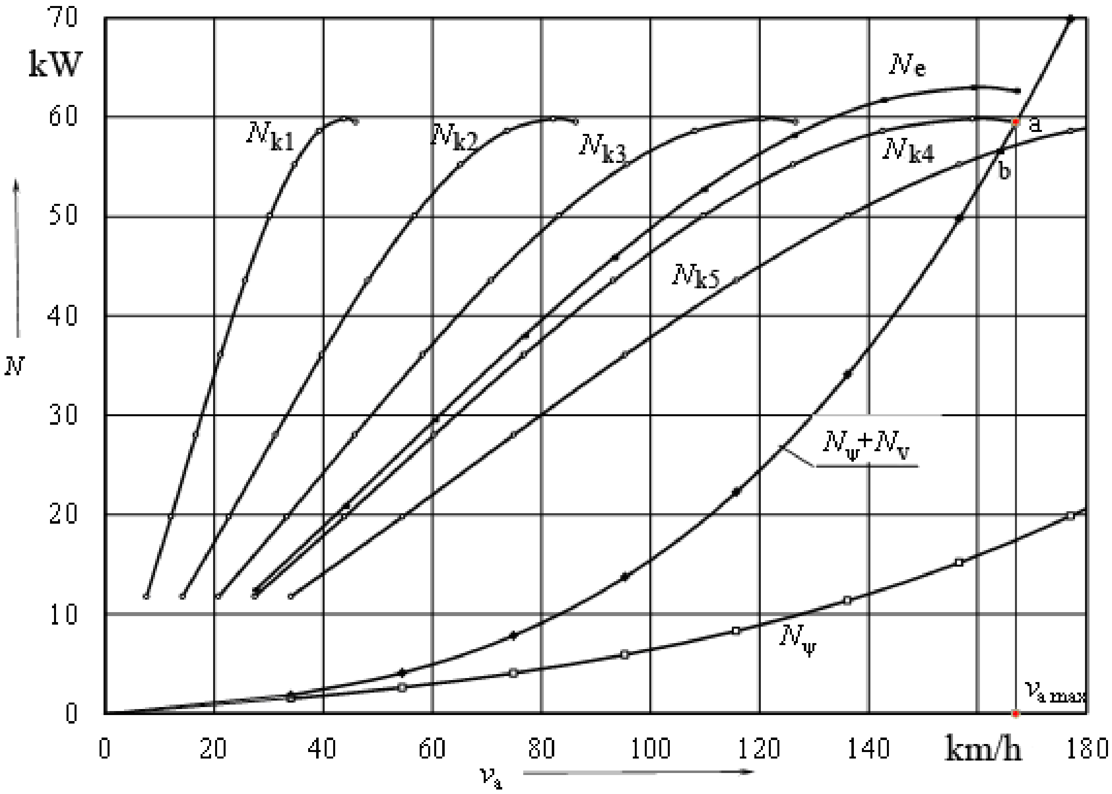


Fig. 4.13. **Determination of the maximum speed of the vehicle**

In this case, the power spent to overcome the maximum road resistance is equal to the sum $N_{\psi_{max}} = N_\psi + N_z$ and is determined from the power balance equation

$$N_k = N_\psi + N_z + N_v = N_{\psi_{max}} + N_v \rightarrow N_{\psi_{max}} = N_k - N_v .$$

The graphic determination of the capacities N_k and N_z is performed on the scale of the ordinate N (see Fig. 4.14) . The value of the maximum road resistance ψ_{max} is calculated based on the obtained power value $N_{\psi_{max}}$

$$N_{\psi_{max}} = \frac{G_a \cdot \psi_{max} \cdot v_1}{13} \rightarrow \psi_{max} = \frac{13 \cdot N_{\psi_{max}}}{G_a \cdot \psi_{max} \cdot v_1} .$$

Vehicle movement with speed v_1 (see Fig. 4.14) is possible in the second, third, fourth and fifth gears. The power supplied to the drive wheels is $N_{k2} > N_{k3} > N_{k4} > N_{k5}$, therefore the power reserve in the second gear is the largest. It is obvious that it is in this gear that the vehicle will overcome the greatest movement resistance at the speed v_1 .

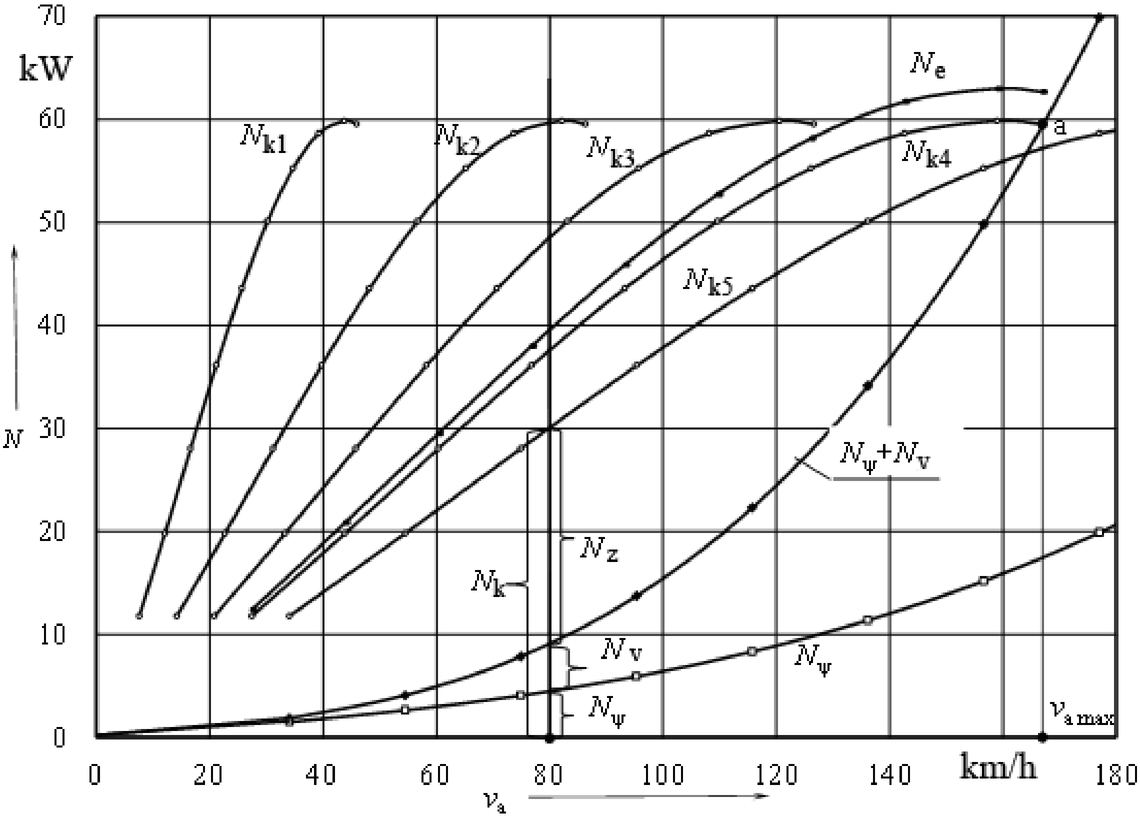


Fig. 4.14. **Determination of the maximum road resistance that a vehicle can overcome at a speed of v_1 ($v_1 = 80$ km/h)**

3 . *Determining the engine power required for the smooth movement of the vehicle.* The movement of the vehicle at a uniform speed, less than the maximum speed that is possible in the given conditions, occurs when the engine is operating at a partial characteristic. On the power balance graph, all dependencies correspond to the operation of the engine on the external speed characteristic (Fig. 4.15).

In order to use it to determine the engine power required for uniform movement (see Fig. 4.15 – 100 km/h), it is first necessary to determine what power is required on the drive wheels for movement at this speed. This power is determined from the condition of uniform movement $N_k = N_\psi + N_v$, as a point of intersection of the vertical from the given speed with the graph $N_\psi + N_v = f(v_a)$. In Figure 4.15, the dashed line shows the dependence $N_k = f(v_a)$ during engine operation on the partial characteristic. It should be noted that this power N_k does not depend on the gear in which the vehicle moves at a given speed. This is explained by the fact that the power on the drive wheels during uniform movement depends only on the resistance of the movement.

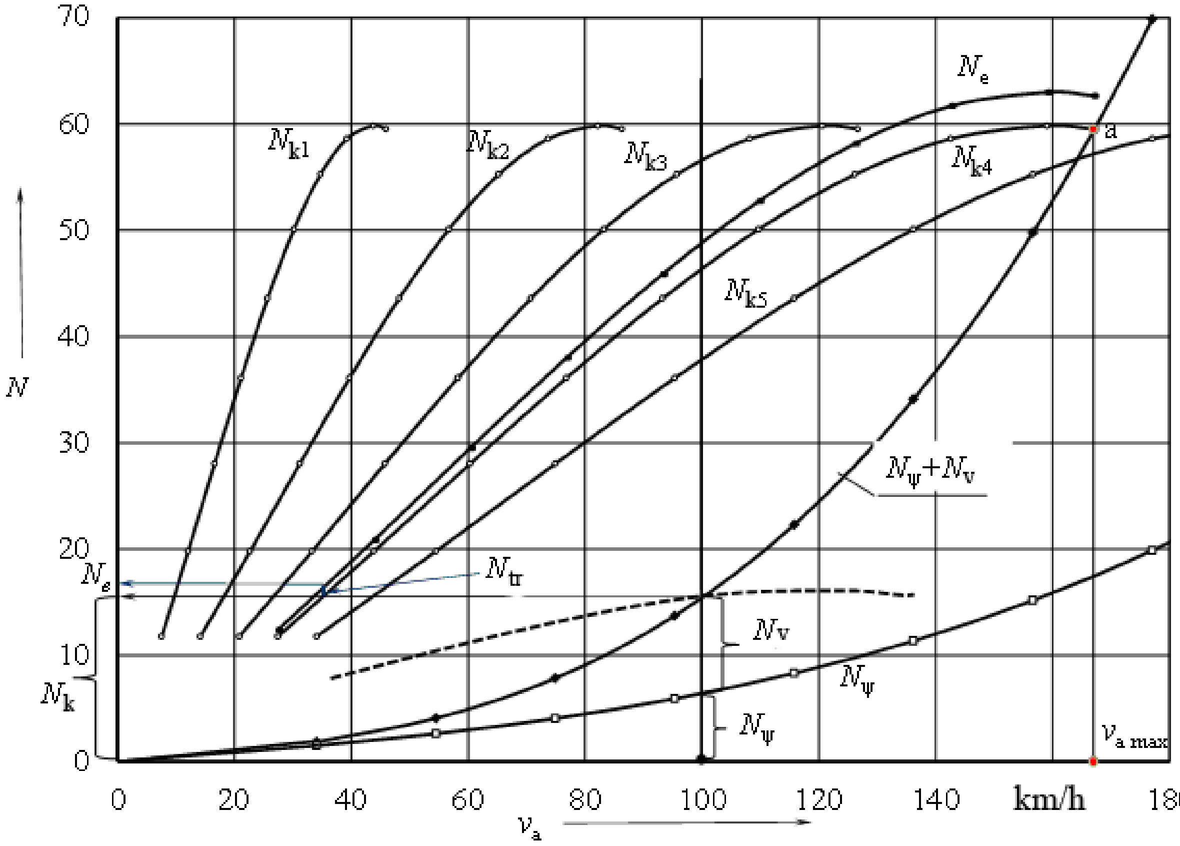


Fig. 4.15. **Determination of vehicle engine power at a constant speed of 100 km / h**

In order to find the engine power necessary for the smooth movement of the vehicle, it is necessary to add the power N_{tr} to the power N_k, which is consumed in the transmission when this power is transferred to the drive wheels. The graphical addition of N_{tr} to N_k is carried out on the graph of the gear for which the dependence of N_e is constructed at full fuel supply (see Fig. 4.15 - the fourth gear). From the point of intersection of the vertical and the dependence $N_{,}$ the horizontal to the ordinate axis determines the required engine power.

4 . *Determination of the maximum possible acceleration of the vehicle in a given gear and speed.* The maximum acceleration of the vehicle at a given gear and speed is possible if the entire power reserve N_z is used to overcome the power of the acceleration resistance N_j . The value of the power reserve N is determined graphically (see Fig. 4.14), taking into account that

$$N_k = N_\psi + N_v + N_z \rightarrow N_z = N_k - N_\psi - N_v .$$

Equating $N_z = N_j$ and expanding N_j, we get

$$N_z = \frac{G_a}{g} \cdot \delta_{vr} \cdot \frac{v_a}{3600} \cdot j_{a\,max},$$

from where we can calculate the maximum possible acceleration of the vehicle at this speed (see Fig. 4.14 v_a=80 km/h)

$$j_{a\,max} = \frac{3600 \cdot N_z \cdot g}{G_a \cdot \delta_{vr} \cdot v_a}.$$

Figure 4.14 shows that movement at this speed is possible in the second, third, fourth and fifth gears. The power supplied to the drive wheels $N_{k2} > N_{k3} > N_{k4} > N_{k5}$, therefore, the power reserve N_z the second gear is the largest. It is obvious that it is in this gear that the vehicle will have the maximum possible acceleration at a speed of v_1.

<u>5 . *Determination of engine power at a given speed of movement in any gear.*</u> During the acceleration of the vehicle with full fuel supply, at each value of the speed, the engine develops a certain effective power N_e. At the same vehicle speed, the engine develops different effective power in different gears. In order to determine the power of the engine at a given speed (see Fig. 4.16 – 90 km/h), it is necessary to draw a perpendicular to the dependence $N_k = f(v_a)$ corresponding to the gear on which the vehicle is moving (see Fig. 4.16 – N_{k3}, N_{k4}, N_{k5}).

Is easiest to determine the effective power of the engine when the vehicle is moving in the gear for which the dependence $N_e = f(v_a)$ is constructed – in Figure 4.16, this is the fourth gear. At the point of intersection *a* of the perpendicular with the dependence $N_{k4} = f(v_a)$ the power on the driving wheels is determined. To this power, add the power consumed in the transmission - segment N_{tr4} and determine the effective power of the engine N_{e4} on the ordinate N. In Figure 4.16, all components of the power balance are marked with curly brackets.

The effective power of the engine when the vehicle is moving in any gear is determined in the same sequence. When moving in the third gear, the power on the drive wheels N_{k3} is determined by point *b* . We look for the same power value N_k at the horizontal intersection at point *d* with the dependence $N_{k4} = f(v_a)$, determine the power consumed in the transmission N_{tr3} . The effective power of the engine N_{e3} when moving in third gear at a given speed is determined on the ordinate N.

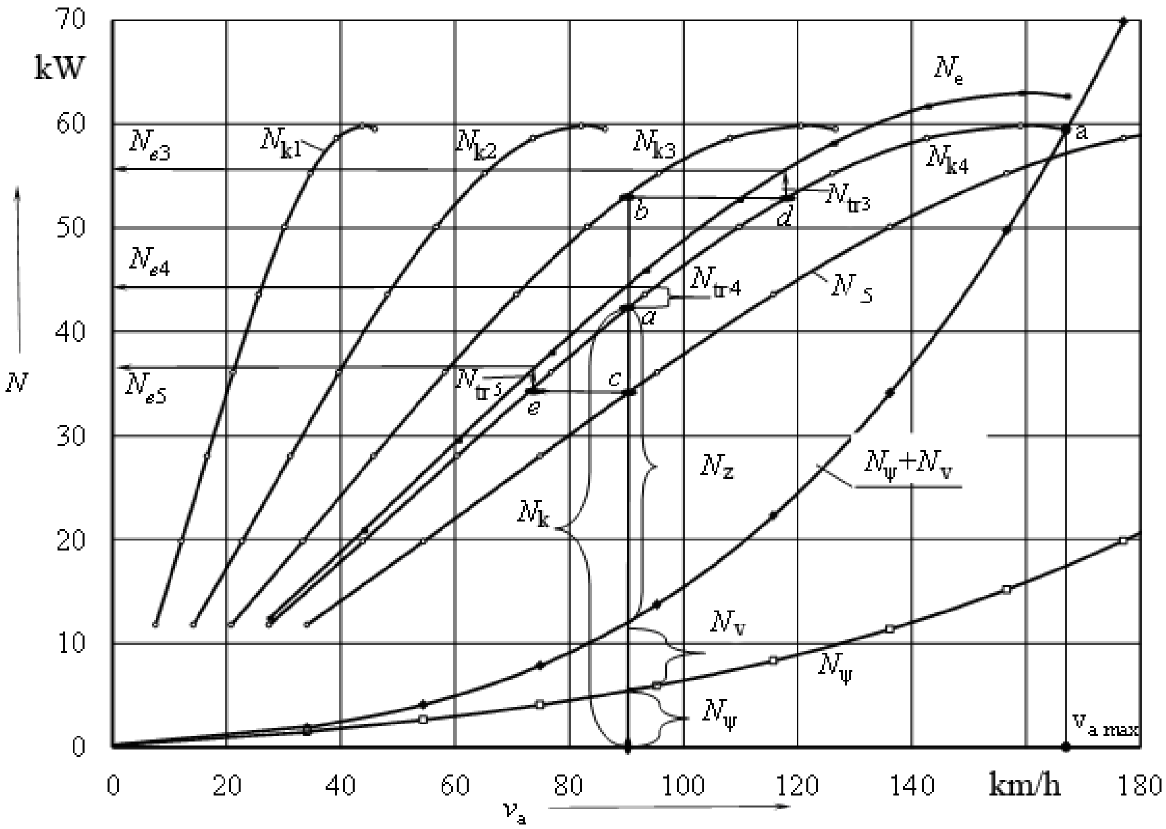


Fig. 4.16. **Determination of engine power at a given vehicle speed**

The effective power of the engine N_{e5} is determined in the same way . Point *c* corresponds to the power on the drive wheels N_{k5} . At the point *e*, the ordinate between the dependences N_e and N_{k4} characterizes the power N_{tr5} .

The horizontal to the ordinate *N* determines the effective power of the engine N_{e5} .

The difference in the effective power of the engine when the vehicle is moving at the same speed is due to the fact that it develops a different torque at a different frequency of rotation of the crankshaft.

6 . *Determining the speed of the vehicle in any gear for a given effective engine power.* If the engine develops a certain power, then the movement of the vehicle in different gears occurs at different speeds (see Fig. 4.17). Determining the speed of the vehicle at a given engine power is the inverse of the problem discussed above.

To determine the speed of the vehicle, it is necessary to know the power on the driving wheels in the corresponding gears N_{k1}, N_{k2}, N_{k3}, N_{k4}, N_{k5} . If the effective power of the engine N_e is given , the power on the drive wheels when moving in the fourth gear N_{k4} is determined by the point *a* of the intersection of the vertical and the dependence $N_{k4} = f(v_a)$.

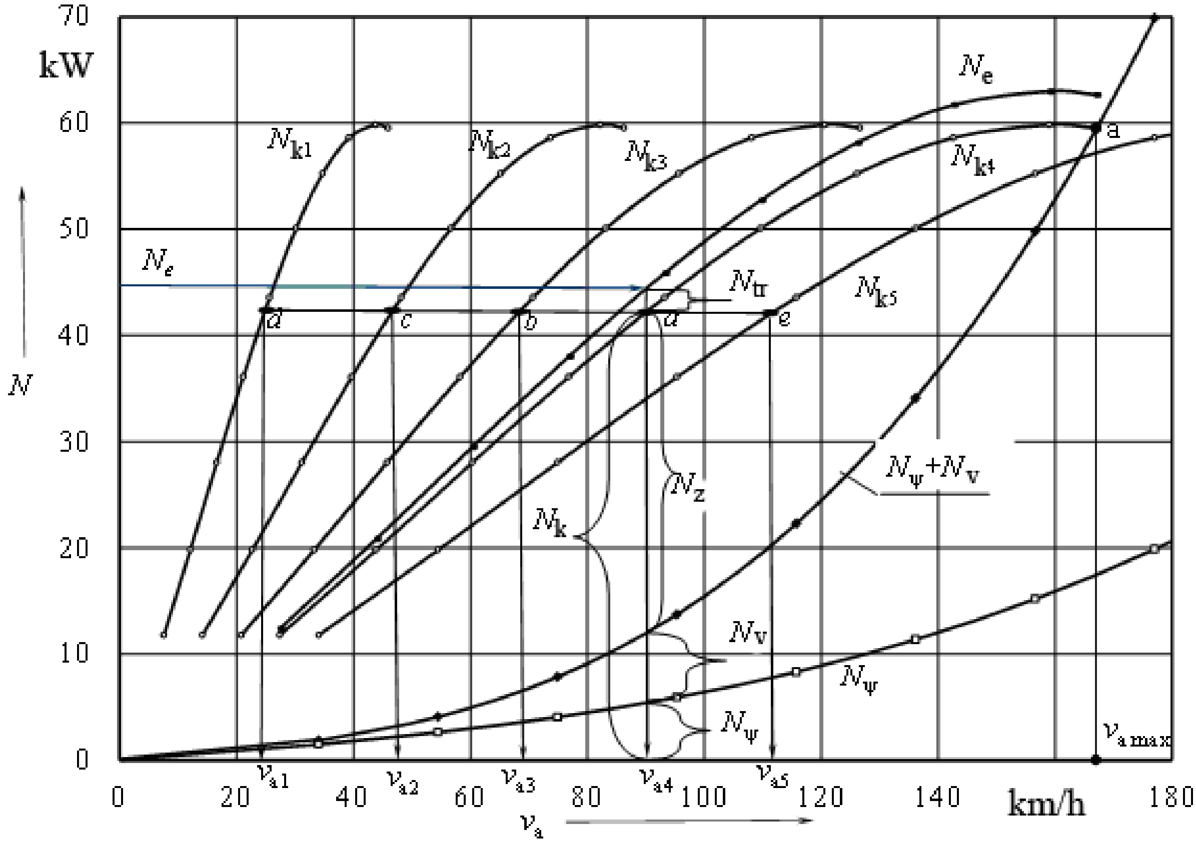


Fig. 4.17. **Determining the speed of the vehicle at a given engine power**

At the same time, the vertical extended to the abscissa axis indicates the speed of the vehicle v_{a4}, if the fourth gear is engaged.

Since the effective power of the engine N_e the same, the power delivered to the drive wheels in different gears is also the same, i.e. $N_{k1} = N_{k2} = N_{k3} = N_{k4} = N_{k5}$. Their values (points *b* , *c* , *d* and *e*) determines the horizontal drawn through point *a* . At the same time, vertical lines drawn to the abscissa from points *b* , *c* , *d* and *e* indicate the values of the corresponding speeds v_{a1}, v_{a2}, v_{a3} and v_{a5}.

4.8.3. Engine power utilization factor

Engine power utilization factor is the ratio of the power required for uniform movement of the vehicle at a given speed to the power developed by the engine on the drive wheels at the same speed and full fuel supply

$$u_N = \frac{N_\psi + N_v}{N_k} = \frac{N_\psi + N_v}{N_e \cdot \eta_{tr}}. \qquad (4.48)$$

The larger the ratio, the more efficiently the engine is used, but the power reserve is smaller, which means that the acceleration is less possible.

4.9. Acceleration of the vehicle

In the conditions of operation, the vehicle moves unevenly for the most part. In city conditions, the vehicle moves: evenly - 15...25%; coasting and with braking - 30...40%; overclocking - 30...45%.

Acceleration of the vehicle is determined by its reception. *Acceptance* of a vehicle is called its ability to quickly increase the speed of movement and is characterized by indicators:

1) acceleration during acceleration j_a, m/s ^{2};
2) acceleration time t_p, s;
3) acceleration path S_p, m.

4.9.1. Acceleration of the vehicle during acceleration

The amount of acceleration during acceleration of the vehicle is determined from the force balance equation

$$P_k = P_\psi + P_v + P_j \Rightarrow P_j = P_k - P_\psi - P_v; \tag{4.49}$$

$$\frac{G_a}{g}\delta_{vr} \cdot j_a = P_k - P_\psi - P_v; \tag{4.50}$$

$$j_a = \frac{P_k - P_\psi - P_v}{G_a \cdot \delta_{vr}} \cdot g. \tag{4.51}$$

It is convenient to perform the analysis of the vehicle reception according to the graph of accelerations during acceleration

$$j_a = \frac{D - \psi}{\delta_{vr}} \cdot g. \tag{4.52}$$

When constructing the graph, we make the following assumptions:

1) the vehicle accelerates on a horizontal road with a hard surface;
2) there is no wheel slippage;
3) acceleration occurs with full fuel supply.

Taking into account these assumptions, the equation can be used to calculate the acceleration

$$j_a = \frac{D - f}{\delta_{vr}} \cdot g. \qquad (4.53)$$

The graph of the acceleration of a passenger vehicle in each gear is shown in Figure 4.18a. On the section 0 → v_{amin} – the vehicle accelerates in first gear with clutch slippage.

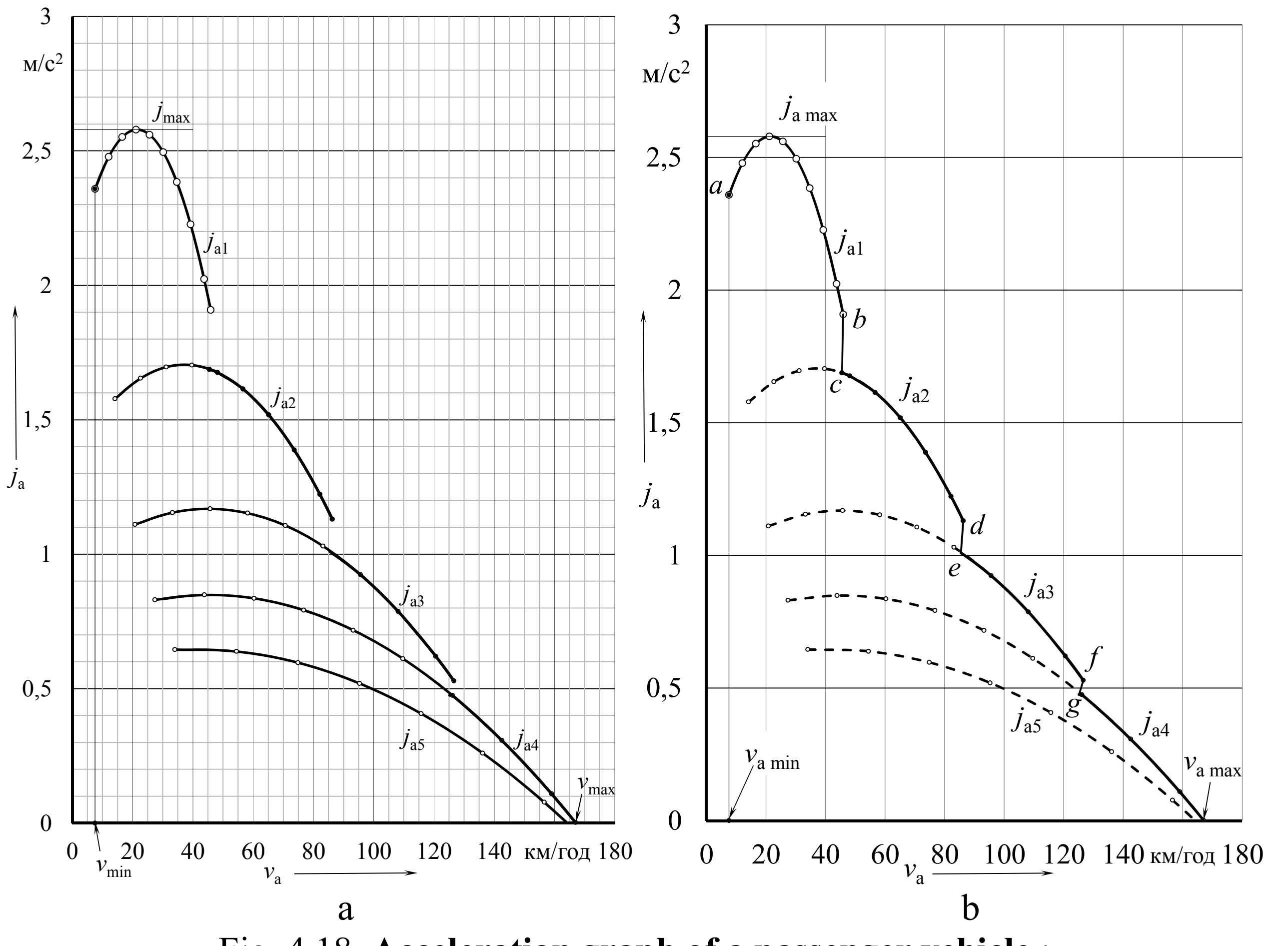


Fig. 4.18. **Acceleration graph of a passenger vehicle** :
a – in the entire range of speeds; b – during acceleration with maximum acceleration; v_{amin} – speed of movement corresponding to the minimum stable revolutions of the crankshaft; v_{amax} – maximum speed of movement

In this case, the dynamic factor is uncertain, but this area is very small and therefore may not be taken into account when determining the acceleration parameters of the vehicle. The acceleration of the vehicle in all other gears is calculated from the minimum possible speed to the maximum. Since the movement of the vehicle at a speed that is less than critical is unstable, therefore acceleration is not carried out at it. To ensure the best handling of the vehicle, that is, to accelerate with the maximum possible acceleration, it is necessary to rationally choose the speed at which the gear is switched from lower to higher.

Figure 4.18b shows that it is rational to switch from the first gear to the second at the vehicle speed v_b, which corresponds to the point b, because further acceleration is impossible. Acceleration in the second gear will start from the speed v_c, which corresponds to the point c.

When switching gears, the engine is disconnected from the drive wheels and the vehicle rolls with acceleration

$$j_{\text{п}} = \frac{D_{\text{n}} - f}{\delta_{\text{vr}}} \cdot g, \tag{4.54}$$

where j_{n} – acceleration of the vehicle when switching gears;

D_{n} – dynamic factor of the vehicle when switching gears.

Since when switching gears, the torque M_{k} is not supplied to the drive wheels, therefore the total traction force P_{k} is zero and the dynamic factor of the vehicle is determined by the dependence

$$D_{\text{n}} = \frac{P_{\text{k}}(=0) - P_{\text{v}}}{G_{\text{a}}} = -\frac{P_{\text{v}}}{G_{\text{a}}}, \tag{4.55}$$

where P_{v} – force of air resistance at the speed of the vehicle during gear shifting.

Since the dynamic factor (4.55) has a negative value, therefore, the acceleration (4.54) will acquire a negative value, that is, the vehicle moves with deceleration during gear shifting. Therefore, the speed of the vehicle will decrease by the time of gear shifting

$$\Delta v_{\text{n}} = j_{\text{n}} \cdot t_{\text{n}}, \tag{4.56}$$

where Δv_{n} is the decrease in the speed of the vehicle during gear shifting (has a negative value), m/s;

t_{n} is the time of gear switching.

The gear shifting time depends on the driver's qualification and the design of the gearbox, and its values are determined by the interval t_{n} = 0.5 s...1.5 s. Usually, when calculating vehicle acceleration indicators, the switching time is taken as equal to 0.8 s.

Acceleration in the second gear will start from the speed (4.57), which corresponds to the point c, the value of which is equal to the sum

$$v_c = v_b + \Delta v_{\text{n}}. \tag{4.57}$$

The decrease in the speed of the vehicle during gear shifting is greater, the greater the air resistance, i.e. at high speeds.

The rational speed of the vehicle to start switching and the speed at which the acceleration of the vehicle begins in the next gear are shown in Figure 4.18b. The fifth gear of this vehicle is not used at maximum acceleration. This is a special, economical transmission.

The graph of truck accelerations is presented in Figure 4.19. Due to the large value of δ_{vr} in the first gear, the acceleration of the vehicle during acceleration in the first and second gears may differ little.

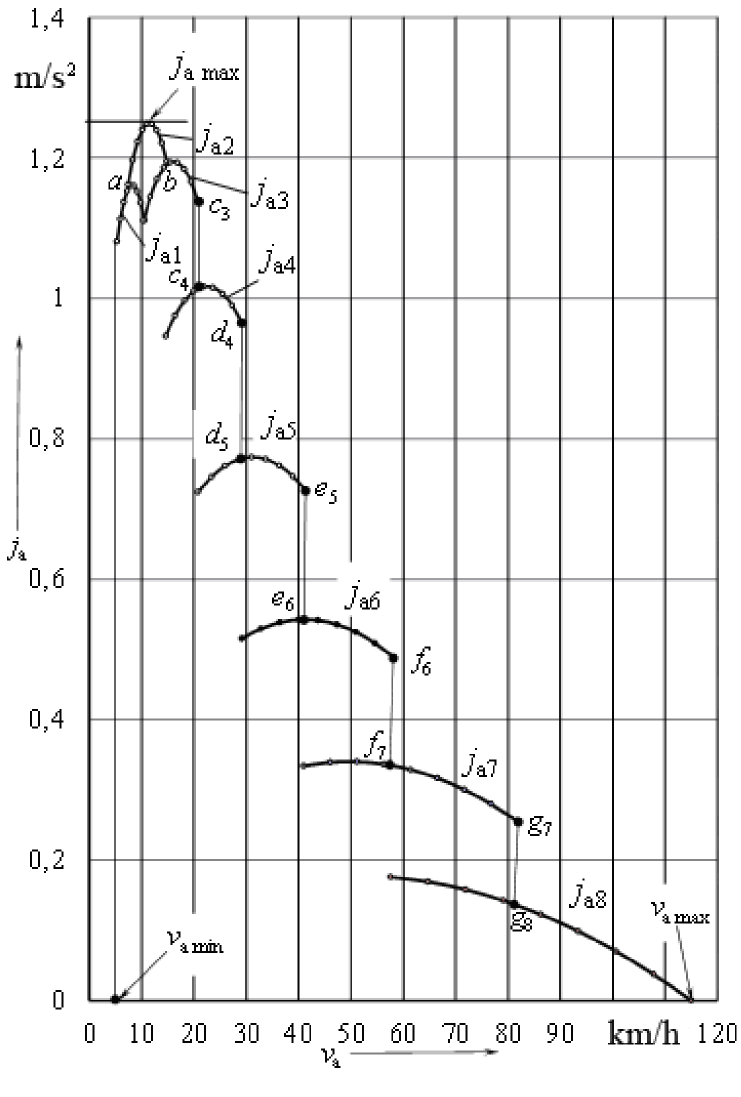


Fig. 4.19. **Truck acceleration schedule**

For this reason, with large values of the gear ratios of the first gear, the acceleration of the truck in the second gear is higher than in the first. In this case, when accelerating a truck on a road with little road resistance, it is rational to start in the second gear in the gearbox. And the first gear on such vehicle is not used for acceleration, but for overcoming maximum road resistance.

t_p and the acceleration path S_p in a given range of speeds are more informative indicators of the vehicle responsiveness .

4.9.2. Vehicle acceleration time

The vehicle acceleration time can be determined experimentally by measurement or numerically by solving the differential equation of the vehicle motion. Also, several graph-analytical methods are proposed for the theoretical determination of the time t_p and the acceleration path s_p. The method of E.O. Chudakova and Yakovleva M.O. consists in the fact that the estimated speed interval is divided into small sections. At the same time, it is assumed that on each section the vehicle accelerates with a constant acceleration, which has an average value for this section. An explanation of the use of this method is presented in Figure 4.20.

For each section, the average acceleration j average is calculated .

$$j_{sr} = 0,5\left(j_n + j_k\right), \tag{4.58}$$

where j_n and j_k are the acceleration at the beginning of the section and at the end of the section.

At the same time, the speed at the beginning and at the end of the section is connected by a known dependence

$$v_{kj} = v_n + j_{sr} \cdot \Delta t, \tag{4.59}$$

where Δt is the time of passage of the section.

Let's determine from the equation (4.59) the time of passage of each section

$$\Delta t = \frac{v_{kj} - v_n}{j_{sr}}. \tag{4.60}$$

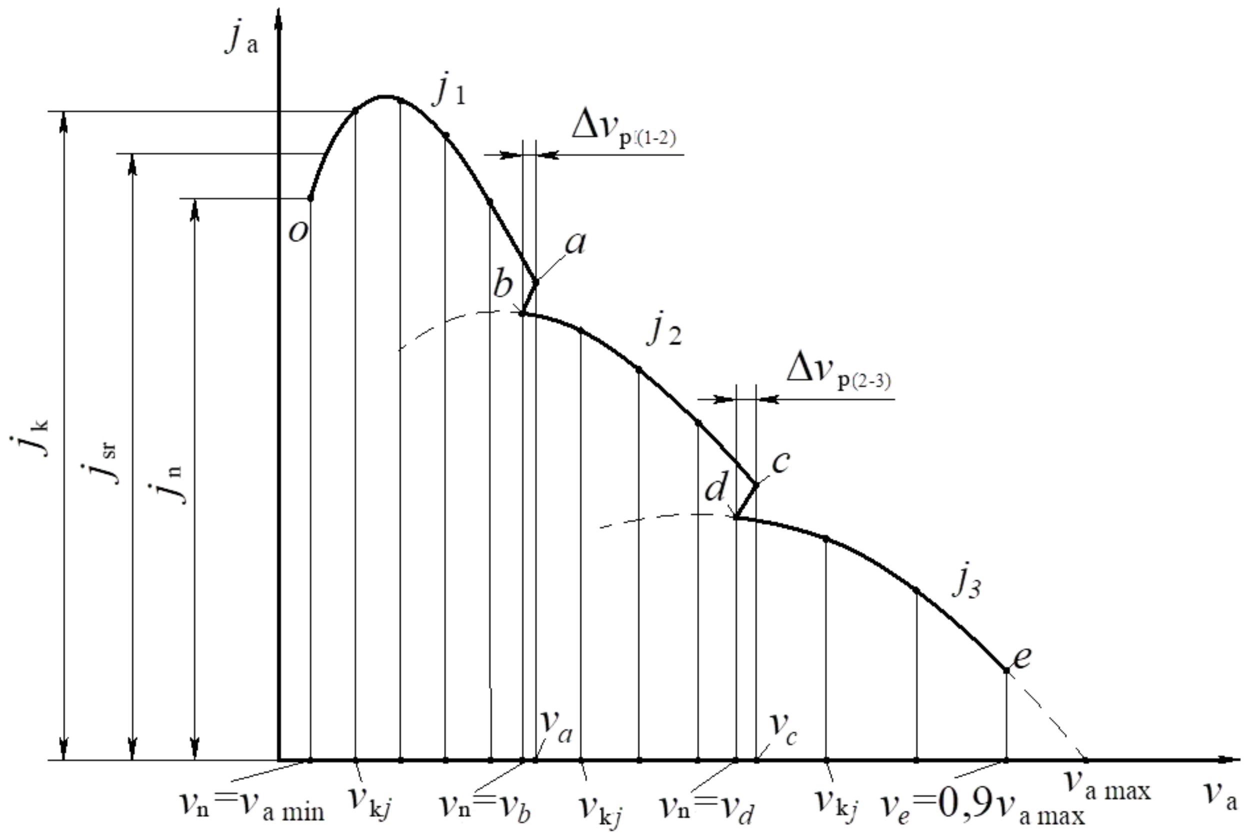


Fig. 4.20. **Scheme of breakdown of the acceleration range of the vehicle into sections** : v_n i v_{kj} - speed at the beginning of the section and at the end of the acceleration section

The acceleration time of the vehicle in the i -th gear will be obtained by adding up the time of passing all the acceleration sections in this gear:

$$\tau_i = \Delta t_1 + \Delta t_2 + \Delta t_3 + ... + \Delta t_k, \qquad (4.61)$$

where Δt_1, Δt_2, Δt_3, … Δt_k is the time of passage of the section (k is the number of the section during acceleration on this gear).

Vehicle acceleration time t_r over the entire calculated range as a sum

$$t_p = \tau_1 + t_{p(1\text{-}2)} + \tau_2 + t_{p(2\text{-}3)} + ... + \tau_{n_к}, \qquad (4.62)$$

where τ_1, τ_2, … τ_{nk} is the acceleration time at each gear in the gearbox;

n_k - the number of gears in the gearbox, during which acceleration occurs;

$t_{p(1\text{-}2)}$, $t_{p(2\text{-}3)}$ – the time of gear shifting from the first to the second and from the second to the third, respectively.

The graph of the acceleration time of the vehicle has the form shown in Figure 4.21.

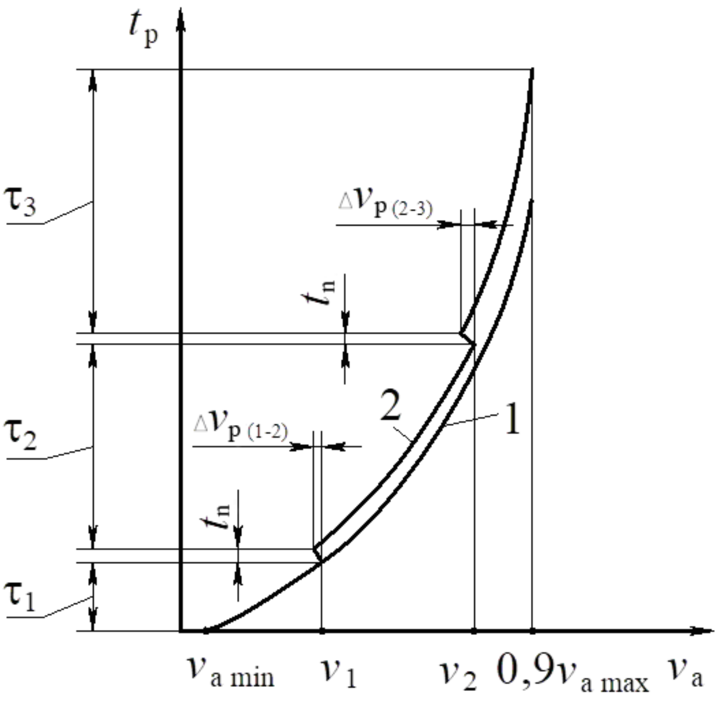


Fig. 4.21. **Vehicle acceleration time schedule** :
1 – without taking into account the time of gear shifting;
2 – taking into account the time of gear shifting

To assess the impact of the gear shifting time, the figure shows the dependences of the acceleration time of the vehicle with the gear shifting time taken into account and without the gear shifting time taken into account, i.e. with a stepless gearbox.

The control time of acceleration to the set speed is used to assess the vehicle efficiency. Acceleration control time: t_{p100} – for passenger vehicle – acceleration time to a speed of 100 km/h; t_{p60} – for trucks – acceleration time to a speed of 60 km/h.

4.9.3. Way of acceleration of the vehicle

The acceleration path of the vehicle is defined as the sum of the acceleration paths in each section of the range. The path traveled by the vehicle on each section from v_n to v_{kj} is determined by the known relationship (4.63), assuming that the vehicle moves at an average speed on this section

$$\Delta s = v_{sr} \cdot \Delta t\,, \tag{4.63}$$

where $v_{sr} = 0,5\left(v_n + v_{kj}\right)$– the average speed of movement in the acceleration section.

The distance traveled by the vehicle during acceleration in the i -th gear in the gearbox

$$s_i = \Delta s_1 + \Delta s_2 + \Delta s_3 + ... + \Delta s_k . \quad (4.64)$$

The distance traveled by the vehicle during acceleration:

$$S_{\text{p}} = s_1 + s_{\text{p(1-2)}} + s_2 + s_{\text{p(2-3)}} + ... + s_{n_{\text{к}}} , \quad (4.65)$$

where $s_{\text{p(1-2)}}$, $s_{\text{p(2-3)}}$ is the distance traveled by the vehicle during gear shifting from the first to the second and from the second to the third, respectively.

The distance traveled by the vehicle at each switch is determined by the known relationship (4.66), assuming that the vehicle moves with an average speed v_{psr} :

$$s_{\text{p}} = v_{\text{psr}} \cdot t_{\text{n}} = \left(v \ + 0,5 \cdot \Delta v_{\text{p}}\right) \cdot t_{\text{n}} , \quad (4.66)$$

where v is the speed of the vehicle at the moment of switching off the gear, m/s.

The graph of the acceleration path of the vehicle has the form presented in Figure 4.22. The control path of acceleration to the set speed is used to evaluate the vehicle reception.

Acceleration control path: S_{p100} – for passenger vehicle – acceleration time to a speed of 100 km/h; S_{p60} – for trucks – acceleration time to a speed of 60 km/h.

When constructing graphs of the time and path of acceleration of the vehicle, calculations are performed from the speed of $v_{\text{a min}}$ to the speed of 0.9 $v_{\text{a max}}$. Because at a speed close to the maximum, the reception of the vehicle decreases significantly, and the acceleration time and distance increase significantly.

To compare the reception of different vehicle, it is convenient to use the resulting graph of acceleration characteristics - the dependence of acceleration time on the distance traveled (see Fig. 4.23). It can be seen from the figure that in the same driving time t_1 , vehicle 1 will cover a distance s_1 , which is more than the distance s_2 covered by vehicle 2.

This shows that vehicle 1 has better dynamics. But during time t_2 , vehicle 2 will catch up with vehicle 1, as evidenced by the equality $s_1 = s_2$, and then overtake it. This indicates a higher top speed of vehicle 2.

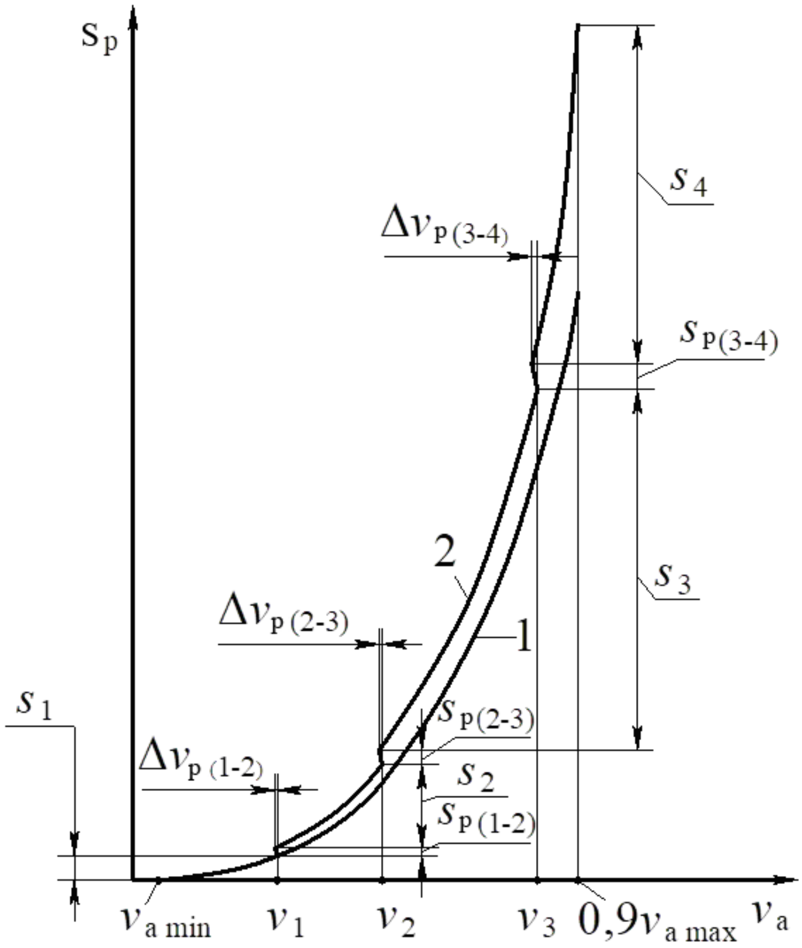


Fig. 4.22. **Acceleration path schedule** :
1 – without taking into account the time of gear shifting;
2 – taking into account the time of gear shifting

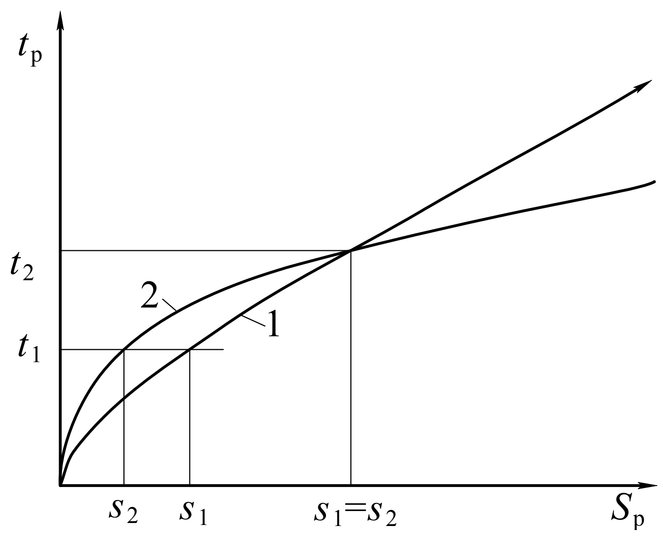


Fig. 4.23. **Acceleration characteristics of the vehicle** :
1 – characteristics of a highly dynamic vehicle; 2 – characteristics of a vehicle with low dynamics, but high maximum speed

Control questions

1. Name the gauges and indicators of traction and speed properties of the vehicle.
2. Write the equation of motion of the vehicle.
3. What factors affect the value of the coefficient of consideration of the rotating masses of the vehicle?
4. What is the equation of the traction balance and the graph of the traction characteristics of the vehicle called?
5. Write down the necessary and sufficient conditions for the continuous movement of the vehicle.
6. What is called the dynamic factor and the graph of the dynamic characteristics of the vehicle?
7. Write the equation of motion of the vehicle in dimensionless form.
8. What is called a dynamic vehicle passport?
9. How is the condition of the absence of skidding of the driving wheels determined when overcoming the road resistance ψ ?
10. What is the vehicle power balance equation and the vehicle power balance graph?
11. Write down the equations of the vehicle during acceleration and during uniform motion.
12. Explain the sequence of construction of graphs of acceleration, time and path of acceleration of a vehicle?
13. What structural parameters and operational factors affect the traction and speed properties of the vehicle?

TOPIC 5

FUEL EFFICIENCY OF THE VEHICLE

5.1. Meters and indicators of fuel efficiency vehicle

The fuel efficiency of a vehicle is its ability to rationally use fuel energy during operation in various conditions.

5.1.1. Fuel efficiency meters

The fuel efficiency of a vehicle largely depends on *the engine efficiency indicators* :

Q_t – hourly fuel consumption, kg/h – mass of fuel consumed by the engine in one hour of operation with a given load;

g_e - specific fuel consumption, g/(kW · h) - mass of fuel consumed by the engine in one hour of operation per unit of power.

Vehicle fuel efficiency meter:

q_s – highway fuel consumption, kg/100 km, and q_{s1} l/100 km – mass (volume) of fuel consumed by the vehicle engine to cover 100 km of the road.

Meters of the level of organization of the transport process:

q_{t} – specific cost of cargo transportation, g/tkm;

q_{n} – specific cost of passenger transportation, g/km.

5.1.2. Vehicle fuel efficiency indicators

The fuel efficiency of a vehicle is evaluated by the following indicators:

1 – control fuel consumption;

2 – fuel consumption in the main driving cycle on the road;

3 – fuel consumption in the urban driving cycle on the road;

4 – fuel consumption in the urban cycle on the stand;

5 – fuel characteristic of steady motion;

6 – fuel-speed characteristics on a main hilly road.

Indicator 1 is fuel consumption at a given speed of movement on a straight horizontal road in a higher gear.

Indicators 2 and *3* are the fuel consumption obtained on a straight horizontal road with regulated traffic modes imitating highway (urban) operational modes.

Indicator 4 is the fuel consumption obtained on the stand with a regulated driving mode that simulates urban operating mode.

Indicator 5 is the dependence of road fuel consumption on the speed of steady movement in a higher gear.

Indicator 6 is the dependence of road fuel consumption and driving speed on the permitted speed when driving on a main hilly road with a given longitudinal profile.

Indicator 1 is used to indirectly assess the technical condition of the vehicle. Indicator 5 is used for a comparative assessment of the fuel economy of similar vehicle, and the remaining indicators are used to determine the average fuel consumption in typical driving conditions.

5.1.3. Analysis of engine and vehicle fuel efficiency meters

The hourly fuel consumption Q_t, kg/h, is determined experimentally on the bench and characterizes the fuel consumption in steady engine mode. Figure 5.1 shows ESChE (External Speed Characteristics of the Engine) with a graph of hourly fuel consumption. To determine the hourly fuel consumption, the engine operates on a measuring stand at a specified load and shaft rotation frequency, while the amount of fuel consumed in 1 hour of operation is measured. At a given shaft rotation frequency n_e, the fuel supply and the load on the engine M_e are gradually increased so that the shaft rotation frequency remains constant until the fuel supply reaches a maximum. With a given load, the engine runs for some time t (several seconds) and the amount of fuel consumed Q (several grams) is measured . Based on the obtained data, the hourly fuel consumption, kg/h, is calculated using the equation $Q_t = 3.6 \cdot Q / t$. The hourly fuel consumption value Q_t is recorded on the ESChE graph at the corresponding rotation frequency n_e. This is repeated for several values of n_e.

The specific fuel consumption g_e, g/(kW · h), is calculated for each value of n_e according to the corresponding values of N_e and Q_t according to the equation

$$g_e = \frac{Q_t}{N_e} \cdot 1000. \qquad (5.1)$$

The specific fuel consumption allows you to assess the quality of the engine design and the perfection of the organization of its work process.

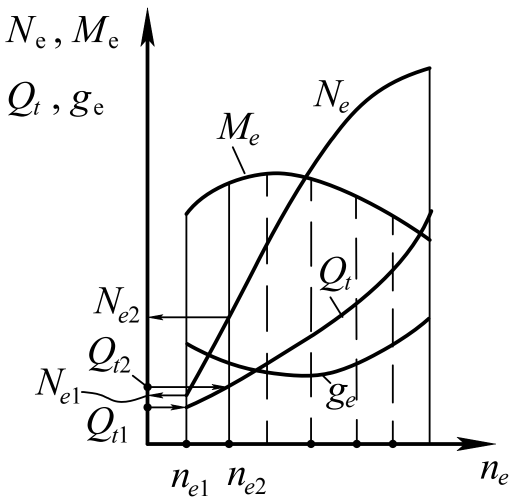


Fig. 5.1. **External speed characteristics of the engine**

For modern internal combustion engines, g_e is within

$$g_{e\min} = \begin{cases} 260\,\text{g/(kW}\cdot\text{h)}...310\,\text{g/(kW}\cdot\text{h)} - \text{ICE}; \\ 195\,\text{g/(kW}\cdot\text{h)}...230\,\text{g/(kW}\cdot\text{h)} - \text{Diesel.} \end{cases}$$

Road fuel consumption q_s is determined experimentally:
– the amount of spent fuel Q per distance S_a is measured ;
– and is calculated according to the equation

$$q_s = 100\frac{Q}{S_a}, \tag{5.2}$$

where Q – amount of spent fuel, kg;
S_a – distance traveled by the vehicle, km.

If the vehicle is in steady motion, the equation is valid

$$Q = Q_t \cdot t, \tag{5.3}$$

where t is the driving time of the vehicle.

In this case, it is possible to determine the road fuel consumption q_s, kg/100 km, using the equation

$$q_s = 100 \cdot \frac{Q_t \cdot t}{S_a} = 100\frac{Q_t}{v_a} = 100\frac{g_e \cdot N_e}{1000 v_a} = \frac{g_e \cdot N_e}{10 v_a}, \tag{5.4}$$

where v_a is the speed of steady motion, km/h.

Or in volumetric units q_{sl}, l/100 km,

$$q_{sl} = \frac{g_e \cdot N_e}{10 v_\text{a} \cdot \rho_\text{t}}, \tag{5.5}$$

where ρ_t is fuel density, kg/l.

The density of gasoline depends on its octane number and is in the range $\rho_\text{g} \approx$ 0.76 kg/l ... 0.78 kg/l. The density of diesel fuel depends on its type (summer, winter) - $\rho_\text{d} \approx$ 0.83 kg/l ... 0.86 kg/l.

The measurer of the level of organization of the transport process is the road fuel consumption in grams per unit of transport work. For example: for cargo transportation, in grams per ton-kilometer - g/tkm, and for passenger transportation in grams per passenger-kilometer - g/pkm.

$$q_\text{t} = \frac{q_s \cdot 1000}{100 G_\text{t}} = \frac{10 q_s}{G_\text{t}}, \tag{5.6}$$

where G_t is the useful load on the vehicle when carrying out transport work (tons of cargo or the number of passengers).

5.1.4. Fuel consumption equation

The fuel consumption equation can be obtained if in the equation (5.5) of road fuel consumption q_{sl} the effective power of the engine N_e is expressed in terms of the power on the driving wheels N_k and then it is replaced according to the power balance equation (4.40) by the sum $N_\psi + N_\text{v} + N_j$. After simple transformations, we get the fuel consumption equation in the form of equation (5.7).

$$q_{sl} = \frac{g_e \cdot N_e}{10\rho_\text{t} \cdot v_\text{a}} = \frac{g_e \cdot N_\text{k}}{10\rho_\text{t} \cdot v_\text{a} \cdot \eta_\text{tr}} = \frac{g_e}{10\rho_\text{t} \cdot v_\text{a}} \cdot \frac{N_\psi + N_\text{v} + N_j}{\eta_\text{tr}} =$$

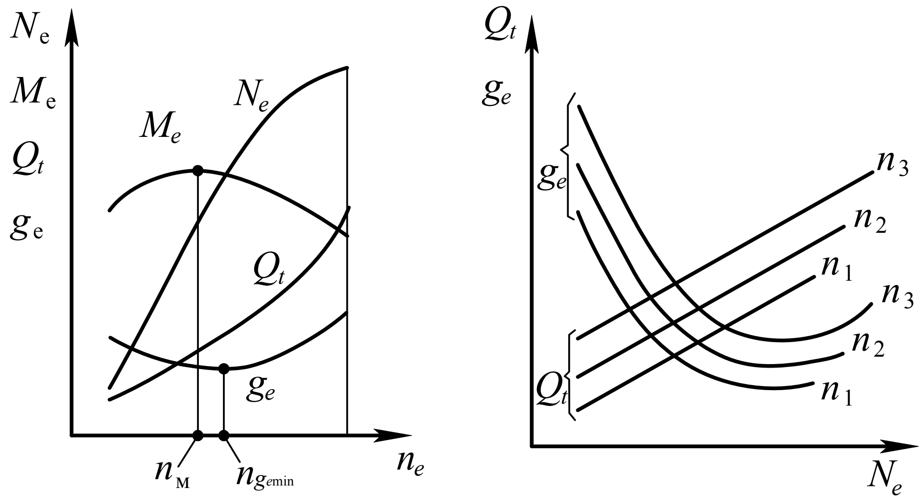
$$= \frac{g_e}{10\rho_\text{t} \cdot v_\text{a}} \cdot \frac{G_\text{a} \cdot \psi \dfrac{v_\text{a}}{3600} + \dfrac{\text{k}_\text{v} \cdot F \cdot v_\text{a}^3}{46656} + \dfrac{G_\text{a}}{g} \delta_\text{vr} \cdot j_\text{a} \dfrac{v_\text{a}}{3600}}{\eta_\text{tr}} = \ . \tag{5.7}$$

$$= \frac{g_e}{36000\rho_\text{t} \cdot \eta_\text{tr}} \cdot \left(G_\text{a} \cdot \psi + \frac{\text{k}_\text{v} \cdot F \cdot v_\text{a}^2}{13} + \frac{G_\text{a}}{g} \cdot \delta_\text{vr} \cdot j_\text{a} \right)$$

The fuel consumption equation determines the dependence of road fuel consumption on the specific fuel consumption of the engine g_e, speed v_a, acceleration j_a of movement, vehicle loading G_a and movement resistance ψ.

Using the fuel consumption equation, it is possible to find q_{sl} for given driving conditions, if the dependence is known $g_e = f\left(N_e, n_e\right)$.

The dependence of specific fuel consumption $g_e = f\left(N_e, n_e\right)$ is called *the load characteristic*. The load characteristic of the engine (Fig. 5.2b) is obtained by recalculating the parameters of the ESChE (Fig. 5.2a).



and b

Fig. 5.2. **Engine characteristics** :
a – external high-speed; b - loading

The specific fuel consumption of the engine depends on the power utilization factor. The dependence of the specific fuel consumption on the coefficient of engine power utilization at different crankshaft rotation frequencies is shown in Figure 5.3.

Professor I.S. Schlippe proposed an approximate method for determining g_e for the entire engine operating range

$$g_e = g_N \cdot k_u \cdot k_n, \tag{5.8}$$

where $g_N = (1.15...1.05)\, g_{e\,min}$ – specific fuel consumption at maximum power;

k_u – coefficient that takes into account the dependence $g_e = f(u)$;

k_n – coefficient that takes into account the dependence $g_e = f(n)$.

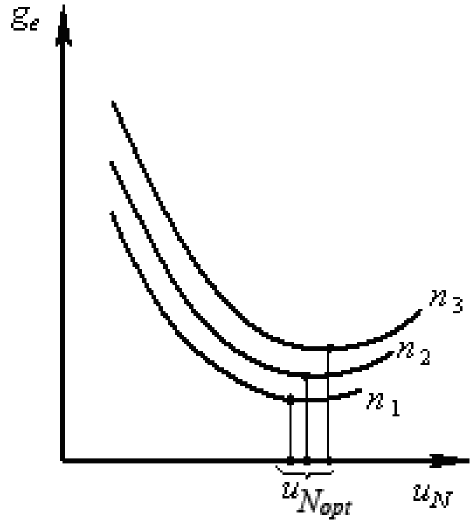


Fig. 5.3. **Influence of engine operation mode on specific fuel consumption**

The coefficient that takes into account the effect of engine shaft rotation frequency on fuel consumption is determined experimentally and can be described by a polynomial

$$k_n = 1.25 - 0{,}99\frac{n_e}{n_N} + 0.98\cdot\left(\frac{n_e}{n_N}\right)^2 - 0.24\cdot\left(\frac{n_e}{n_N}\right)^3.$$

The extremum of this polynomial $k_{n\,\min} = 0.9544$ at $n_e = 0.67\, n_N$.

The coefficient that takes into account the dependence of fuel consumption on engine load is determined for different engines.

For engines with spark ignition, this coefficient describes the following polynomial:

$$k_u = 3.27 - 8.22\frac{N_e}{N_{\max}} + 9.13\cdot\left(\frac{N_e}{N_{\max}}\right)^2 - 3.18\cdot\left(\frac{N_e}{N_{\max}}\right)^3.$$

The extremum of this polynomial $k_{u\,\min} = 0.8977$ at $N_e = 0.72\, N_{\max}$.

For diesel engines, this coefficient describes a polynomial of the form

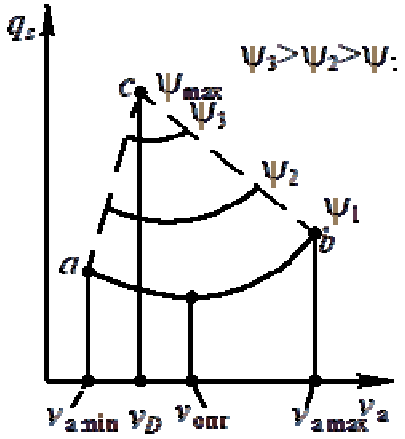
$$k_u = 1.2 + 0.14\frac{N_e}{N_{\max}} - 1.8\cdot\left(\frac{N_e}{N_{\max}}\right)^2 + 1.46\cdot\left(\frac{N_e}{N_{\max}}\right)^3.$$

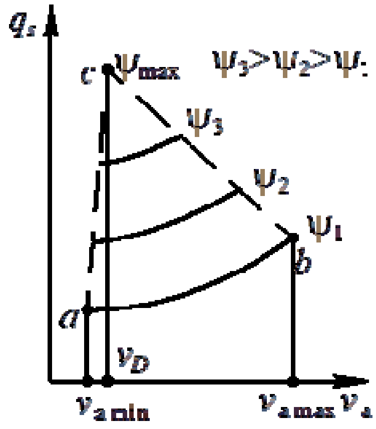
The extremum of this polynomial $k_{u\,\min} = 0.9069$ at $N_e = 0.78\, N_{\max}$.

5.2. Fuel economy characteristics of the vehicle

5.2.1. Analysis of fuel economy parameters vehicle characteristics

The fuel economy characteristic of the vehicle allows you to analyze the relationship between fuel consumption and driving conditions. Proposed by Academician E.O. Chudakov's graph of fuel economy characteristics is shown in Figure 5.4. Each dependence on the graph characterizes road fuel consumption when the vehicle moves with a different value of constant speed on roads with a different resistance coefficient. The graphs (Fig. 5.4) show the characteristics of two vehicle when moving in one of the gears in the box. Similar characteristics can be obtained for each transmission.





a b

Fig. 5.4. **Fuel economy characteristics of the vehicle** :
a – with a spark-ignition engine; b - with diesel

For vehicle with engines with spark ignition, the dependence of $q_s = f(v_a)$ can have a minimum at the speed v_{opt}. The value of the speed v_{opt} decreases with an increase in the road resistance coefficient (Fig. 5.4a). For vehicle with a diesel engine, road fuel consumption increases in the entire range of constant speed values on the road with a given drag coefficient (Fig. 5.4b) .

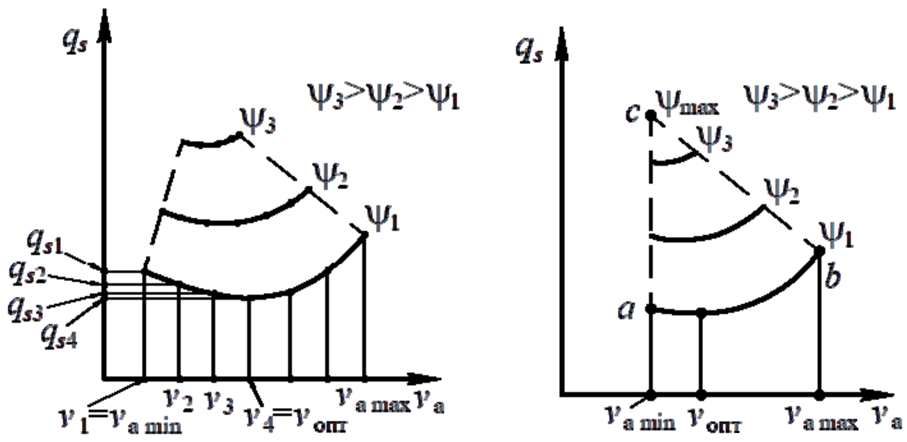
In the case of $D = \psi_{max}$, the vehicle can move only at the critical speed v_D, while the fuel consumption reaches a maximum.

Each point of the line $a – c$ connecting the first points of the dependencies $q_s = f(v_a)$ corresponds to the fuel consumption when driving at minimum stable speeds on roads with different ψ .

When the movement resistance increases, the v_{min} value increases. At the same time, the movement of the vehicle occurs at the coefficient of utilization of engine power $u_N < 100\%$. The b - c curve connecting the endpoints of the q_s dependencies $= f(v_a)$ of the fuel-economy characteristic, corresponds to the maximum possible speeds on roads with different resistance, therefore it characterizes the fuel consumption at the engine power utilization factor $u_N = 100\%$.

5.2.2. Construction of fuel economy characteristics of the vehicle

The fuel-economy characteristic of the vehicle is built according to the results of experimental studies or according to the results of the calculation (Fig. 5.5). When experimentally determining the parameters of the characteristic on the measured area with the resistance coefficient ψ_1, the vehicle moves at a constant speed $v_1 = v_{a\,min}$ and the road fuel consumption q_{s1} is measured (Fig. 5.5a).



a b

Fig. 5.5. **Construction of the fuel-economy characteristics of the vehicle** :
a – characteristic parameters; b - a simplified view of the characteristic

Then, on the same section, the vehicle moves at a constant speed v_2 and the road fuel consumption q_{s2} is measured . The experiment is also carried out for other values of constant speed v_3, v_4, ... $v_{a\,max}$ and the corresponding values of road fuel consumption q_{s3}, q_{s4}, ... are measured . The measurements are repeated on other sections of the road with different coefficients. Based on the obtained results, a schedule of fuel economy characteristics is constructed.

The constructed characteristic makes it possible to determine the on-road fuel consumption of a vehicle at a given speed on a road with a known ψ .

The parameters of the vehicle fuel economy characteristics can be obtained by calculation. One of the calculation methods has the following sequence:

1) select the gear number in the gearbox for which the characteristic is being built (further, all the selected parameters must correspond to the parameters of the vehicle movement in this gear);

2) are given by the value ψ ;

3) set 5 ... 7 vehicle speed values;

4) for each speed value, calculate the power value N_{ψ}, N_{v} , the corresponding engine power $N_e = (N_{\psi} + N_{\text{v}})/\eta_{\text{tr}}$ and the frequency of rotation of the crankshaft $n_e = v_{\text{a}} \cdot u_0 \cdot u_{\text{k}} / (0.377 \cdot r_{\text{k}})$;

5) determine the coefficient of engine power utilization $u_N = (N_{\psi} + N_{\text{v}})/(N_e \eta_{\text{tr}})$;

6) define n_e / n_N;

7) determine the coefficients k_u and k_n;

8) determine the specific fuel consumption of the engine $g_e = g_N k_u k_n$;

9) determine the travel cost $q_{sl} = \dfrac{g_e \cdot \left(N_{\psi} + N_{\text{v}}\right)}{10\rho_{\text{t}} \cdot \eta_{\text{tr}} \cdot v_{\text{a}}}$.

5.3. Factors affecting the fuel economy of a vehicle

5.3.1. Design factors affecting fuel efficiency

The fuel efficiency of a vehicle is significantly affected by:

1) *engine type* . A diesel vehicle is more economical because:

– in engines with spark ignition $g_{e\,\text{min}}$ = 260 g/(kW · h)...310 g/(kW·h);

– for diesel engines $g_{e\,\text{min}}$ = 195 g/(kW · h) ...230 g/(kW · h);

– dependence $g_e = f(u)$ in diesel engines is less than in engines with spark ignition;

– fuel density for diesel is greater than that of gasoline, which affects road fuel consumption

$$q_{sl} = \frac{g_e}{36000\rho_{\text{t}} \cdot \eta_{\text{tr}}} \cdot \left(G_{\text{a}} \cdot \psi + \frac{\text{k}_{\text{v}} \cdot F \cdot v_{\text{a}}^2}{13} + \frac{G_{\text{a}}}{g} \cdot \delta_{\text{vr}} \cdot j_{\text{a}} \right);$$

2) *improving the organization of the engine work process* :
– electronic ignition system;
– direct gasoline injection;
– regulation of gas distribution phases;
– use of supercharging with intermediate air cooling;
- disconnection of part of the cylinders;
– engine with variable working volume;
– reduction of power consumption for the drive of auxiliary units (electric fan drive);
– adiabatic working process of the engine;
3) *increasing the coefficient of use of ICE* ;
4) *selection of gear ratios of the transmission* :
– direct transmission;
– an accelerating (economical) transmission has a gear ratio i_{ku} , which is a function of two parameters:
– motor shaft rotation frequency;
– engine load;
5) *reducing the weight of the vehicle* ;
6) *improvement of aerodynamic properties of the vehicle* ;
7) *improvement of tire design* ;
8) *increasing the efficiency of the transmission* ;
9) *increase in payload* .

5.3.2. Operational factors affecting the fuel efficiency of the vehicle

1) *the speed of the vehicle* . The optimal speed in terms of fuel consumption is determined by the interval

$$v_{\text{opt}} \approx \begin{cases} 10\ \text{m/s} \ldots 12\ \text{m/s} - \text{for passenger cars;} \\ 7\ \text{m/s} \ldots 8\ \text{m/s} - \text{for trucks and buses.} \end{cases}$$

2) *utilization ratio of the truck's carrying capacity* ;
3) *selection of the optimal transmission* ;
4) *driving style* : uniformity of movement; use of kinetic and potential energy; dynamic overcoming of climbs;
5) *technical condition of the vehicle:* engine condition (ignition system, valve clearances, power system); undercarriage (tire pressure, wheel mounting angles).

Control questions

1. Name the gauges and indicators of fuel efficiency of the engine, vehicle and organization of the transport process?
2. What is the relationship between the specific fuel consumption of the engine and the road fuel consumption of the vehicle?
3. Write the equation for fuel consumption by a vehicle while moving.
4. How does its load and speed mode affect the specific fuel consumption of the engine?
5. What is the fuel economy characteristic of a vehicle?
6. Describe how the fuel economy characteristic of a vehicle is built based on the results of experimental studies?
7. Name the sequence of the calculation method of building the fuel-economy characteristics of the vehicle.
8. What design factors affect fuel efficiency?
9. What operational factors affect fuel economy?

TOPIC 6

TRACTION CALCULATION OF THE VEHICLE

6.1. Purpose of vehicle traction calculation

The purpose of the traction calculation of the vehicle is to determine the main parameters of the engine and transmission N_{max}, n_N, u_0, u_{kp}, u_p, which provide the vehicle with the maximum speed v_{max} on normal roads and the possibility of driving on roads with increased resistance.

When calculating the traction of a newly designed vehicle, it is necessary to have actions with the following groups of parameters:

1) parameters set by the technical specifications (TS) for the project;
2) selected parameters ;
3) calculated parameters.

Parameters set by the TS:

1) type of vehicle: passenger car, truck or bus (coach);
2) cargo capacity (passenger capacity): q , n ;
3) maximum speed $v_{a\,max}$;
4) road resistance coefficient ψ_v, which the vehicle can overcome at maximum speed $v_{a\,max}$;
5) the maximum coefficient of road resistance ψ_{max} , which the vehicle can overcome in a lower gear ;
6) minimum stable speed $v_{a\,min}$;
7) engine type;
8) type of transmission.

Selectable parameters:

1) weight of the vehicle in the equipped state: G_0;
2) flow factor: $W_v = k_v \cdot F_a$;
3) weight distribution along the axes : G_{01} and G_{02}, G_1 and G_2;
4) engine crankshaft revolutions at maximum power n_N, (n_0, n_v) ;
5) Transmission efficiency: η_{tr}.

Calculated parameters:

1) maximum engine power N_{max};
2) gear ratio of the main gear , u_0;
3) gear ratios of the gearbox: u_i, i – number of gears;
4) gear ratio of an additional gearbox (multiplier, demultiplier, transfer box).

6.2. Determination of vehicle parameters to be selected

As a result of the analysis of the operational and technical characteristics of the closest analogues, the following are established and substantiated: the layout diagram of the vehicle, gross weight, number of axles, including driving axles, distribution of gross weight among the axles, base, height of the center of gravity above the supporting surface, overall height and track of the front wheels and on their basis - the area of the frontal surface of the vehicle, the type and size of the tires, and behind it - the static radius of the wheel with the load, the type and speed of the engine, the coefficient of air resistance and the efficiency of the transmission. At the same time, the main trends in the development of vehicle designs are taken into account: reducing fuel consumption and reducing the equipped mass. When choosing the layout scheme of passenger vehicle, it is taken into account that in recent years front-wheel drive vehicle have been developed in especially small and small classes.

First, the equipped weight of the vehicle is determined (the weight of the vehicle with fuel and equipment, but without the driver and passengers).

The curb weight of passenger vehicle depends on the class and group. The working volume of the engine is the main parameter that determines the belonging of the vehicle to the corresponding class and group, for the designed vehicle it is accepted by analogy. The curb weight of a passenger vehicle is selected from the appropriate range according to table 6.1. Smaller values of the equipped weight (Table 6.1) are accepted for front-wheel drive vehicle, and larger values for rear-wheel drive vehicle.

Table 6.1 – **Curb mass ranges of passenger car according to class and group**

Class	Group	Working volume of the engine V_L, l	Equipped mass m_o, kg
1	1	to 0.849	to 699
	2	0.85 ÷ 1.099	700 ÷ 864
2	1	1.1 ÷ 1.299	865 ÷ 989
	2	1.3 ÷ 1.499	965 ÷ 1139
	3	1.5 ÷ 1.799	1115 ÷ 1239
3	1	1.8 ÷ 2.499	1240 ÷ 1319
	2	2.5 ÷ 3.499	1390 ÷ 1609
4	1	3.5 ÷ 4.999	1610 ÷ 2020
	2	more than 5	not regulated

For trucks, the estimated curb weight of the vehicle is determined based on the statistical data of the analysis of the closest analogues, based on the carrying capacity coefficient

$$k_g = \frac{q_n}{m_{0p}}, \tag{6.1}$$

where q_{n} is the carrying capacity of the analogue vehicle;

$m_{0\text{p}}$ is the equipped mass of the analogue vehicle.

The curb weight of the designed truck is determined by the average value of the carrying capacity coefficient of three similar vehicles according to the equation

$$m_0 = \frac{q}{k_{\text{gs}}}, \tag{6.2}$$

where $k_{\text{gs}} = \text{S}\, k_{\text{g}}/3$ is the average value of the carrying capacity coefficient.

The curb weight of the bus depends on its nominal capacity. At the same time, the nominal capacity n means the number of passengers carried by the bus under normal traffic conditions (not during peak hours). The curb weight of the bus is determined by the specific curb weight of the analog bus

$$\alpha_{m_0} = \frac{m_{0\text{n}}}{n_{\text{n}}}, \tag{6.3}$$

where $m_{0\text{n}}$ is the equipped mass of the analogue bus;

n_{n} is the nominal number of passengers of the analogue bus.

The curb weight of the designed bus is determined by the average value of the specific curb weight of similar buses according to the equation

$$m_0 = n \cdot \alpha_{m_0\text{c}}, \tag{6.4}$$

where $\alpha_{m_0\text{c}} = \Sigma\alpha_{m0}/3$ – average value of the specific equipped mass of analogue buses.

The total weight of the vehicle can be determined by the equations:

– for trucks

$$m_{\text{a}} = m_0 + m_{\text{gr}} + \left(m_{\text{ch}} + m_{\text{b}}\right) \cdot \left(n_{\text{c}} + 1\right); \tag{6.5}$$

– city buses

$$m_a = m_0 + (m_{ch} + m_b) \cdot (n_c + n_r + 1); \qquad (6.6)$$

– passenger vehicle and long-distance buses

$$m_a = m_0 + (m_{ch} + m_b) \cdot (n_c + 1), \qquad (6.7)$$

where m_0 is the equipped mass of the vehicle (the mass of the vehicle with refueling and equipment, but without the driver and passengers), kg;

m_{gr} – load weight (carrying capacity), kg;

m_{ch} =75 kg – weight of the driver or passenger;

n_c и n_r – the number of seats for passengers and for standing, respectively;

m_b – weight of baggage, kg.

The weight of the luggage of the driver and passenger of trucks and city buses is equal to 5 kg, buses for intercity transport - 15 kg and passenger vehicle - 10 kg per person.

The distribution of the total mass between the axles of a passenger vehicle depends on its layout. With a classic layout, the rear axle is taken as (0.52 – 0.55)· m_a , with a front-wheel drive layout (0.43 – 0.47)· m_a , with a rear engine – (0.56 – 0.60)· m_a.

Determining the distribution of the weight of a truck on its axles is performed on the condition of full use of the load capacity of the tires, compliance with the norms limiting the maximum permissible load on the axle, and statistical data. At the same time, it is taken into account that according to the permissible load per axle, vehicle are divided into two groups: A and B. For vehicle of group A, the load per axle is allowed up to 100 kN, group B - up to 60 kN. The permissible load on one bridge of a balancing suspension depends on the distance between the axes of adjacent bridges and is 70% - 100% of the load that is allowed on a single bridge with a distance between bridges of 1 m - 2.5 m, respectively. An increase in the load on the driving axle leads to an increase towing weight and in accordance with the increase in its passability. But at the same time, the carrying capacity decreases. The weight of the drive axle, the wheels of which have double tires, is recommended to be taken in vehicle of group A (0.65 – 0.70)· m_a , and in vehicle of group B – (0.7 – 0.80)· m_a. A separate group consists of off-road vehicle, the load on the axles of which is usually not regulated. The movement of such vehicle on public roads is generally prohibited.

6.3. Determination of maximum engine power

Determination of the maximum engine power N_{max} is performed in two stages.

The first stage. At the first stage, the power N_v is determined, necessary for smooth movement of the vehicle at maximum speed. The balance of the vehicle power when moving at maximum speed

$$N_k = N_\psi + N_v;$$

$$N_e \cdot \eta_{тр} = \frac{G_a \cdot \psi_v \cdot v_{a\,max}}{3600} + \frac{k_v \cdot F \cdot v^3_{a\,max}}{46656}. \quad (6.8)$$

Engine power at maximum speed $N_e = N_v$

$$N_v = \frac{G_a \cdot \psi_v \cdot v_{a\,max}}{3600 \cdot \eta_{tr}} + \frac{k_v \cdot F \cdot v^3_{a\,max}}{46656 \cdot \eta_{tr}}, \quad (6.9)$$

where $G_a = m_a \cdot g$ – gravitational force of the full mass vehicle;

ψ_v – coefficient of total road resistance at the maximum speed of the vehicle ($\psi_v = f_v = f_0 + 5{,}4 \cdot 10^{-7} \cdot v^2_{a\,max}$).

The coefficient of rolling resistance at low speed can be approximately taken for vehicle f_0 = 0.01 and for trucks f_0 = (0.015...0.02).

The second stage. At the second stage, the maximum power of the engine is determined. The maximum power of the engine is a function of the calculated power N_v, the type of internal combustion engine and the crankshaft speed n_N chosen according to the prototype

$$N_{e\,max} = f(N_v, n_N). \quad (6.10)$$

Case 1 – gasoline engines without a maximum speed limiter (passenger vehicle engines).

In this case, the maximum speed of the vehicle is limited by the power of the engine. At the same time, the maximum effective power of the engine is equal to

$$N_{e\,max} = \frac{N_v}{A_1 \frac{n_v}{n_N} + A_2 \left(\frac{n_v}{n_N}\right)^2 - \left(\frac{n_v}{n_N}\right)^3}, \quad (6.11)$$

where n_v is the rotation frequency of the engine shaft at the maximum speed of the vehicle;

$A_1 = A_2 = 1$ – empirical coefficients characterizing the engine type.

It is obvious that in this case the engine shaft rotation frequency at the maximum vehicle speed n_v is the maximum engine shaft rotation frequency. For modern designs of internal combustion engines without a limiter of the maximum rotation frequency of the engine shaft, the ratio $n_v = n_{e\max} = (1.01...1.05) \cdot n_N$ is characteristic . It is also obvious that the maximum engine power in this case is equal to the maximum effective engine power $N_{\max} = N_{e\max}$.

Case 2 – gasoline engines with a maximum speed limiter (engines of trucks and powerful vehicle), as well as diesels.

In this case, the maximum speed of the vehicle is limited by the maximum frequency of rotation of the engine shaft. At the same time, equality is fair

$$n_v = n_{\max} = n_o = n_N, \quad (6.12)$$

where n_o – rotation frequency of the engine shaft at which the limiter of the maximum rotation frequency is activated.

It is obvious that the power of the engine when moving at maximum speed is the maximum power of the engine, since the limiter of the maximum speed of rotation prevents the increase of effective power and a fair equality

$$N_v = N_{\max}, \quad (6.13)$$

where $N_{\max}$ is the maximum engine power corresponding to the shaft rotation frequency n_N.

The maximum effective engine power $N_{e\max}$ would be achieved at the shaft rotation frequency $\psi_{\max}$, if there was no speed limiter. Its value can be determined by the calculated power $N_v = N_{\max}$ and the value of the frequency n_N selected according to the analogue of the given type of engine

$$N_{e\max} = \frac{N_v}{A_1 \frac{n_v}{n_N^*} + A_2 \left(\frac{n_v}{n_N^*} \right)^2 - \left(\frac{n_v}{n_N^*} \right)^3}, \quad (6.14)$$

where n_N^* – theoretical rotation frequency of the engine shaft, at which the maximum effective power $N_{e\max}$ would be achieved in the absence of a limiter of the maximum rotation frequency (for diesel n_N^*=(1.0...1.1)·n_N; for gasoline engines with a limiter maximum frequency n_N^*=(1.1...1.25)·n_N).

Empirical coefficients characterizing diesel A_1 = 0.5, A_2 = 1.5.

6.4. Determination of gear ratios of the transmission

6.4.1. Determination of the gear ratio of the main gear u_o

The gear ratio of the main gear is determined from the condition of the vehicle moving at maximum speed at the maximum frequency of rotation of the engine shaft.

The dependence of the speed of the vehicle on the frequency of rotation of the engine shaft is known

$$v_a = 0.377 \frac{n_e \cdot r_k}{u_o \cdot u_k}. \quad (6.15)$$

When the vehicle is moving at maximum speed, the equality $v_a = v_{a\,max}$ and $n_e = n_{e\,max}$ take place . Given this, we will determine the gear ratio of the main gear from the equation of the vehicle speed

$$u_o = 0{,}377 \frac{n_{e\max} \cdot r_k}{v_{a\max} \cdot u_{kv}}, \quad (6.16)$$

where u_{kv} – higher calculated gear ratio of the gearbox, at which the maximum speed of the vehicle is reached.

In known vehicle designs, the above estimated transmission ratio of the gearbox is accepted depending on the structural scheme of the gearbox, the type of vehicle and the type of engine:

– for permanent gearboxes of passenger vehicle and light trucks with a gasoline engine u_{kv} = 1;

– for permanent gearboxes of vehicle with a diesel engine u_{kv} = 0.6 ... 0.8;

– for two-shaft gearboxes of front-wheel drive passenger vehicle u_{kv} = 0.6 ... 0.9 (a smaller value for vehicle with a diesel engine);

– for two-shaft gearboxes of passenger vehicle with a rear engine configuration u_{kv} = 0.94 ... 1.06.

The calculated value of the transmission ratio of the main gear u_o is then refined from the condition of an integer value of the number of teeth of engagement.

6.4.2. Determination of gear ratios of the gearbox

6.4.2.1. Determination of the gear ratio of the first stage of the gearbox

The gear ratio of the first stage of the gearbox (first gear) is selected from the following conditions:

– the ability to overcome the maximum road resistance with a given road resistance coefficient ψ_{max};

– maneuvering capabilities with a given minimum stable speed $v_{a\,min}$;

– absence of skidding of the driving wheels when overcoming the maximum road resistance.

The gear ratio of the first gear, provided that it is possible to overcome the maximum road resistance. In this case, the vehicle develops the maximum dynamic factor

$$D_{max} = \frac{P_{k1}^{max} - P_v}{G_a}. \qquad (6.17)$$

Overcoming the maximum road resistance occurs at a constant speed, since the vehicle is no longer able to accelerate. At the same time, the equality is valid $D_{max} = \psi_{max}$. Taking this into account and assuming that at the speed of the vehicle at which the maximum road resistance is overcome, the force of air resistance is little different from zero, it is possible to write

$$\frac{P_{k1}^{max}}{G_a} = \psi_{max}. \qquad (6.18)$$

Revealing the value of the total traction force on the driving wheels, we get

$$\frac{M_{e\,max} \cdot u_{1\psi} \cdot u_o \cdot \eta_{tr}}{G_a \cdot r_d} = \psi_{max}, \qquad (6.19)$$

where will we get

$$u_{1\psi} = \frac{G_a \cdot \psi_{max} \cdot r_d}{M_{e\,max} \cdot u_o \cdot \eta_{tr}}, \qquad (6.20)$$

where $u_{1\psi}$ is the gear ratio of the first gear, which provides the vehicle with the ability to overcome the given maximum road resistance.

The gear ratio of the first gear under the condition of ensuring the possibility of maneuvering with a given minimum stable speed $v_{\text{a min}}$ is determined by the equation

$$u_{1v} = 0.377 \frac{n_{e\min} \cdot r_{\text{k}}}{u_{\text{o}} \cdot v_{\text{a min}}}, \tag{6.21}$$

where u_{1v} is the gear ratio of the first gear, which provides the possibility of maneuvering with the specified minimum speed $v_{a\ \min}$.

$n_{e\ \min}$ – the minimum steady frequency of rotation of the crankshaft of the engine, rpm;

$v_{\text{amin}} =< (4 \ldots 5)$ km/h – maneuvering speed.

Usually we take $n_{e\min} = (0.16 \ldots 0.18)\, n_N$.

Gear ratio of the first gear, provided there is no skidding of the driving wheels when overcoming the maximum road resistance

$$D_{\varphi} \geq D_{\max} = \psi_{\max}. \tag{6.22}$$

For a rear-wheel drive vehicle, the dynamic factor is the adhesion utilized

$$D_{\varphi} = \frac{P_{\varphi 2}}{G_{\text{a}}} = \frac{G_2 \cdot m_{z2} \cdot \varphi_x}{G_{\text{a}}} = \frac{P_{\text{к max}}}{G_{\text{a}}} = \frac{M_{e\max} \cdot u_{1\varphi} \cdot u_0 \cdot \eta_{\text{tr}}}{G_{\text{a}} \cdot r_{\text{d}}},$$

where do we get

$$u_{1\varphi} = \frac{G_2 \cdot m_{z2} \cdot \varphi_x \cdot r_{\text{d}}}{M_{e\max} \cdot u_0 \cdot \eta_{\text{tr}}}. \tag{6.23}$$

The procedure for selecting the gear ratio of the first gear of the gearbox:

a) compare the values of $u_{1\psi}$ and u_{1v} and choose the larger value;

b) compare the value of the gear ratio chosen under point "a" with the value $u_{1\varphi}$ and choose a smaller value;

c) the value of the transmission ratio of the gearbox, selected according to point "b", is assigned the designation $u_{\text{k}1}$.

6.4.2.2. Determination of the number of gearbox stages

The number of steps is one of the main characteristics of the gearbox and is determined by its range, type and purpose of the vehicle. Gearbox range D_k is a ratio

$$D_k = \frac{u_{k1}}{u_{kv}}. \tag{6.24}$$

In general, the more steps in the gearbox, the better the traction characteristics of the vehicle. It should be noted that an increase in the number of steps leads to an increase in the price of the box and causes difficulties for the driver in choosing the desired gear. Therefore, non-automatic transmissions for vehicle have 4 to 6 gears and for trucks 5 to 12. Automatic transmissions can have more gears. Such gearboxes for trucks have 16-24 gears. The dependence of the number of steps in the gearbox on its range is shown in table 6.2.

Table 6.2 – **Dependence of the number of stages of the gearbox on its range**

Parameter	Value						
Box range D_k	3.1–4.7	5.7–8.5	7.9–9.35	8–10	9.2–13.5	13–19,4	17–24.7
Number of degrees, n_k	4	5	6	8	10	16	20

If the gear ratio of the first gear is more than 8.5, the overall dimensions of the gearbox of the designed vehicle will be too large. To reduce the dimensions of the gearbox, an additional gearbox is added to the vehicle transmission.

In this case, the gear ratio of the first gear defines the equation

$$u_{1\psi} = \frac{G_a \cdot \psi_{max} \cdot r_d}{M_{e\max} \cdot u_o \cdot u_{rn} \cdot \eta_{tr}}, \tag{6.25}$$

where u_{rn} *is* the gear ratio of the lower gear of the additional gearbox.

As a rule, the gear ratio of the lower gear of the additional gearbox is within u_{rn} = 1.8 ... 2.3.

6.4.2.3. Determination of gear ratios of stages intermediate gears of the box

The determination of the gear ratios of the stages of the intermediate gears of the box is usually performed so that their values are distributed according to a geometric progression or a harmonic series:

a) Gear ratios of gearbox *distributed according to geometric progression:*

$$u_{ki} = \sqrt[n_k - 1]{u_{k1}^{n_k - i}}, \tag{6.26}$$

where n_k – number of gears in the box (except for the accelerating gear and the reverse gear);

i – serial number of the transmission.

In this case, the possibility of engine operation during acceleration of the vehicle is ensured in the same mode on all gears in the gearbox, which is mainly for trucks .

b) Transmission numbers of gearbox *distributed according to the harmonic series:*

$$u_{ki} = \frac{u_{k1}}{\frac{i-1}{m_v - 1} \cdot \left(\frac{u_{k1}}{u_{kv}} - 1 \right) + 1}, \tag{6.27}$$

where m_v – gear number in the gearbox, at which $v_{a\,\max}$ is reached (this is the gear number to which the gear ratio $u_{k\,v}$ corresponds).

In this case, an increase in speed during acceleration of the vehicle is ensured by the same amount in each gear in the gearbox, which is mainly for passenger vehicle.

c) Determination of the gear ratio of the accelerating (economical) transmission of the gearbox.

ratio of the accelerating (economical) transmission of the gearbox is determined from the condition of ensuring minimum road fuel consumption during long-term movement of the vehicle at a constant speed, with the so-called "cruising" speed.

$$u_e = \frac{n_{ge\min} \cdot r_k}{v_{ek} \cdot u_0}, \tag{6.28}$$

where $n_{ge\,\min} = (0.6...0.7) \cdot n_N$ – crankshaft rotation frequency at which the engine operates with the lowest specific fuel consumption;

v_{ek} – vehicle speed at which it is desirable to have the lowest road fuel consumption.

Usually, the value of the "cruising" speed of movement is taken according to the permissible speed, for example v_{ek} = 90 km/h, or v_{ek} = 130 km/h.

d) Determination of the gear ratio of the reverse gear.

The value of the gear ratio of the reverse gear is determined for constructive reasons when developing the design of the gearbox. It usually has a meaning

$$u_r = (0.9...1.3) \cdot u_{k1} . \quad (6.29)$$

6.4.2.4. Determination of the gear ratio of the transfer case boxes

Usually, there are two gears in the transfer case: higher and lower. The higher one is specified, and the lower one is determined from the condition of wheel slippage

$$u_{rn} = \frac{G_a \cdot \varphi_x \cdot r_d}{M_{e\max} \cdot u_{k1} \cdot u_0 \cdot \eta_{tr}}, \quad (6.30)$$

where φ_x = 0.8 – adhesion utilized coefficient.

The found value must be checked by the value of the minimum steady speed obtained at a given value of u_{rn}.

$$v_{a\,min} = 0.377 \frac{n_{e\min} \cdot r_k}{u_0 \cdot u_{k1} \cdot u_{rn}}, \quad (6.31)$$

where $n_{e\,min}$ is the minimum stable revolutions of the crankshaft of the engine;

$v_{a\,min}$ = (2 – 3) km/h – the recommended minimum speed.

The minimum stable revolutions of the crankshaft of the engine are $n_{min} = 0.5\, n_N$.

Control questions

1. State the types of traction calculation and their purpose.
2. What groups of parameters are determined during the traction calculation of the vehicle?

3. Describe how the maximum engine power is determined.
4. Describe the sequence of determining the gear ratios of the transmission.
5. Write a equation to determine the gear ratio of the main gear.
6. Write the equations for determining the gear ratio of the first stage of the gearbox.
7. How do you determine the number of gears in a gearbox?
8. How are the gear ratios of the intermediate gears in the box determined?
9. How are the economy and reverse gear ratios determined?
10. How are transfer case ratios determined?

TOPIC 7

BRAKING DYNAMICS OF THE VEHICLE

7.1. The process of braking a vehicle and the equation of motion of a vehicle during braking

The process of braking a vehicle is the creation of artificial resistance to the rotation of the wheels. At the same time, the vehicle kinetic energy is spent on performing friction work in the braking mechanisms and the tire's contact patch with the road. The heat generated at the same time dissipates in the surrounding atmosphere.

The ability of a vehicle to perform braking with maximum efficiency and with the efficiency necessary to control traffic, to remain in a braked state in place, and to move at a constant speed on long descents is called *braking properties of the vehicle* . The braking properties of a vehicle determine not only its braking dynamics, but also its general dynamics of movement.

Types of braking:

– *emergency braking* – braking with the maximum possible efficiency. Goal: stopping the vehicle as soon as possible (3%...5% of all braking);

- *service braking* - braking with the efficiency necessary to control the movement of the vehicle. The goal is to reduce the speed of the vehicle (95%...97% of all braking).

Braking techniques:

– braking only by the braking system;

– braking only by the engine;

– joint braking by the braking system and the engine;

– braking by the periodic action of the braking system.

When the vehicle brakes, the fuel supply stops and the traction force $R_k = 0$. Natural friction in the tires and air resistance P_v reduce the speed of the vehicle and, as a result, the force of inertia appears. At the same time, if the clutch is not turned off, then the moment of resistance in the engine M_r is transmitted to the driving wheels from the engine crankshaft, which causes the appearance of a rotational resistance force P_r on them . After pressing the brake pedal, the braking force P_t appears on the wheels.

In this case, the resistance to the rotation of the wheels and, accordingly, the movement of the vehicle is formed not only by natural friction in the tires, but also by the resistance of the engine, which is connected to the driving wheels, and by artificially created friction in the braking mechanisms. As a result of resistance to wheel rotation, longitudinal reactions R_{x1} , R_{x2} , directed against the vehicle's speed vector, occur in the spot of contact with the supporting surface of the road (Fig. 7.1).

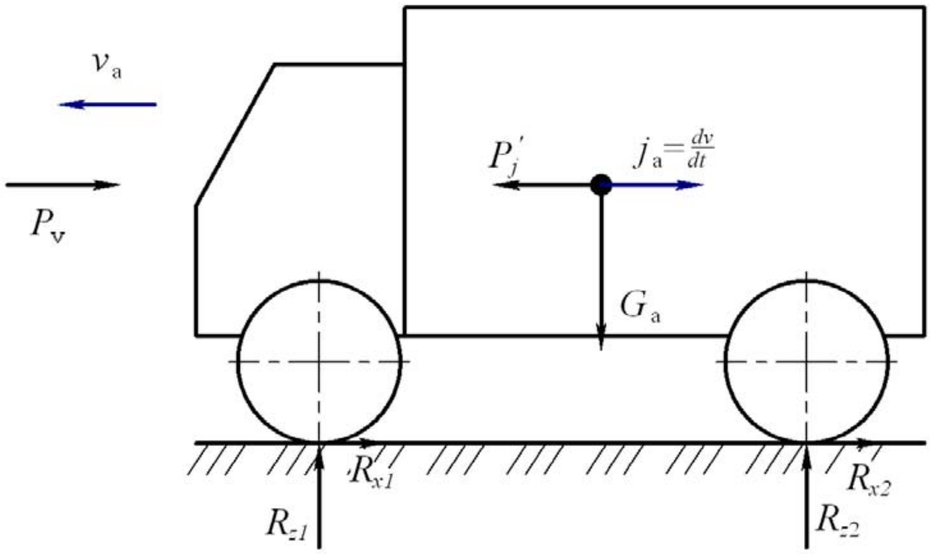


Fig. 7.1. **Diagram of the forces acting on the vehicle during braking**

The equation of the vehicle motion during braking connects the forces that cause a change in its dynamic state - a decrease in speed and inertial forces that oppose this change. Usually, the vehicle braking process is considered on a horizontal road.

The sum of the forces causing a decrease in the speed of the vehicle

$$\sum P = R_{x1} + R_{x2} + P_v. \tag{7.1}$$

The forces that cause the vehicle speed to change and the inertial forces that result from the change in speed are always equal but oppositely directed

$$\sum P = P_{j_T}, \tag{7.2}$$

where $P_{j_T} = m_a \cdot \left(-dv/dt\right)$ is the force of inertia of the forward moving parts of the vehicle during braking.

We denote the negative acceleration of the vehicle " $-\frac{dv}{dt}$" by the identifier $j_{\text{т}}$ and call it *deceleration* , and we define the mass of the vehicle as the ratio $\frac{G_a}{g}$, then

$$P_{j_{\text{T}}} = \frac{G_{\text{a}}}{g} \cdot j_{\text{T}} . \tag{7.3}$$

Taking into account the values of the forces, equality (7.2) will take the form

$$R_{x1} + R_{x2} + P_{\text{v}} = \frac{G_{\text{a}}}{g} \cdot j_{\text{T}} , \tag{7.4}$$

or

$$R_{x1} + R_{x2} + P_{\text{v}} - \frac{G_{\text{a}}}{g} \cdot j_{\text{T}} = 0 , \tag{7.5}$$

where $R_{x1} = \frac{M_{\text{T1}}}{r_{\text{d}}} + P_{f1} - \frac{J_{\text{k1}}}{r_{\text{d}} \cdot r_{\text{k}}} \cdot \left(-\frac{dv}{dt} \right)$ – longitudinal reaction to the wheels of the front axle;

$R_{x2} = \frac{M_{\text{T2}}}{r_{\text{d}}} + P_{f2} - \frac{J_{\text{k2}} + J_e \cdot u_{\text{tr}}^2 \cdot \eta_{\text{tr}}}{r_{\text{d}} \cdot r_{\text{k}}} \cdot \left(-\frac{dv}{dt} \right) + \frac{M_{\tau} \cdot u_{\text{tr}}}{r_{\text{д}} \cdot \eta_{\text{tr}}}$ – longitudinal reaction on the wheels of the rear axle connected to the engine.

Taking into account the values of the reactions R_{x1}, R_{x2} and what $-\frac{dv}{dt} = j_{\text{T}}$ equation (7.5) will take the form

$$\frac{M_{\text{T1}} + M_{\text{T2}}}{r_{\text{d}}} + \sum P_f - \frac{J_{\text{k1}}}{r_{\text{d}} \cdot r_{\text{k}}} \cdot j_{\text{T}} - \frac{J_{\text{k2}} + J_e \cdot u_{\text{tr}}^2 \cdot \eta_{\text{tr}}}{r_{\text{d}} \cdot r_{\text{k}}} \cdot j_{\text{T}} + \frac{M_{\tau} \cdot u_{\text{tr}}}{r_{\text{d}} \cdot \eta_{\text{tr}}} + P_{\text{v}} - \frac{G_{\text{a}}}{g} \cdot j_{\text{T}} = 0. \tag{7.6}$$

denote

$P_{\text{T}} = P_{\text{T1}} + P_{\text{T2}} = \frac{M_{\text{T1}}}{r_{\text{d}}} + \frac{M_{\text{T2}}}{r_{\text{d}}}$ – braking power of the vehicle;

$P_f = \sum P_f = P_{f1} + P_{f2}$ – the rolling resistance of the vehicle;

$P_{j_T} = \frac{G_a}{g} \cdot j_T = z$ – inertial force of forward moving parts of the vehicle;

$P_{J_k+J_e} = \frac{\sum J_k + J_e \cdot u_{tr}^2 \cdot \eta_{tr}}{r_d \cdot r_k} \cdot j_T$ – the total inertial force of the vehicle rotating masses applied to the brake wheels;

$P_\tau = \frac{M_\tau \cdot u_{tr}}{r_d \cdot \eta_{tr}}$ is the braking force of the engine applied to the braking wheels (M_τ – moment of resistance to rotation of the crankshaft in engine braking mode).

Taking into account the accepted notations, the equation of motion of the vehicle under braking will take the form

$$P_T + P_f - P_{J_k+J_e} + P_v + P_\tau - P_{j_T} = 0 . \tag{7.7}$$

When braking the vehicle with the engine disconnected from the wheels, the equation of motion will take the form

$$P_T + P_f - P_{J_k} + P_v - P_{j_T} = 0 , \tag{7.8}$$

where $P_{J_k} = \frac{\sum J_k}{r_d \cdot r_k} \cdot j_T$ is the force of inertia of the rotating masses of the wheels.

With emergency braking, that is, with full use of the grip properties of the wheels with the road, it can be assumed that $P_v \approx 0$ then

$$P_{T\max} + P_f - P_{J_k} - P_{j_T\max} = 0 . \tag{7.9}$$

At the same time, the maximum total longitudinal reaction is formed on the wheels, equal to the force of adhesion of the wheels of the vehicle with the road P_φ, i.e. $R_{x\max} = P_\varphi = P_{T\max} + P_f - P_{J_k}$, accordingly, it can be written

$$R_{x\max} - P_{j_T\max} = 0 . \tag{7.10}$$

Assuming the equality of the adhesion utilized coefficients of the wheels of the front and rear axles $\varphi_{x1} = \varphi_{x2} = \varphi_{x\max}$, it can be assumed that

$$R_{x\max} = P_{\varphi 1} + P_{\varphi 2} = \left(R_{z1} + R_{z2}\right) \cdot \varphi_x = G_a \cdot \varphi_x .$$

Taking into account the accepted assumption and, revealing $P_{j_{\text{T}}\max}$, let's transform equation (7.10) into the form

$$G_{\text{a}} \cdot \varphi_{x\max} - G_{\text{a}} \cdot z_{\max} = 0. \tag{7.11}$$

After dividing equation (7.11) by the weight of the vehicle G_{a}, we get the equation of the vehicle motion when braking with maximum efficiency in the dimensionless form of the record

$$\varphi_{x\max} - z_{\max} = 0. \tag{7.12}$$

7.2. Meters and indicators of braking properties of the vehicle

As measures of the effectiveness of braking properties (by analogy with traction dynamics), the following are used: deceleration j_{T}; braking time t_{T}; braking distance S_{T}. Indicators of braking efficiency are:

– maximum deceleration during emergency braking $j_{\text{T}\max}$;
– minimum braking time at a given speed;
– the minimum braking distance at a given speed.

7.2.1. Maximum deceleration of the vehicle

The maximum deceleration of the vehicle during emergency braking is calculated from the equation of motion presented in the relative form

$$\varphi_{x\max} = z_{\max} = \frac{j_{\text{T}\max}}{g}. \tag{7.13}$$

At the adhesion utilized coefficient $\varphi_{x\max} = z_{\max} = 0.8$ the maximum possible deceleration is equal to $j_{\text{T}\max} = 7.85$ m/s^2.

7.2.2. Emergency braking time

The time of emergency braking of the vehicle is determined by the maximum deceleration $j_{\text{T}\max}$, expressing it as a negative acceleration $-dv/dt$

$$-\frac{dv}{dt} = g \cdot z_{\max}. \tag{7.14}$$

Let's turn equation (7.14) into a differential equation with separate variables

$$dt = -\frac{1}{g \cdot z_{\max}} dv. \tag{7.15}$$

To determine the braking time, let's integrate the left and right parts of equation (7.15) by the corresponding variables:

$$\int_{0}^{t_{\text{T}}} dt = -\frac{1}{g \cdot z_{\max}} \int_{v_{\text{n}}}^{v_{\text{k}}} dv, \tag{7.16}$$

where 0 and v_{n} are the lower limits of integration corresponding to the initial time and initial speed of braking;

t_{T} and v_{k} are the upper limits of integration, corresponding to the braking time and the final braking speed.

Assuming that $z_{\max}$ does not change during the braking time, we determine the braking time

$$t_{\text{T}} = -\frac{1}{g \cdot z_{\max}} \left(v_{\text{k}} - v_{\text{n}} \right). \tag{7.17}$$

When braking to a complete stop, given that $v_{\text{k}} = 0$, the braking time becomes significant

$$t_{\text{T}} = \frac{v_{\text{n}}}{g \cdot z_{\max}}. \tag{7.18}$$

In equation (7.18), all components are represented in the SI system. To use the dimension of speed in km/h, let's rewrite equation (7.18) in the form

$$t_{\text{T}} = \frac{v_{\text{an}}}{3.6 \cdot g \cdot z_{\max}}, \tag{7.19}$$

where v_{an} is the initial braking speed, km/h.

The minimum time for braking the vehicle to a complete stop with an initial speed of v_{an} = 80 km/h, with a adhesion utilized coefficient $\varphi_{x\max} = z_{\max} = 0.8$, according to equation (7.19) is equal to $t_{\text{T}} = 2.83$ s.

7.2.3. Vehicle braking distance

The braking distance S_T of the vehicle is determined from the differential equation

$$dS = v \cdot dt. \tag{7.20}$$

Substitute the value of *dt* from equation (7.15) into it

$$dS = -\frac{1}{g \cdot z_{max}} v \cdot dv. \tag{7.21}$$

Let's integrate this equation within the appropriate limits

$$\int_0^{S_T} dS = -\frac{1}{g \cdot z_{max}} \int_{v_n}^{v_k} v \cdot dv, \tag{7.22}$$

and determine the value of the braking distance when braking from v_n to v_k

$$S_T = -\frac{1}{2 \cdot g \cdot z_{max}} \left(v_k^2 - v_n^2\right) \text{ or } S_T = \frac{1}{2 \cdot g \cdot z_{max}} \left(v_n^2 - v_k^2\right). \tag{7.23}$$

If the speed of the vehicle is expressed in km/h, then the braking distance

$$S_T = \frac{1}{2 \cdot 3.6^2 \cdot g \cdot z_{max}} \left(v_{an}^2 - v_{ak}^2\right). \tag{7.24}$$

When braking to a complete stop $v_{ak} = 0$, in this case

$$S_T = \frac{v_{an}^2}{26 \cdot g \cdot z_{max}}. \tag{7.25}$$

The minimum braking distance of the vehicle to a complete stop from the initial speed $v_{an} = 80$ km/h, with the adhesion utilized coefficient $\varphi_{x\,max} = z_{max} = 0.8$, according to equation (7.25) is equal to $S_T = 32$ m.

When deriving the equations for the braking distance and time, the assumption was made that the movement of the vehicle is uniformly decelerated, but this is not the case for real vehicles, so $\varphi_{x\,max}$ is not equal to z_{max}, and the values of S_T and t_T differ from the theoretically obtained values by 10-40%. To take this circumstance into account, we introduce the empirical coefficient of reduction adhesion utilized k_e into the equation (7.19) and (7.25), which is within

$$k_e = \begin{cases} 1.1...1.2 - \text{cars and trucks based on them;} \\ 1.\ 21...1.4\ -\ \text{trucks and buses (coaches).} \end{cases}$$

With this in mind, the vehicle braking distance and braking time determines the equations

$$t_T = \frac{k_e \cdot v_{an}}{3.6 \cdot g \cdot \varphi_{x\max}} \text{ and } S_T = \frac{k_e \cdot v_{an}^2}{26 \cdot g \cdot \varphi_{x\max}}. \quad (7.26)$$

For a car at $v_{an} = 80$ km/h, $\varphi_{x\max} = 0.8$ and $k_e = 1.1$, the braking time is equal to $t_T = 3.11$ s and distance is equal to $S_T = 34.5$ m.

7.2.4. Vehicle braking diagram

For a comprehensive assessment of the vehicle braking dynamics, a graphic representation of the dependence of the deceleration j_T on time *t is used* (Fig. 7.2). Sometimes schedule $j_T = f(t)$ is supplemented with the dependence of the braking force $P_T = f(t)$, speed during braking $v_a = f(t)$ and effort on the brake pedal $P_p = f(t)$ from the time of braking. This graph is called the *brake diagram of the vehicle.*

During the movement of the vehicle, the driver notices a danger on the road (point A) and from this moment the countdown of the time and distance of his stop begins. The vehicle stopping time consists of the driver's reaction time and braking time. The driver's reaction time depends on a number of factors: qualification, age, state of health, etc. Usually, the driver's reaction time determines the interval of 0.2...1.5 s. The driver's reaction time is the time it takes him to make a decision to brake and move his foot to the brake pedal. In calculations, the reaction time is taken as 0.8 s. After the driver presses the brake pedal, the creation of braking moments on the wheels of the vehicle occurs after some time. The delay time of the braking system depends on the type of brake drive and the type of braking mechanisms. It is usually in the range of 0.05...0.07 s for disc brakes and 0.15...0.2 for drum brakes in the case of a hydraulic brake drive. With a pneumatic brake drive, the delay time is 0.2...0.4 s. The build-up time characterizes the rate of growth of the vehicle deceleration after the braking mechanisms have started to create a braking moment. The rise time depends on the type of brake drive: for a hydraulic drive - 0.05...0.2 s, for a pneumatic drive - 0.15 ... 0.6 s .

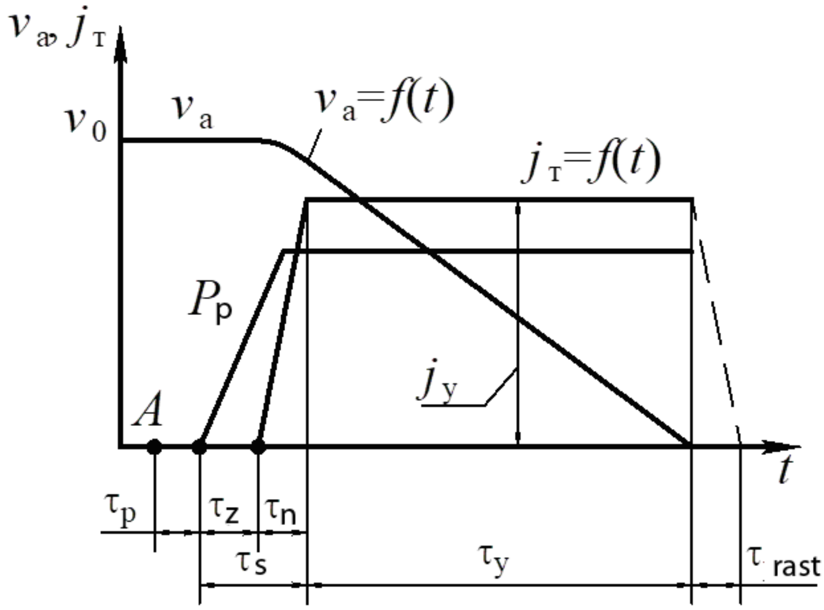


Fig. 7.2. **Vehicle braking diagram:**
$v_{an} = v_0$ – initial braking speed; τ_p – driver reaction time;
τ_z – delay time; τ_n – rise time; τ_s – activation time;
τ_y – braking time with constant deceleration j_y;
τ_{rast} – deceleration time

With the help of the braking diagram, you can determine the braking distance of the vehicle, that is, the distance traveled by the vehicle from the moment of applying force to the brake pedal until it stops

$$S_T = \frac{v_0}{3.6}\left(\tau_z + 0.5 \cdot \tau_n\right) + \frac{k_e \cdot v_0^2}{26 \cdot g \cdot \varphi_{x\max}}. \quad (7.27)$$

The stopping distance of the vehicle, that is, the distance traveled by the vehicle from the moment of detection of danger (point A) until it stops

$$S_o = \frac{v_0}{3.6}\left(\tau_p + \tau_z + 0.5 \cdot \tau_n\right) + \frac{k_e \cdot v_0^2}{26 \cdot g \cdot \varphi_{x\max}}. \quad (7.28)$$

For a passenger vehicle, the braking distance at v_{an} = 80 km/h, $\varphi_{x\ max}$ = 0.8 and k_e = 1.1 taking into account the activation time of the braking system (τ_z = 0.05 s, τ_n = 0.12 s) equals S_T = 36.95 m, and the stopping distance taking into account the driver's reaction time τ_p = 0.8 s is equal to S_o = 54.72 m.

7.2.5. Static characteristics of vehicle braking control

The static characteristic of brake control is the dependence of the constant deceleration of the vehicle on the steady force on the brake pedal (Fig. 7.3). The static characteristic determines the *braking controllability of the vehicle* - the ability to create a deceleration of the vehicle in accordance with the driver's control signals. The evaluation of the braking controllability of the vehicle is performed according to the value of the coefficient of efficiency of the braking control.

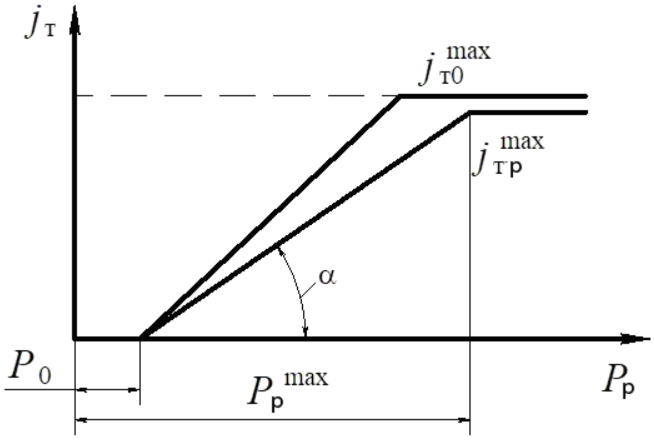


Fig. 7.3. **Static characteristics of braking control vehicle**

The coefficient of efficiency of brake control (braking system) is defined as a ratio

$$k_{\text{ту}} = \frac{j_{\text{a}}}{P_{\text{p}} - P_0}; \left[\frac{\text{m/s}^2}{\text{N}}\right] = \left[\frac{1}{\text{kg}}\right], \qquad (7.29)$$

where P_{p} is a constant value of the force on the brake pedal;

P_0 is the insensitivity force of the braking system.

Since the ratio is valid $\varphi_{x0} > \varphi_{x\text{p}}$ (with a decrease in the normal load on the wheel, the adhesion utilized coefficient increases), of course, under the condition $\varphi_{x\max} = z_{\max}$, then the ratio is also valid

$$j_{\text{т0}\max} = \varphi_{x0} \cdot g > j_{\text{тр}\max} = \varphi_{x\text{р}} \cdot g\ , \qquad (7.30)$$

where $j_{\text{t0 max}}$, $j_{\text{tp max}}$ – the maximum deceleration of the vehicle without load and with full load, respectively;

φ_{x0}, $\varphi_{x\text{p}}$ – coefficient of adhesion of vehicle wheels, respectively, without load and with full load.

Given the equation (7.30) in accordance with the equation (7.29), it can be asserted

$$k_{\text{ту}}^{0} > k_{\text{ту}}^{\text{р}}, \qquad (7.31)$$

where $k_{\text{ту}}^{0}$, $k_{\text{ту}}^{\text{р}}$ is the coefficient of efficiency of the braking control of the vehicle, respectively, without load and with full load.

The greater the coefficient of efficiency of brake control, the less force on the pedal to form the given (and in particular the maximum) deceleration of the vehicle. However, the upper limit $k_{\text{ту}}^{0}$ is limited by the deterioration of the regulation (dosage) of the deceleration for the vehicle without load, and the lower limit $k_{\text{ту}}^{\text{р}}$ is determined by the force on the brake pedal that is allowed. In the design of the vehicle, they strive to ensure a change in the coefficient of effectiveness of the brake control when the degree of load changes within

$$0.026\ \frac{\text{m/s}^2}{\text{N}} \le k_{\text{ту}} \le 0.045\ \frac{\text{m/s}^2}{\text{N}}.$$

The static characteristics of the braking control should have a zone of insensitivity in the range P_0 = 45...100 N. At the same time, compensation for the weight of the driver's foot is ensured. The maximum permissible force on the pedal is regulated by the standards: for passenger vehicle $[P_{\text{p}}^{\max}]$ = 500 N, and for trucks $[P_{\text{p}}^{\max}]$ = 700 N. However, to increase the comfort of braking control, the value of the force on the pedal that achieves the maximum deceleration of the vehicle $P_{\text{п}}^{\max}$ = 120 – 300 N is recommended.

7.3. Distribution of normal reactions on the wheels during vehicle braking and their influence on the braking process

7.3.1. Optimal distribution of braking forces on the axles vehicle

The assumption that the braking forces reach the maximum possible clutch values at the same time on all wheels is of significant importance for the accuracy of determining the constant deceleration and braking distance . To comply with this assumption, it is necessary that the distribution of braking forces between the wheels of different axles of the vehicle satisfies the equation (7.32).

$$\frac{P_{т1}}{P_{т2}} = \frac{R_{x1\max}}{R_{x2\max}} = \frac{P_{\varphi1}}{P_{\varphi2}} = \frac{R_{z1} \cdot \varphi_{x1}}{R_{z2} \cdot \varphi_{x2}}, \qquad (7.32)$$

where P_{t1}, P_{t2} – braking forces on the wheels of the front and rear axles;

$R_{x\,1\max}$, $R_{x\,2\max}$ – maximum longitudinal reactions on the wheels of the front and rear axles;

$P_{\varphi\,1}$, $P_{\varphi\,2}$ – wheel adhesion utilized forces of the front and rear axles;

$\varphi_{x\,1}$, $\varphi_{x\,2}$ – adhesion utilized coefficients of the wheels of the front and rear axles.

If we assume that $\varphi_{x\,1} = \varphi_{x\,2} = = \varphi_{x}$, then the equality

$$\frac{P_{т1}}{P_{т2}} = \frac{R_{z1}}{R_{z2}}. \qquad (7.33)$$

The optimality of such a distribution is explained by two circumstances:

– the adhesion coefficient reaches its maximum at a certain amount of slip $s_{кр}$, which corresponds to the transition of longitudinal forces to the maximum possible due to adhesion. If such a sliding value s_{cr} is achieved simultaneously on all wheels, then the braking force on each of them will be the maximum possible;

– with this sliding, the ability of the wheel to resist lateral forces is still quite high (see Fig. 7.4 at $s_{kp} = 0.24$; $\varphi_{x\max} = 0.86$; $\varphi_{y} = 0.35$).

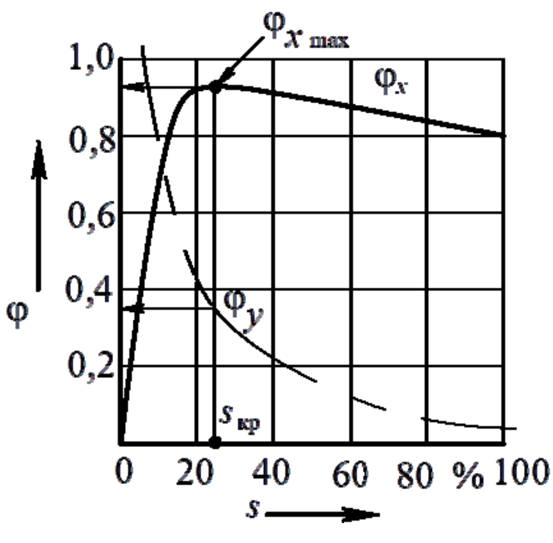


Fig. 7.4. **φ - *s* diagram**

7.3.2. Distribution of normal wheel reactions during vehicle braking

To determine the optimal distribution of braking forces, it is necessary to know the normal reactions acting on the wheels of each of the axles during braking. To determine the normal reactions on the wheels during braking with full use of the traction properties of the wheels, that is, in accordance with equation (7.32), we use the scheme presented in Figure 7.5.

Since the force of air resistance decreases quickly and its value is small, it can be taken as zero. The equation of the moments relative to the point o_2 assuming that $P_v = 0$:

$$\Sigma M_{o_2} = 0;\ R_{z1} \cdot L - P_{j_\text{т}\max} \cdot h_g - G_\text{a} \cdot b = 0\,. \qquad (7.34)$$

Considering that $P_{j\,\text{т}\max} = R_{x\,1\max} + R_{x\,2\max}$, we get

$$R_{z1} = G_\text{a} \cdot \frac{b}{L} + \left(R_{x1\max} + R_{x2\max}\right) \cdot \frac{h_g}{L}. \qquad (7.35)$$

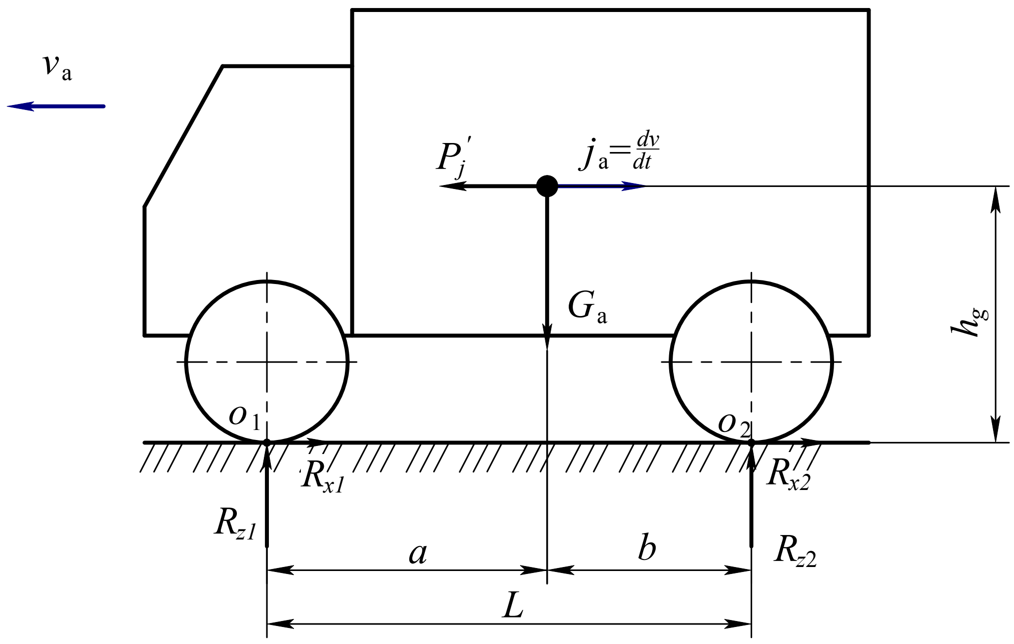


Fig. 7.5. **Scheme for determining the normal reactions on the axles when braking a vehicle with full use of adhesion utilizeds properties of wheels**

Considering that the maximum longitudinal reactions on the wheels are equal to their traction forces with the road, let's transform equation (7.35) into the form

$$R_{z1} = G_{\text{a}} \cdot \frac{b}{L} + \left(R_{z1} \cdot \varphi_{x1\max} + R_{z2} \cdot \varphi_{x2\max}\right) \cdot \frac{h_g}{L} =$$
$$= G_{\text{a}} \cdot \frac{b}{L} + G_{\text{a}} \cdot z_{\max} \cdot \frac{h_g}{L} = G_{\text{a}} \left(\frac{b}{L} + z_{\max} \cdot \frac{h_g}{L} \right). \quad (7.36)$$

Compiling the equation of moments relative to the point o $_1$, after similar transformations we obtain the normal reaction on the rear axle of the vehicle

$$R_{z2} = G_{\text{a}} \left(\frac{a}{L} - z_{\max} \cdot \frac{h_g}{L} \right). \quad (7.37)$$

It is obvious that when braking the vehicle, the normal reaction on the wheels of the front axle increases, and on the wheels of the rear axle it decreases by the same amount. The change in the normal reactions on the axles during braking is determined by the ratio of the height of the center of gravity to the base of the vehicle and the coefficient of friction during braking. At the same time, the optimal distribution of normal reactions according to equation (7.33) determines equality

$$\frac{P_{\text{т}1}}{P_{\text{т}2}} = \frac{R_{z1}}{R_{z2}} = \frac{G_{\text{a}} \left(\frac{b}{L} + z_{\max} \cdot \frac{h_g}{L} \right)}{G_{\text{a}} \left(\frac{a}{L} - z_{\max} \cdot \frac{h_g}{L} \right)} = \frac{b + z_{\max} \cdot h_g}{a - z_{\max} \cdot h_g}. \quad (7.38)$$

All quantities included in the right-hand side of the equation are variables. Coordinates *a* , *b* and h_g change with changes in the load on the vehicle, and the braking ratio z_{max} depends on braking conditions and tire properties.

It should be noted that the actual distribution of braking moments (and, therefore, braking forces) between the wheels of different axles in each specific vehicle depends on the design features of the braking system (for example, on the size of the brake discs and the diameters of the cylinders of the front and rear brake mechanisms).

7.3.3. Vehicle braking force distribution coefficient

From the equation (7.38), it is clear that in order for the vehicle to brake with maximum deceleration under any road conditions and under any load, it is necessary that the braking forces on the wheels of the front and rear axles should always be proportional to the normal reactions on these axes In this case, it is believed that the distribution of braking forces is optimal. A vehicle's braking system may be designed so that there is a constant ratio between the braking forces of the front and rear wheels, or it may have devices that vary this ratio so as to approach the ideal ratio. In this case, it is considered that the distribution of braking forces of the vehicle is real without regulation and with regulation.

The distribution of braking forces on the wheels when braking a vehicle is usually characterized by the coefficient of distribution of braking forces β_t . The coefficient of distribution of braking forces is defined as the ratio of the braking force on the front axle to the braking force of the vehicle

$$\beta_t = \frac{P_{t1}}{P_t}, \qquad (7.39)$$

where P_{t1} is the braking force of the front axle of the vehicle;

P_t is the braking force of the vehicle.

With an ideal distribution of braking forces, braking of the vehicle can occur with the maximum possible braking forces on the wheels of the front and rear axles, which are equal to the forces of their adhesion to the surface. In this case, the distribution coefficient β_t is called optimal β_{opt} .

$$\beta_{opt} = \frac{P_{t1\max}}{P_{t\max}}. \qquad (7.40)$$

The maximum value of the braking forces in equation (7.40) is obtained at the maximum value of the adhesion utilized coefficient. We will assume that the values of the maximum adhesion utilized coefficients of the wheels of the front and rear axles are the same and equal braking ratio $\varphi_{x1\max} = \varphi_{x2\max} = z_{\max}$. Taking into account the accepted assumption, we get

$$\beta_{opt} = \frac{R_{z1} \cdot \varphi_{x1\max}}{G_a \cdot z_{\max}} = \frac{G_a\left(b + z_{\max} \cdot h_g\right) \cdot \varphi_{x1\max}}{G_a \cdot L \cdot z_{\max}} = \frac{b + z_{\max} \cdot h_g}{L}, \qquad (7.41)$$

The optimal coefficient of distribution of braking forces of the vehicle β_{opt} depends on its design (base L), coordinates of the center of gravity

(b and h_g) and on the value of the maximum braking ratio z_{max} which is equal to the maximum adhesion utilized coefficient $\varphi_{x\ max}$ on the given road. It should be remembered that the values of the maximum braking ratio z_{max} vary from 0.1 to 0.9 depending on road conditions.

7.3.4. Dependence of the optimal vehicle braking force distribution coefficient on the clutch coefficient

As it was established above, the coefficient of distribution of the braking forces of the vehicle β_{opt} for the specified parameters of the vehicle is a variable value depending on the adhesion utilized coefficient. Let's transform equation (7.41) into the form

$$\beta_{opt}^{(theoretical)} = \frac{b}{L} + \frac{h_g}{L} \cdot \varphi_{x\max}. \tag{7.42}$$

It is obvious that this is the equation of a straight line that does not pass through the origin. It should be noted that without special automatic devices in the braking system of the vehicle, the coefficient of distribution of braking forces is a value independent of the adhesion utilized coefficient. In this case, the coefficient of distribution of braking forces at the specified parameters of the vehicle has a constant value β_{const}.

Figure 7.6 shows a graph of the dependence of the coefficients of the distribution of braking forces for the optimal distribution (theoretical) and in the absence of automatic devices in the braking system.

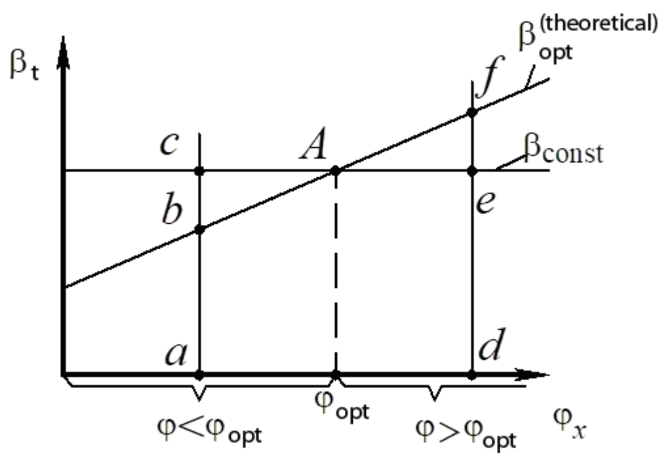


Fig. 7.6. **Dependence of the coefficient of distribution of braking forces**

The line β_{opt} is the locus of points that determine the value of the coefficient of distribution of braking forces, at which all wheels of the vehicle will reach the maximum value of traction on the road with any coefficient of adhesion. For the line β_{const}, there is only one value of the coefficient of adhesion φ_{opt}, at which all wheels of the vehicle will reach the maximum value in terms of adhesion.

For the case $\varphi < \varphi_{opt}$

$$ac = \beta_{const}; ab = \beta_{opt}; ac > ab \rightarrow \beta_{const} > \beta_{opt}.$$

$$P_{t1} = \beta_t \cdot P_t \rightarrow P_{t1const} > P_{t1opt}.$$

Since in this case the braking force on the front wheels of the vehicle with a constant distribution of braking forces is greater than the optimal value, they are blocked first.

For the case $\varphi > \varphi_{opt}$

$$ed = \beta_{const}; fd = \beta_{opt}; ed < fd \rightarrow \beta_{const} < \beta_{opt}.$$

$$P_{t1} = \beta_t \cdot P_t \rightarrow P_{t1const} < P_{t1opt}.$$

Since in this case the braking force on the front wheels of a vehicle with a constant distribution of braking forces is less than the optimal value, the rear wheels are blocked first.

7.3.5. The graph of the distribution of braking forces of the vehicle

The distribution of braking forces between the axles of the vehicle is given in the form of the dependence of the braking force on the rear axle from the braking force on the front axle $P_{t2} = f(P_{t1})$. A graphical presentation of this dependence is shown in Figure 7.7.

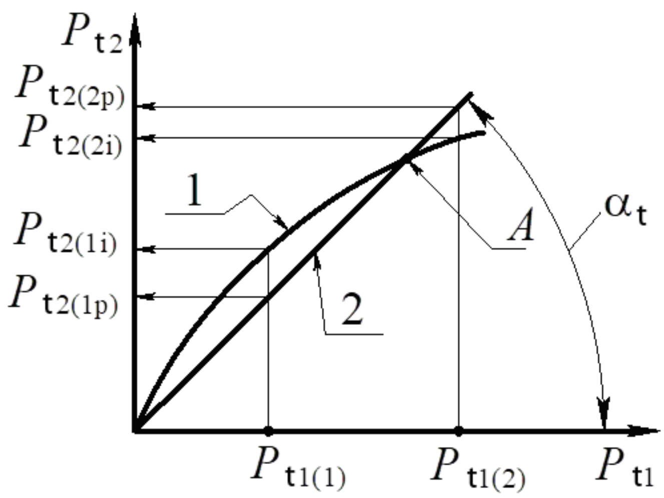


Fig. 7.7. **Distribution of braking forces between vehicle axles :**
1 – optimal (ideal) distribution; 2 – real distribution

With the optimal distribution of braking forces between the axles, the dependence of the braking force on the rear axle from the braking force on the front axle is determined from the equation

$$P_{t2max} = P_{tmax} - P_{t1max} . \quad (7.43)$$

According to the definition of the optimal coefficient of distribution of braking forces (7.40) can be written

$$P_{tmax} = \frac{P_{t1max}}{\beta_{opt}}, \quad (7.44)$$

taking into account (7.43), let's transform the equation (7.44) into the form

$$P_{t2max} = \frac{P_{t1max}}{\beta_{opt}} - P_{t1max} = P_{t1max}\left(\frac{1}{\beta_{opt}} - 1\right) = P_{t1max}\left(\frac{1-\beta_{opt}}{\beta_{opt}}\right). \quad (7.45)$$

Curve 1 in Figure 7.7, constructed according to dependence (7.45) for the optimal distribution coefficient, is called *the graph of the ideal distribution of braking forces.*

A real braking system, which does not have a brake force regulator, ensures the distribution of braking forces between the vehicle axles according to the dependence $P_{t2} = P_{t1} \cdot \text{tg}\, \alpha_t$. The angle of inclination α_t of dependence 2 (Fig. 7.7) is determined by the design and parameters of the braking mechanisms of the front and rear axles.

The graph of the distribution of the vehicle braking forces makes it possible to analyze the degree of approximation of the real distribution to the optimal (ideal) distribution. So, if the braking force $P_{t1(1)}$ acts on the front axle, then the force $P_{t2(1p)}$ acts on the rear axle with the real distribution , and with the ideal one - the force $P_{t2(1i)}$. It is obvious that with the given braking efficiency, the rear wheels of the vehicle do not use the possibility of the clutch with the real distribution of braking forces, since the braking force is less than the ideal value. This allows us to conclude that in the case of limiting the maximum braking efficiency by the clutch conditions at the braking force on the front axle $P_{t1(1)}$, the wheels of this axle will be locked first. If the braking force $P_{t1(2)}$ acts on the front axle , then the force $P_{t2(2p)}$ acts on the rear axle with the real distribution , and with the ideal one - the force $P_{t2(2i)}$. It is obvious that with a given braking efficiency, the rear wheels of the vehicle are braked more than the clutch is possible with the actual distribution of braking forces, since the braking force on the rear axle is greater than the ideal value of the braking force. This allows us to conclude that in this case the wheels of the rear axle will be locked first. It is also clear from the graph (Fig. 7.7) that if the vehicle brakes at $P_{t1(A)}$, then the real distribution of braking forces corresponds to the optimal (ideal) distribution.

7.3.6. Regulation of the distribution of braking forces of the vehicle

If the wheels of the front axle lock first, the vehicle loses controllability. If the rear wheels are locked first, the vehicle loses stability, which is very dangerous. Therefore, in order to improve braking properties, automatic devices or systems for regulating the distribution of braking forces of the vehicle are included in the design of the braking system.

The goal of adjusting the distribution of braking forces of the vehicle is to approach the ideal distribution. The ideal distribution of the vehicle braking forces is determined by its design and depends not only on the clutch ratio, but also on the vehicle load, as the coordinates of the center of gravity b and h_g change . Figure 7.8 shows the dependences of the ideal distribution of braking forces of the vehicle in loaded 1 and unloaded 2 states. This means that the "ideal" regulator should change the ratio in the equation (7.38) depending on the degree of loading of the vehicle. One of the ways to adjust the braking forces is to adjust the pressure of the working fluid (liquid or air) in the brake circuits of the front and rear wheels.

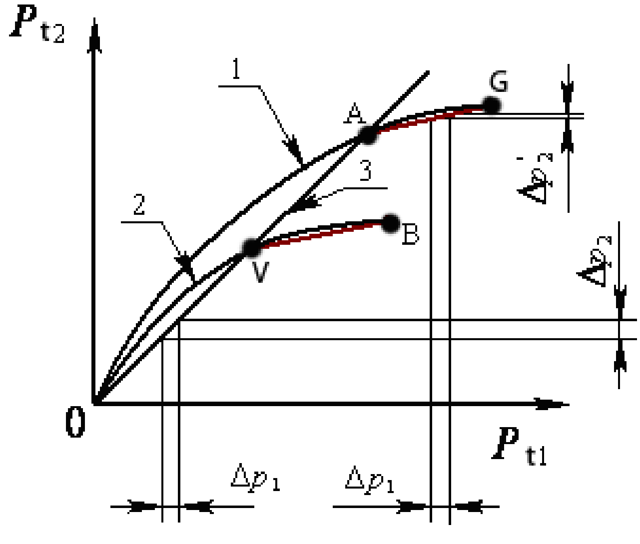


Fig. 7.8. **Regulation of distribution of braking forces between axles vehicle :** Δp_1, Δp_2 – pressure increase in the front and back braking circuits

The dependence of the actual distribution of braking forces depends on the regulation law. In Figure 7.8, the broken lines 0AG and 0VB reflect the real distribution of the vehicle braking forces, respectively, in a loaded and unloaded state. Points A and B correspond to switching on the regulator at different loading conditions.

After turning on the regulator, the increase in pressure in the brake circuit of the front wheels by Δp_1 causes an increase in the pressure in the circuit of the rear wheels by $\Delta p'_2$, less than before turning on the regulator Δp_2 . Branches AG and VB are called *regulatory branches of the characteristics of the distribution of braking forces.* Adjusting the distribution of braking forces allows you to avoid the primary blocking of the rear wheels and prevent the loss of stability of the vehicle.

7.3.7. Adjusting the braking torque on the wheel

The ideal distribution of the vehicle braking forces ensures the simultaneous achievement of the values of longitudinal reactions on all wheels, as much as possible by the clutch. It should be noted that this adjustment does not exclude the possibility of locking the wheels due to an increase in the braking moments created by the braking mechanisms. Locking the wheels leads to a loss of controllability and stability of the vehicle. Automatic adjustment of the braking torque is used to prevent the wheels from locking in different braking conditions. Such systems, according to their functional purpose, are called *anti-lock braking systems* (ABS). Any ABS, regardless of its design (Fig. 7.9), as an automatic control system, consists of: sensors of wheel rolling parameters; control unit; executive mechanisms (for brake systems with hydraulic and pneumatic drives, they are called *pressure modulators*).

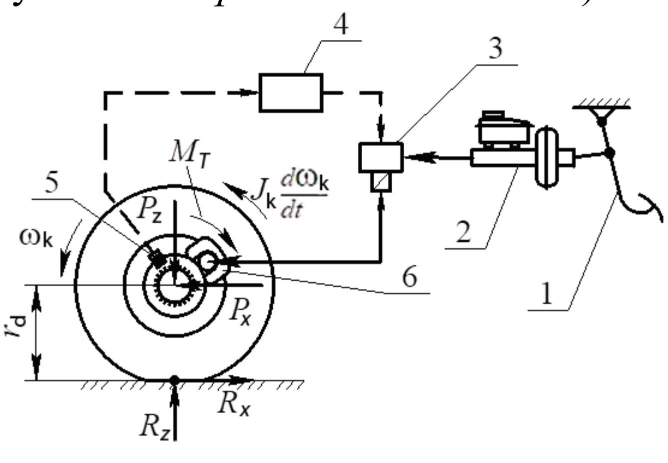


Fig. 7.9. **Structural diagram of ABS**

Figure 7.9 shows the diagram of the hydraulic brake system, which includes the brake pedal 1, the main brake cylinder 2, the working brake cylinder 6 and ABS elements: pressure modulator 3, control unit 4, wheel speed sensor 5.

To find out when and how the ABS pressure modulator works, consider the equation of the moments acting on the braking wheel with maximum use of the traction properties of the tire

$$R_x \cdot r_\text{d} + J_\text{k} \cdot \frac{d\omega_\text{k}}{dt} - M_\text{T} = 0, \tag{7.46}$$

where $R_x = P_\varphi$ is the force of wheel adhesion to the supporting surface;

$\frac{d\omega_\text{k}}{dt} = \varepsilon_\text{k}$ – angular deceleration of the wheel;

J_k – moment of inertia of the wheel;

M_T is the braking torque created by the braking mechanism.

The first term in equation (7.46) represents the moment of wheel adhesion to the support surface $M_\varphi = R_x \cdot r_\text{d}$. Given that we are considering the case of $R_x = P_\varphi$, the adhesion utilized moment can be expressed as $M_\varphi = R_z \cdot \varphi_x \cdot r_\text{d} = (R_z \cdot r_\text{d}) \cdot \varphi_x$. Assuming that R_z and r_d do not change during braking, it is possible to assert the value of the adhesion utilized moment M_φ proportional to the value φ_x. That is, the dependence $M_\varphi = f(s)$ can be expressed as $\varphi_x = f(s)$, taking into account the proportionality factor ($R_z \cdot r_\text{d}$) along the ordinate axis (Fig. 7.10). To analyze the balance of the moments acting on the braking wheel, figure 7.10 also shows the dependence of the braking moment on the wheel slip $M_\text{T} = f(s)$.

When you press the brake pedal 1 (Fig. 7.9), fluid pressure is formed in the main brake cylinder 2. From the main cylinder of the master cylinder 2, liquid under pressure flows through the normally closed valves of the modulator 3 into the working brake cylinder 6. This causes an increase in the braking torque M_T (section 0 - *a* , see Fig. 7.10). In this section, the braking moment M_T is greater than the moment of adhesion M_φ of the wheel to the support surface, which causes a decrease in the angular speed of the wheel ω_k and the speed of the wheel v_k and the vehicle v_a (see Fig. 7.11). At the same time, the angular deceleration of the wheel ε_k increases and its coefficient of longitudinal slip *s* (see Fig. 7.10).

If the force on the pedal is such that the braking moment on the wheel does not exceed the value determined by point *a* (Fig. 7.10), then the pressure modulator 3 is not activated. It should be noted that at the same time the ABS is functioning - the sensors measure the wheel rolling parameters, and the control unit evaluates their values and, according to the algorithm, makes a decision - not to turn on the pressure modulator.

Figure 7.10 shows that as the braking torque M_T increases , wheel slip s increases . At critical sliding s_{kp}, the longitudinal adhesion utilized coefficient φ_x takes on a maximum value and, accordingly, the adhesion utilized moment is equal to $M_{\varphi\max}$. The purpose of ABS operation is to ensure control of the braking torque M_T so that the wheel slip s acquires values close to the critical slip s_{kp} . At the same time, it is ensured that the wheel is not blocked and the maximum possible braking force is provided under the condition of wheel adhesion. In the absence of wheel locking, the controllability and stability of the vehicle during braking is preserved.

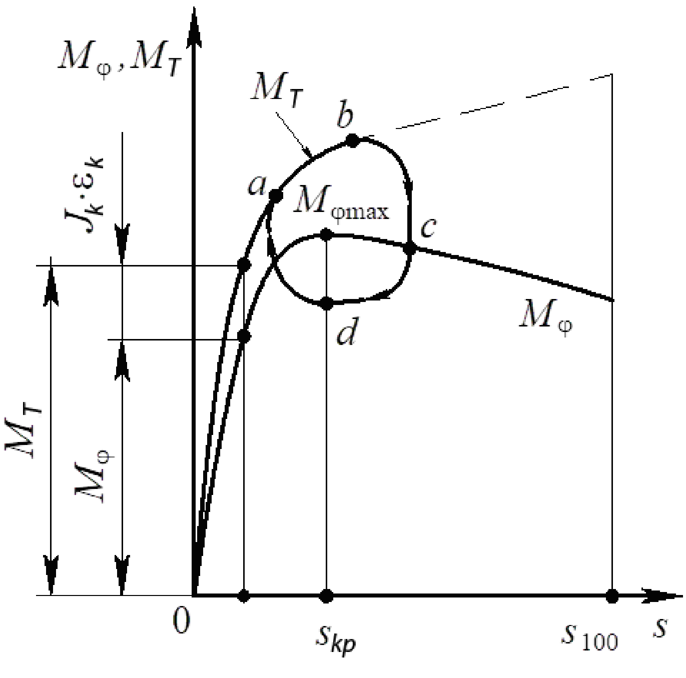


Fig. 7.10. **Change of torques on the brake wheel during ABS operation**

Let's consider how ABS with wheel angular deceleration control ε_k provides regulation of wheel slip s near s_{kp} , and therefore regulation of braking torque M_T near $M_{\varphi\max}$.

Let's rewrite equation (7.46) in the form

$$M_{\varphi} + J_k \cdot \varepsilon_k - M_T = 0, \qquad (7.47)$$

and solve it with respect to the angular deceleration of the wheel ε_k

$$\varepsilon_k = \frac{M_T - M_\varphi}{J_k}. \tag{7.48}$$

Figure 7.10 shows that the ordinate between the curves M_T and M_φ characterizes the moment of inertia of the wheel $M_j = J_k \cdot \varepsilon_k$. Since the moment of inertia of the wheel J_k is a constant value, it can be assumed that the ordinate between the curves M_T and M_φ characterizes the angular deceleration of the wheel ε_k. As the braking torque M_T increases, the angular deceleration ε_k of the wheel also increases. It is impossible to measure the sliding of the wheel s_{kp}, and the angular deceleration of the wheel ε_k is calculated in the control unit based on the data of the wheel speed sensor 5. Especially fast deceleration of the wheel ε_k increases in the section *a* – *b* (Fig. 7.10), where the difference $M_T - M_\varphi$ increases sharply due to a decrease in M_φ. A sharp increase in the angular deceleration of the wheel indicates that the wheel slip has become slightly more than the critical s_{kp} . Therefore, at the braking moment M_T, corresponding to the value at point *b,* the control unit forms and sends *a command to the modulator to reduce the pressure in the working brake cylinder*. Point *b* corresponds to the first command, according to which the braking torque M_T decreases and at point *c* becomes equal to the clutch torque M_φ. According to (7.48), the wheel deceleration $\varepsilon_k = 0$. The lack of wheel deceleration serves as the basis (point *c*) for sending a second command to the modulator to *maintain constant pressure in the working brake cylinder* . But due to the inertia of the brake hydraulic drive, the pressure decreases somewhat, which means that the braking torque M_T also decreases . In this case, the numerator in equation (7.48) becomes negative, the angular deceleration of the wheel changes its sign and the wheel begins to accelerate due to the inertia of the vehicle. The maximum acceleration of the wheel is achieved at s_{kp} , since the moment M_φ has a maximum value, and the moment M_T is minimal. The third *pressure increase command corresponds to point d* . At the same time , the braking torque *Mt* increases to point *a* and the ABS operation cycle repeats.

Figure 7.11 shows the change in vehicle movement parameters during braking. The angular speed of the wheel ω_k when braking without

ABS rapidly decreases and at the moment of time τ_1 the wheel stops rotating - it is blocked.

When the anti-lock system is functioning, the wheel angular velocity ω_{kABS} cyclically decreases–increases close to the value of the wheel angular velocity ω_{ks} with ideal moment control with critical wheel slip. When the speed of the vehicle decreases to a certain value v_{min}, the anti-locking system is disabled, and the wheel is locked at the moment of time τ_2 . At this moment, the speed of the vehicle is low, and the locking of the wheels will not cause a violation of stability and controllability, since the vehicle will stop already at the moment of time τ_3 . Disabling the ABS at the end of braking is necessary for the vehicle to come to a complete stop, because if the ABS is working, the wheel will never stop turning, for example when driving on a uphill.

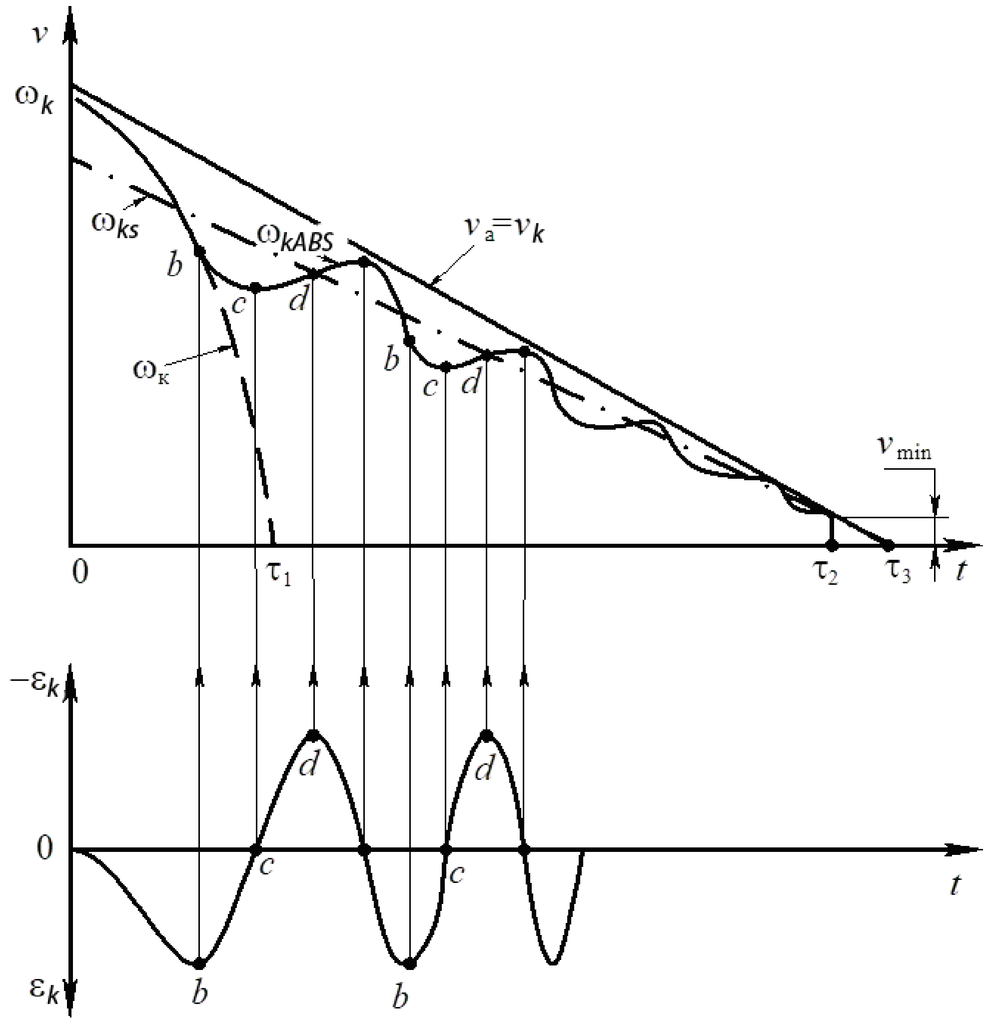


Fig. 7.11. **Graph of dependence of movement parameters vehicle from time to time when braking**

The braking distance of the vehicle and the stopping distance of the vehicle during ABS operation can be determined by analogy with equations (7.27) and (7.28) through the k_{ABS} coefficient in equations (7.49) and (7.50). The k_{ABS} coefficient varies in the range 1.05...1.3.

$$S_{\text{T}} = \frac{v_0}{3.6}\left(\tau_{\text{z}} + 0.5\cdot\tau_{\text{n}}\right) + \frac{k_e \cdot k_{ABS} \cdot v_0^2}{26\cdot g\cdot \varphi_{x\max}}. \tag{7.49}$$

$$S_{\text{o}} = \frac{v_0}{3.6}\left(\tau_{\text{p}} + \tau_{\text{z}} + 0.5\cdot\tau_{\text{n}}\right) + \frac{k_e \cdot k_{ABS} \cdot v_0^2}{26\cdot g\cdot \varphi_{x\max}}. \tag{7.50}$$

Or from equation (7.51) due to the maximum braking ratio $z_{\max}$ and the coefficient of use of adhesion forces ε_{ABS} during ABS operation. The coefficient of use of adhesion forces ε_{ABS} for any ABS used on any road, in accordance with UN requirements (Rules 13, appendix 13) should be in the range of 0.75...1.

$$S_{\text{o}} = \frac{v_0}{3.6}\left(\tau_{\text{p}} + \tau_{\text{z}} + 0.5\cdot\tau_{\text{n}}\right) + \frac{v_0^2}{26\cdot g\cdot z_{\max}\cdot \varepsilon_{ABS}}. \tag{7.51}$$

If the value of ε_{ABS} is not known, it is better to use the average value of ε_{ABS} equal to 0.83, as shown by practical calculations for most vehicles.

The multiplication of $z_{\max}\cdot\varepsilon_{ABS}$ in equation (7.51) is nothing more than the braking ratio of the vehicle z_{ABS} during ABS operation.

Control questions

1. What resistances does the vehicle kinetic energy use during braking?
2. What gauges and indicators evaluate the braking properties of a vehicle?
3. Draw and explain the diagram of forces and moments acting on the vehicle during braking.
4. Write the equation of motion of the vehicle when braking.
5. Write the equation of motion of the vehicle during emergency braking.

6. How is the maximum deceleration of the vehicle determined when braking?
7. How is the vehicle emergency braking time determined?
8. How is the path of emergency braking of a vehicle determined?
9. Draw and explain the braking diagram of a vehicle.
10. Draw a static characteristic of the braking control of the vehicle.
11. What characterizes the coefficient of efficiency of braking control?
12. How are normal reactions on vehicle axles determined when braking?
13. Write the condition for the optimal distribution of braking forces on the axles of the vehicle.
14. How is the coefficient of distribution of braking forces determined?
15. The graph of the dependence of the optimal coefficient of distribution of braking forces of the vehicle on the coefficient of adhesion utilized.
16. The graph of the distribution of braking forces of the vehicle.
17. Change of torques on the brake wheel during ABS operation.
18. Graph of the dependence of vehicle movement parameters on time when braking with ABS.

TOPIC 8

VEHICLE STABILITY

8.1. Determination of vehicle stability and its types. Indicators transverse stability

The stability of a vehicle is a set of its qualities that ensure movement in the desired direction without skidding (sliding) or overturning.

Depending on the direction of overturning and sliding, *longitudinal and transverse stability are distinguished* . More likely and therefore more dangerous is a violation of transverse stability, which occurs as a result of the action of lateral forces: centrifugal force, the lateral component of the force of gravity, side wind, impacts on the unevenness of the road.

Indicators of lateral stability:

– critical speed for side slip: $v_{a\varphi}$, km/h;

– critical speed after overturning: v_{aop}, km/h;

– the critical angle of the transverse slope of the road (uphill) for lateral sliding: β_{φ}, degree;

– the critical angle of the transverse slope of the road (uphill) after overturning: β_{op}, degree;

– coefficient of transverse stability, η_p.

When determining the transverse stability of the vehicle, we will assume that the wheels are rigid in the lateral direction, that is, we will not take into account the angle of the lateral deflection of the tires.

Critical speed for sideslip. When a vehicle is moving uniformly in a circle on a horizontal road (Fig. 8.1), side slippage of its wheels can occur as a result of the action of the side force - the centrifugal force at the moment when it becomes equal to the force of the adhesion, i.e.

$$P_y = P_{\varphi y}, \tag{8.1}$$

where P_y is a lateral force (centrifugal force);

$P_{\varphi y}$ is the force of adhesion of the wheels to the road in the lateral direction.

Let us substitute the value of the centrifugal force and the force of adhesion and obtain equation (8.2).

$$\frac{G_a \cdot v_a^2}{13 \cdot g \cdot r_\Theta} = G_a \cdot \varphi_y, \qquad (8.2)$$

where r_Θ is the turning radius.

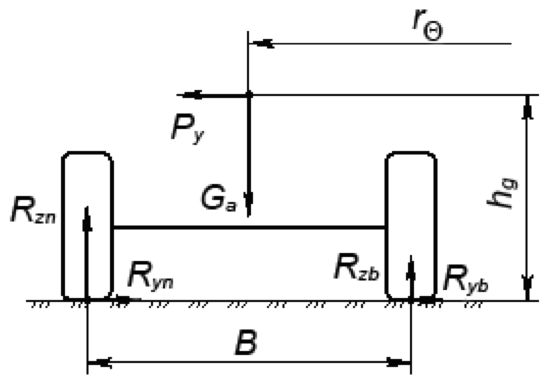


Fig. 8.1. **Scheme for determining the critical speed for stability on a horizontal road**

Given that in this case $v_a = v_{a\varphi}$, we determine the critical speed for side slip from equation (8.2)

$$v_{a\varphi} = 3.6\sqrt{g \cdot r_\Theta \cdot \varphi_y}\,. \qquad (8.3)$$

The critical speed for lateral slippage is the limit speed, after reaching which the vehicle may skid.

A vehicle skid can occur only with an increase in speed, a decrease in the turning radius, or a maximum lateral disturbance (gust of wind, side impact of the wheel on the unevenness of the road, transverse slope).

Critical speed after overturning. When turning on a horizontal road, the transverse force P_y acting on the vehicle can cause not only lateral sliding, but also overturning of the vehicle. Overturning of a vehicle when moving on a horizontal road along a curved trajectory occurs relative to its loaded (external to the center of curvature) wheels (see Fig. 8.1). At the moment of detachment of the wheels (internal in relation to the center of curvature) from the road, their normal reactions $R_{zb} = 0$ and the entire weight of the vehicle is perceived by the outer wheels $R_{zn} = G_a$. In this case, the overturning moment created by the transverse force is balanced by the moment created by the gravity of the vehicle:

$$M_{P_y} = M_{G_a}, \qquad (8.4)$$

where M_{P_y} is the moment created by the centrifugal force;

M_{G_a} – the moment created by the vehicle gravity.

By substituting the values of the moments, we get

$$P_y \cdot h_g = G_a \frac{B}{2}; \tag{8.5}$$

$$\frac{G_a \cdot v_a^2}{13 \cdot g \cdot r_\Theta} \cdot h_g = G_a \cdot \frac{B}{2}. \tag{8.6}$$

Given the fact that in this case $v_a = v_{aop}$, we determine the critical speed of the vehicle by side overturning

$$v_{aop} = 3.6\sqrt{\frac{g \cdot r_\Theta \cdot B}{2 \cdot h_g}}. \tag{8.7}$$

The critical speed for lateral overturning is the limit speed, after reaching which the vehicle may overturn.

When the vehicle is moving on a turn at a critical speed after overturning, it may not overturn. Overturning will occur in the case of minimal lateral disturbance and when the turning radius is reduced or the speed of movement is increased. The overturning of the vehicle is more dangerous than its side sliding, so it is necessary to ensure $v_{a\varphi} < v_{aop}$.

The critical angle of the cross slope of the road for lateral sliding. When a vehicle moves in a straight line on a road with a transverse slope (sidehill, oblique hill), the loss of its lateral stability (Fig. 8.2) is caused by the component of the vehicle gravity parallel to the plane of the slope.

$$P_y = G_a \cdot \sin\beta, \tag{8.8}$$

where β is the angle of the transverse slope of the road (sidehill, oblique hill).

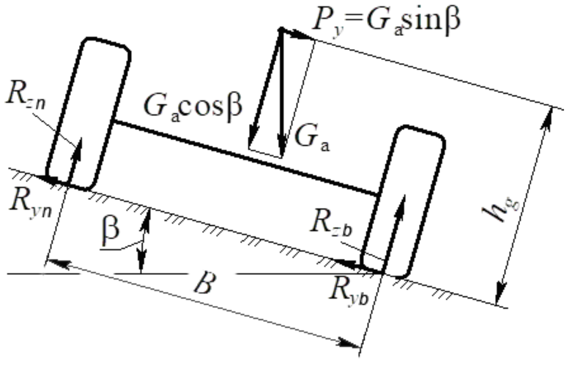


Fig. 8.2. **Scheme for determining the critical angle of the cross slope of the road**

Lateral sliding of the vehicle can begin at the moment when the condition is met

$$P_y = P_{\varphi y}. \tag{8.9}$$

Let us substitute the values of forces and get

$$G_a \cdot \sin\beta = G_a \cdot \cos\beta \cdot \varphi_y. \tag{8.10}$$

Given the fact that in this case $\beta = \beta_\varphi$, we determine the critical angle of the transverse slope of the road for lateral slip β_φ by dividing both parts of equality (8.10) by cos β

$$\text{tg}\,\beta_\varphi = \varphi_y \text{ or } \beta_\varphi = \text{arctg}\,\varphi_y. \tag{8.11}$$

It is obvious that the critical angle of the lateral slope for lateral slip depends only on the coefficient of adhesion of the wheels in the lateral direction. For example, when $\varphi_y = 0.8$, the angle $\beta_\varphi = 39°$ and at $\varphi_y = 1$ angle $\beta_\varphi = 45°$.

The critical angle of the lateral slope of the road sliding is the limit angle at which a vehicle can still move in a straight line on a slope without side sliding of the wheels. Lateral sliding of the vehicle under these conditions will begin under the action of any minimal lateral disturbance.

The critical angle of the cross slope of the road after overturning. When moving in a straight line on a road with a transverse slope, the vehicle may overturn in the event that the overturning moment created by the transverse force P_y is balanced by the restoring moment from the normal component of the vehicle gravity

$$M_{P_y} = M_{G_a}. \tag{8.12}$$

Substitute the values of the moments in (8.12)

$$G_a \cdot \sin\beta \cdot h_g = G_a \cdot \cos\beta \frac{B}{2}. \tag{8.13}$$

Given the fact that in this case $\beta = \beta_{op}$, we determine the critical angle of the transverse slope of the road after overturning by dividing both parts of equality (8.13) by cos β

$$tg\beta_{op} = \frac{B}{2 \cdot h_g}; \rightarrow \beta_{op} = arctg\frac{B}{2 \cdot h_g}. \tag{8.14}$$

The critical angle of the transverse slope of the road after overturning is called the limit angle at which straight-line movement of the vehicle on the slope without overturning is still possible. The overturning of the vehicle in this case can happen with any minimal side impact.

The value of the critical angle of inclination of the road after overturning depends on the design of the vehicle - the ratio of the track of the wheels and the height of the center of gravity. Below are typical values of the critical angle of the road's lateral slope for different types of vehicles.

$$\beta_{op} = \begin{cases} 40° - 50° - \text{cars and trucks based on them;} \\ 30° - 40° - \text{trucks;} \\ 25° - 35° - \text{buses } (\text{coaches}). \end{cases}$$

The overturning of the vehicle is more dangerous than its side sliding, so it is necessary to ensure $\beta_{\varphi} < \beta_{op}$.

<u>*Coefficient of lateral stability of the vehicle.*</u> During the operation of the vehicle, any kind of violation of its lateral stability may occur. The type of loss of lateral stability of the vehicle depends on its design and operating conditions. The coefficient of lateral stability allows you to determine which of the two types of loss of lateral stability (skid or rollover) is more likely in operation.

The coefficient of lateral stability of the vehicle is defined as the ratio of the track of the wheels to the doubled height of its center of gravity

$$\eta_n = \frac{B}{2 \cdot h_g}. \tag{8.15}$$

For example, let's consider the case of a vehicle moving when turning on a horizontal road. For this purpose, we equate the critical speeds for side sliding and overturning

$$v_{a\varphi} = v_{aop}. \tag{8.16}$$

Substitute the corresponding values from (8.3) and (8.7) into (8.16)

$$3.6\sqrt{g \cdot r_{\Theta} \cdot \varphi_y} = 3.6\sqrt{\frac{g \cdot r_{\Theta} \cdot B}{2 \cdot h_g}}. \tag{8.17}$$

From which we find

$$\varphi_y = \frac{B}{2 \cdot h_g} = \eta_n . \tag{8.18}$$

On the basis of analysis (8.18), it can be stated that if the coefficient of adhesion of the wheels to the road in the transverse direction is less than the coefficient of lateral stability, i.e. $\varphi_y < \eta_n$, then with a lateral force, skidding in the transverse direction is more likely than the coefficient of lateral stability, i.e. $\varphi_y > \eta_n$, then the vehicle may overturn without its previous skidding, which is possible on roads with a high coefficient of adhesion. Below are typical values of the lateral stability coefficient for different types of vehicle.

$$\eta_n = \begin{cases} 0.9 - 1.2 - \text{cars and trucks based on them;} \\ 0.55 - 0.80 - \text{ trucks;} \\ 0.5 - 0.6 - \text{buses } (\text{coaches}). \end{cases}$$

8.2. Stability of the vehicle against skidding and overturning while driving on a curved road with a transverse slope

To determine the critical speed during the vehicle movement on a curved road with a transverse slope, we will use the scheme presented in Figure 8.3. Consider the case of uniform movement of a vehicle in a circle with a transverse slope of the road to the center of the turn.

Lateral sliding of its wheels can occur as a result of the action of the total lateral force P_{side}, acting in a plane parallel to the road passing through the center of gravity, at the moment when it becomes equal to the clutch force, i.e.:

$$P_{\text{bok}} = P_{\varphi y} . \tag{8.19}$$

Using the diagram (Fig. 8.3), it is possible to write

$$P_y \cdot \cos\beta - G_a \cdot \sin\beta = \left(G_a \cdot \cos\beta + P_y \cdot \sin\beta\right) \cdot \varphi_y . \tag{8.20}$$

Let's transform the equation (8.20)

$$P_y \cdot \cos\beta - G_a \cdot \sin\beta - G_a \cdot \cos\beta \cdot \varphi_y - P_y \cdot \sin\beta \cdot \varphi_y = 0;$$

$$P_y \cdot \left(\cos\beta - \sin\beta \cdot \varphi_y\right) = G_a \cdot \left(\sin\beta + \cos\beta \cdot \varphi_y\right). \tag{8.21}$$

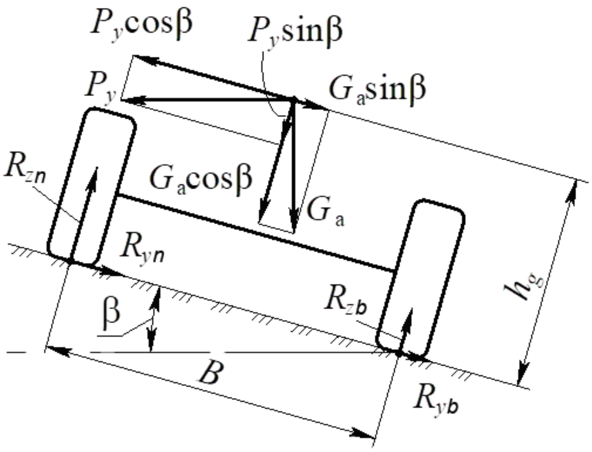


Fig. 8.3. **Scheme for determining the critical speed by stability of a vehicle on a curved road with a transverse slope**

In equation (8.21), let's reveal the value of P_y a centrifugal force

$$\frac{G_a \cdot v^2}{g \cdot r_\Theta} \cdot \left(\cos\beta - \sin\beta \cdot \varphi_y\right) = G_a \cdot \left(\sin\beta + \cos\beta \cdot \varphi_y\right). \qquad (8.22)$$

Considering that in this case $v = v_\varphi$, from equation (8.22) we determine the critical speed for lateral sliding

$$v_\varphi = \sqrt{\frac{r_\Theta \cdot g\left(\sin\beta + \cos\beta \cdot \varphi_y\right)}{\cos\beta - \sin\beta \cdot \varphi_y}}. \qquad (8.23)$$

Let's divide the numerator and denominator of equation (8.23) by cos β and reduce it to the form

$$v_\varphi = \sqrt{r_\Theta \cdot g \frac{\left(tg\beta + \varphi_y\right)}{\left(1 - tg\beta \cdot \varphi_y\right)}}, \qquad (8.24)$$

where v_φ is the critical speed of the vehicle when moving on a road with a transverse slope, m/s.

To determine the critical speed in km/h, let's rewrite the equation (8.24) in the form

$$v_{a\varphi} = 3.6\sqrt{r_\Theta \cdot g \frac{\left(tg\beta + \varphi_y\right)}{\left(1 - tg\beta \cdot \varphi_y\right)}}. \qquad (8.25)$$

The condition of stability of the vehicle after overturning $M_{P_y} = M_{G_a}$, using the scheme presented in Figure 8.3, can be written in the form

$$\left(P_y \cdot \cos\beta - G_a \cdot \sin\beta\right) \cdot h_g = \left(G_a \cdot \cos\beta - P_y \cdot \sin\beta\right) \cdot \frac{B}{2}. \qquad (8.26)$$

Let's open the parentheses in equation (8.26)

$$P_y \cdot \cos\beta \cdot h_g - G_a \cdot \sin\beta \cdot h_g = G_a \cdot \cos\beta \cdot \frac{B}{2} - P_y \cdot \sin\beta \cdot \frac{B}{2}. \qquad (8.27)$$

Given that P_y is a centrifugal force, it is possible to write

$$\frac{G_a}{g} \cdot \frac{v^2}{r_\Theta} = \frac{G_a \cdot (\sin\beta \cdot h_g + \cos\beta \cdot \frac{B}{2})}{\cos\beta \cdot h_g - \sin\beta \cdot \frac{B}{2}}. \qquad (8.28)$$

Considering that in this case $v = v_{op}$, from equation (8.28) we determine the critical speed for lateral overturning

$$v_{op} = \sqrt{\frac{g \cdot r_\Theta \cdot \left(\sin\beta \cdot h_g + \cos\beta \cdot \frac{B}{2}\right)}{\cos\beta \cdot h_g - \sin\beta \cdot \frac{B}{2}}}. \qquad (8.29)$$

We multiply the numerator and denominator of equation (8.29) by 2/cos β and given that $v_{aop} = 3.6\, v_{op}$, we reduce it to the form

$$v_{aop} = 3.6\sqrt{\frac{g \cdot r_\Theta \left(2tg\beta \cdot h_g + B\right)}{2h_g - tg\beta \cdot B}}. \qquad (8.30)$$

Comparing the critical velocities for lateral sliding, determined by equations (8.3), (8.25), and overturning, determined by equations (8.7), (8.30), it is not difficult to establish that at the value of the transverse slope $\beta = 0$, their values are the same. When the lateral slope of the road increases towards the center of the turn, the critical speeds for lateral stability increase. This circumstance is taken into account in road construction to increase the stability of vehicle.

8.3. Stability of the vehicle against overturning taking into account body roll

Under the action of a lateral force, the vehicle body tilts due to the properties of the wheel suspension. This phenomenon is called *body roll* . The roll of the body affects the stability of the vehicle against overturning. Body roll occurs around the roll axis. The roll axis of the vehicle body is a line that connects the roll centers of the front and rear suspensions.

8.3.1. The roll center of the vehicle suspension

The roll center of the suspension depends on the design of its guide device. It should be noted that the position of the roll center of independent suspensions depends on the degree of load, which means that it changes during operation.

The determination of the roll center of independent suspensions is shown in Figure 8.4. The center of the suspension roll is located at the point of intersection of the line connecting the center of the wheel footprint and the instantaneous center of wheel sway with the central longitudinal plane of the vehicle.

The determination of the roll center of an independent suspension with one transverse lever is shown in Figure 8.4a. In this suspension, the instantaneous center of wheel swing coincides with the pivot of the lever. The point of intersection "o" of the central longitudinal plane of the vehicle and the line drawn through the center of the wheel footprint and the pivot of the lever is *the center of the suspension roll* .

The position of the roll center of the independent suspension on two transverse levers (Fig. 8.4b, 8.4c, 8.4d) depends on their location. The instantaneous center of wheel swing in such suspensions is at the point of intersection of the lines drawn through the levers. In suspensions with parallel levers (Fig. 8.4c), such a point is located at infinity. A line drawn from infinity to the center of the wheel footprint lies on the support surface, so the roll center of such a suspension coincides with the level of the support surface. The instantaneous center of wheel sway in McPherson suspension (Fig. 8.4e) is located at the point of intersection of the line drawn through the lever and the line drawn perpendicular to the axis of the rack through the upper hinge.The roll center of the dependent suspension is also determined by the design of its guide device. In a spring suspension with a transverse lever, the roll center is located at the intersection of its axis with the central longitudinal plane of the vehicle .

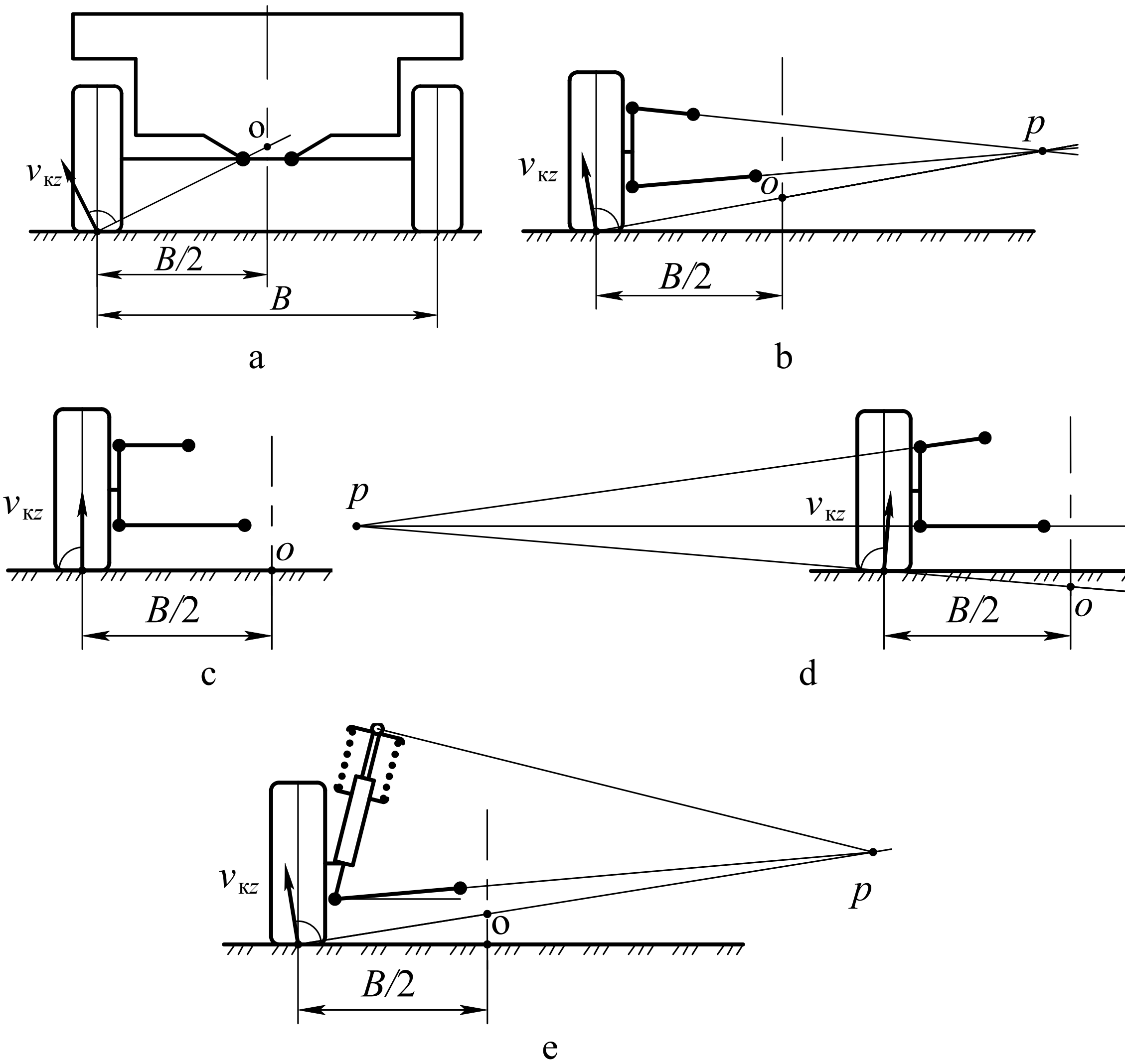
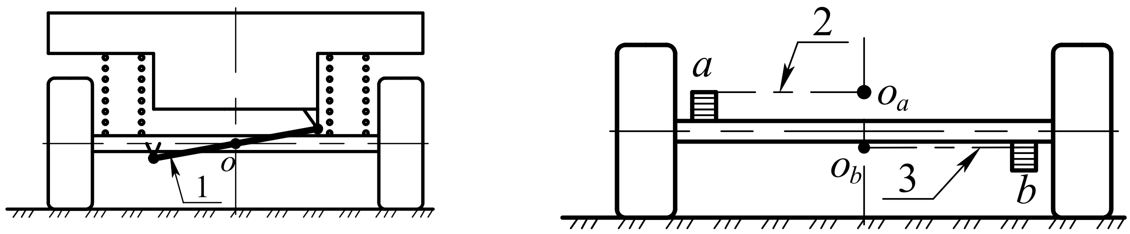


Fig. 8.4. **Determination of the roll center of independent suspensions** :
a - with one lever; b, c, d - with two levers; e – McPherson suspension;
B – vehicle track; $v_{кz}$ is the vector of the instantaneous speed of swinging the wheel around the pole *p* ; *o* is the center of the roll of the suspension



spring suspension with cross lever

spring suspension

Fig. 8.5. **Determination of the roll center of dependent suspensions :**
1 – transverse lever of the suspension; 2, 3 – horizontal from the center of attachment springs to the body to the intersection with the longitudinal plane;
a - during installation springs on top of the bridge beam; *b* – when installing the spring from below on the bridge beam

8.3.2. The roll axis of the vehicle body

The position of the roll axis of the body is shown in Figure 8.6. At the same time, the roll of the body occurs as a result of the side force. When the vehicle is moving in a circle, the lateral force is by its nature a centrifugal force acting at the center of gravity of the sprung mass.

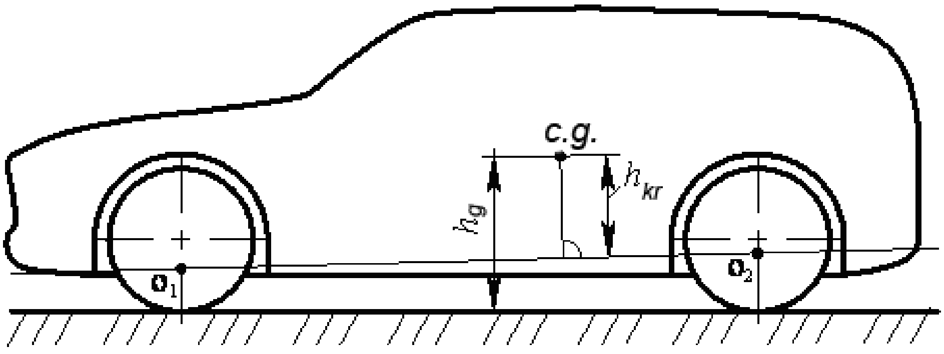


Fig. 8.6. **Here is the roll of the vehicle** :
o_1, o_2 – roll center of the front and rear suspensions, respectively; c.g. – center of gravity of the vehicle body (sprung mass); h_g - height of the center of gravity; h_{kr} is the height of the roll of the vehicle body

At the same time, the body roll height is defined as the distance between the center of gravity and the roll axis. In the case of straight-line movement of the vehicle, the lateral force can be formed by the force of the side wind. At the same time, the lateral force that causes the roll of the body is, by its nature, the total resultant force applied from the air side in the center of the windage of the body.

8.3.3. Critical speed of the vehicle after overturning taking into account body roll

The lateral force P_y , which causes the roll of the body and overturning of the vehicle in the lateral direction, is applied at point *c* (Fig. 8.7). If the lateral force is caused by the centrifugal force due to circular motion , it is the center of gravity of the body (sprung mass). Under the action of a gust of side wind , this is the center of the body's buoyancy.

The lateral force tends to tilt the body on the elastic elements of the suspension. At the same time, an elastic moment is formed in the suspension, which compensates for the overturning moment caused by the lateral force.

Condition of equilibrium of the body on the suspension under the action of centrifugal force

$$M_{\text{podv}} = P_y\left(h_{\text{kr}} - \Delta h\right) + G_{\text{ap}} \cdot \Delta B\,, \tag{8.31}$$

where M_{podv} is the elastic torque of the suspension;

Δh – decrease in the height of the center of gravity;

G_{ap} – gravitational force of the sprung mass of the vehicle (body);

ΔB is the displacement of the center of gravity.

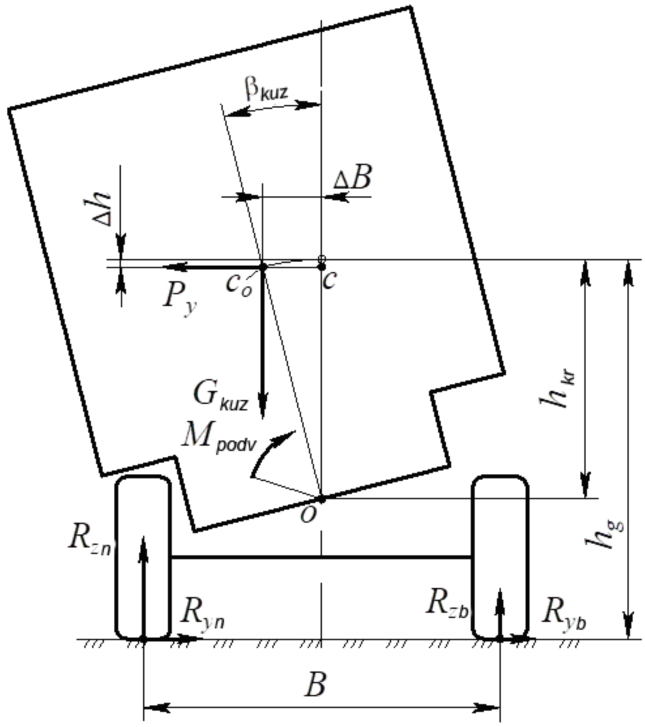


Fig. 8.7. **Scheme for determining the stability of the vehicle against overturning taking into account the suspension**

Mixing the center of gravity Δh and ΔB is easy to determine from the triangle o, c_o, c

$$\Delta h = h_{\text{kr}} \cdot \left(1 - \cos\beta_{\text{kuz}}\right); \tag{8.32}$$

$$\Delta B = h_{\text{kr}} \cdot \sin\beta_{\text{kuz}}\,. \tag{8.33}$$

At small values of the angle β_{kuz} it can be that $\Delta B \approx h_{\text{kr}} \cdot \beta_{\text{kuz}}$ and $\Delta h = 0$.

The spring torque of the suspension M_{podv} is determined by its torsional rigidity

$$M_{\text{podv}} = \left(c_1 + c_2\right) \cdot \beta_{\text{kuz}}\,, \tag{8.34}$$

where c_1 , c_2 – torsional rigidity of the front and rear suspensions, respectively, N·m/rad.

Taking into account the above-mentioned equilibrium equation (8.31) will take the form

$$\left(c_1+c_2\right)\cdot\beta_{\text{kuz}}=P_y\cdot h_{\text{kr}}+G_{\text{ap}}\cdot h_{\text{kr}}\cdot\beta_{\text{kuz}}. \quad (8.35)$$

Where do we determine the roll angle of the body

$$\beta_{\text{kuz}}=\frac{P_y\cdot h_{\text{kr}}}{c_1+c_2+G_{\text{ap}}\cdot h_{\text{kr}}}. \quad (8.36)$$

Mixing the center of gravity to the side defines the equation

$$\Delta B=h_{\text{kr}}\cdot\beta_{\text{kuz}}=\frac{P_y\cdot h_{\text{kr}}^2}{c_1+c_2+G_{\text{ap}}\cdot h_{\text{kr}}}. \quad (8.37)$$

The critical speed of the vehicle after overturning, taking into account body roll, is obtained from equation (8.6), taking into account that the arm of the restoring moment has decreased by ΔB .

$$v_{\text{aop}}=3.6\sqrt{\frac{g\cdot r_{\Theta}\cdot\left(0.5\cdot B-\Delta B\right)}{h_g}}=3.6\sqrt{\frac{g\cdot r_{\Theta}\cdot\left(B-2\cdot\Delta B\right)}{2\cdot h_g}}. \quad (8.38)$$

8.4. The movement of the vehicle when one of the axles is moved

In the first subsection of this section, we considered the stability of the vehicle against skidding of both axles at the same time. But one of the axles actually begins to slide.

Let's consider the peculiarities of the vehicle movement at the moment one of the axles begins to slide. The following condition is necessary for the wheel to roll without skidding (sideways):

$$P_{\varphi}\geq\sqrt{R_x^2+R_y^2},$$

where P_{φ} is the force of wheel adhesion to the road;

R_x – longitudinal reaction of the road on the wheel;

R_y is the lateral reaction of the road on the wheel.

For the case of wheel rolling without slipping, the lateral reaction of the road must meet the condition

$$R_y\geq\sqrt{P_{\varphi}^2-R_x^2}.$$

The lateral reaction R_y , which is added to the wheel and does not cause side slip, is greater, the greater P_φ and the smaller R_x . At $R_x = P_\varphi$ the wheel will start to slide at the slightest side impact.

<u>*Conclusion*</u> : under the specified adhesion conditions (P_φ), ***the wheel is more resistant to side slip if the longitudinal reaction R_x has a smaller value!***

Comparison of lateral stability of driven, driving and brake wheels :

– case 1 – longitudinal reaction on the driven wheel $R_x = P_f$;

– case 2 – longitudinal reaction on the driving wheel $R_x = P_k - P_f$;

– case 3 – longitudinal reaction on the brake wheel $R_x = P_t + P_f$.

It is obvious that the driven wheel is the most resistant against lateral slippage, since the force of resistance to rolling P_f is ten times less than the full traction force P_k on the driving wheel and the braking force P_t on the brake one.

Therefore, in the traction mode of movement, the driving axle of the vehicle is most likely to slip. Let's consider the process of bringing one of the axles of the vehicle with the rear driving wheels (Fig. 8.8 and 8.9).

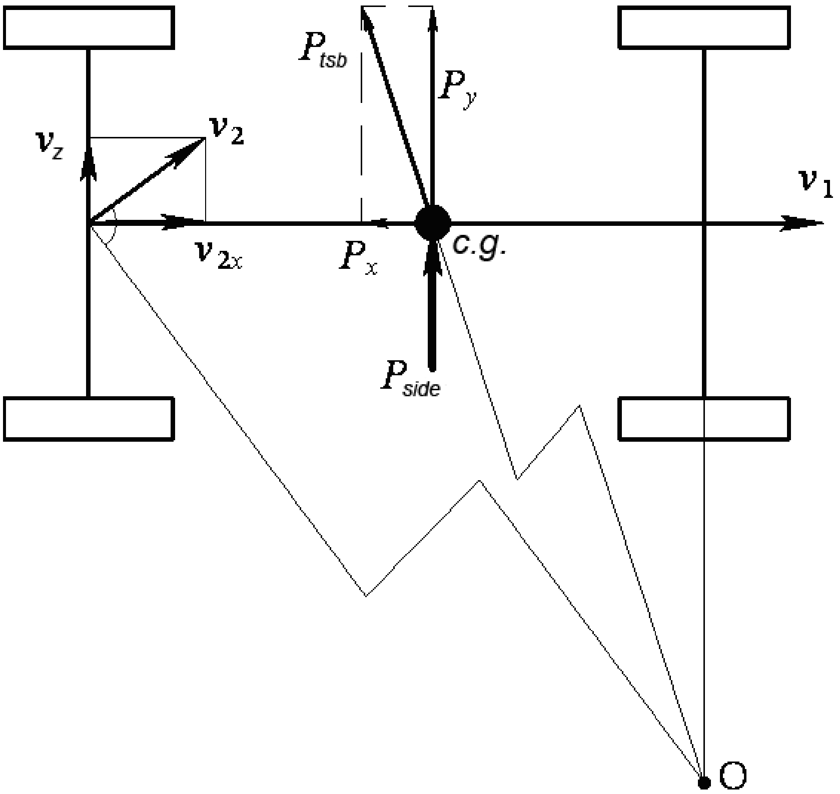


Fig. 8.8. **The scheme of the movement of the vehicle when the rear axle is brought in** : R_{side} – lateral disturbing force; P_{tsb} is the centrifugal force acting on the vehicle; $v_1 = v_{2x}$ – the speed of the front and rear axles along the *x axis* ; v_z is speed bringing in the rear axle; v_2 – the actual speed of the rear axle; O is the instantaneous center of rotation

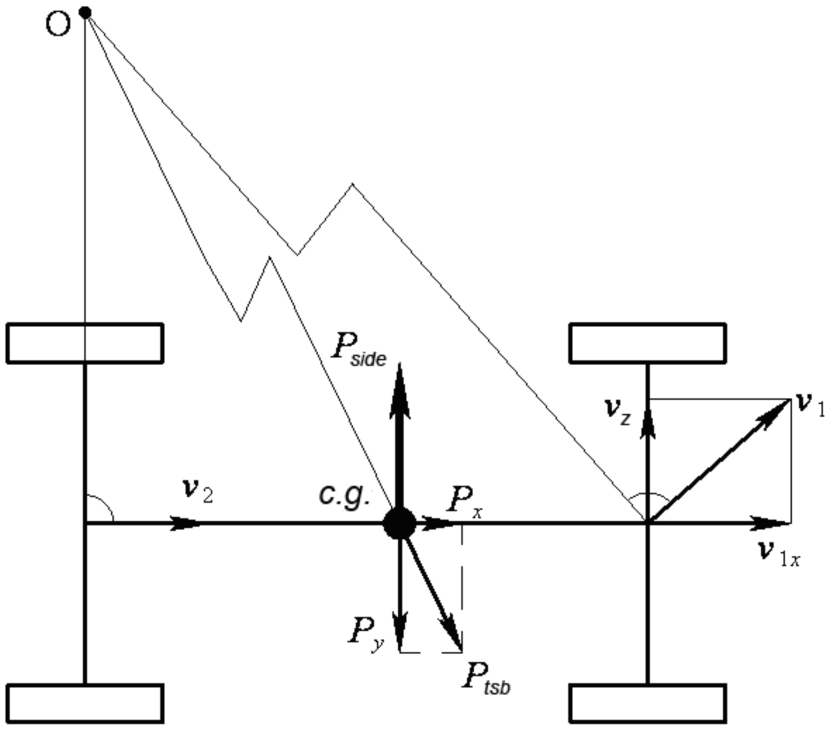


Fig. 8.9. **The scheme of the movement of the vehicle when the front axle is brought in** : $v_{1x} = v_2$ – the speed of the front and rear axles along the *x axis* ; v_z is speed bringing in the front axle; v_1 – front axle speed; other designations see Fig. 8.8

In the event that the rear axle of the vehicle begins to skid, caused by the action of the lateral force P_{side}, its center of gravity moves along a certain curvilinear trajectory with the instantaneous center of rotation at the point O. At the same time, a centrifugal force P_{tsb} , directed from the center O, while the direction of its component P_y coincides with the direction of the force P_{side} , which causes drift. In this case, the force P_{side}, which caused the skidding, consists of P_y and the skidding intensifies, the turn of the vehicle progresses.

Skidding of the rear drive axle is not only more likely, but also more dangerous. To reduce the likelihood of the rear drive axle skidding, it is advisable to reduce the traction force on the wheels. If the skidding of the rear driving axle has begun, it can be extinguished by steering control by turning the steered wheels towards the instantaneous center of rotation O. At the same time, the radius of curvature of the trajectory of movement increases and the centrifugal force decreases.

When the front axle skids (Fig. 8.9), a centrifugal force occurs, directed in the direction opposite to skidding, and prevents the development of skidding. Therefore, the front axle is automatically extinguished .

In the case of the same probability of the front wheel drive axle slipping in a front-wheel drive vehicle and a rear-wheel drive vehicle, the movement of the front-wheel drive vehicle is more stable.

Control questions

1. What is vehicle stability? What types of vehicle stability are there?
2. Name the indicators of transverse stability of the vehicle.
3. How does the transverse slope of the road affect the lateral stability of the vehicle?
4. How does body roll affect the transverse stability of the vehicle?
5. How is the roll center of the suspension determined?
6. What is the roll axis of a vehicle?
7. Draw a diagram of the movement of the vehicle when the front axle is engaged.
8. Draw a diagram of the movement of the vehicle when the rear axle is slipping.

TOPIC 9

THE MOVEMENT OF THE VEHICLE ALONG A CURVILINEAR TRAJECTORY AND ITS CONTROLLABILITY

9.1 . Definition and evaluation indicators of manageability vehicle

Vehicle controllability is a set of properties that determine the characteristics of the vehicle kinematic and power reactions to the driver's control actions during the formation of the movement trajectory.

Maneuverability is related to the safety of driving a vehicle. Loss of controllability is usually manifested in an involuntary deviation of the trajectory and (or) course of the vehicle from the position of the steered wheels set by the driver.

Controllability depends on:

– lateral elasticity of tires;

– stabilization of steered wheels;

– oscillations of steered wheels;

– compliance of the kinematics of the suspension of the steered wheels with the kinematics of the steering drive.

vehicle is evaluated *by the stability of the trajectory control and the stability of the course control* .

Estimated indicators of control stability and units of measurement:

a) stability of trajectory control, points;

b) stability of course management, points;

c) stability of trajectory control during braking, points;

d) stability of directional control during braking, points;

e) the maximum speed of the maneuver, v_{pr}, km/h;

f) the speed of the beginning of the decrease in the stability of the trajectory control, v_{tr}, km/h;

g) the speed of the beginning of the reduction of the stability of course control, v_{course}, km/h.

Indicators "a" and "b" are determined on special roads at the maximum speed v_{max} or on roads of the first category at the permitted speed. The evaluation of indicators "c", "d" takes place during braking from the speed v_n proposed for this category of the vehicle to the speed $v = 0.5\ v_n$ with deceleration $j_a = 0.5\ g$.

The assessment is given in points according to the subjective feelings of the examiner (organoleptic method). Indicators "d" - "g" are determined for critical modes of movement, which consist in the execution of the "shift" and "turn" maneuvers specified by the marking.

9.2. Kinematics of turning a vehicle with rigid wheels

The kinematics of turning a vehicle with steered front wheels depends on the mode of their movement. For the case of the driving rear wheels and the driven front wheels, the kinematics of the vehicle turn is explained by the diagram presented in Figure 9.1. Here, the movement of a vehicle on a turn at low speed is considered. In this connection, the effect of centrifugal forces can be neglected. To ensure rolling of all wheels of the vehicle without slipping or sliding, they must roll in concentric circles with the instantaneous center of rotation at the point O. Therefore, the outer wheel turns to an angle Θ_n, and the inner one - to an angle Θ_v.

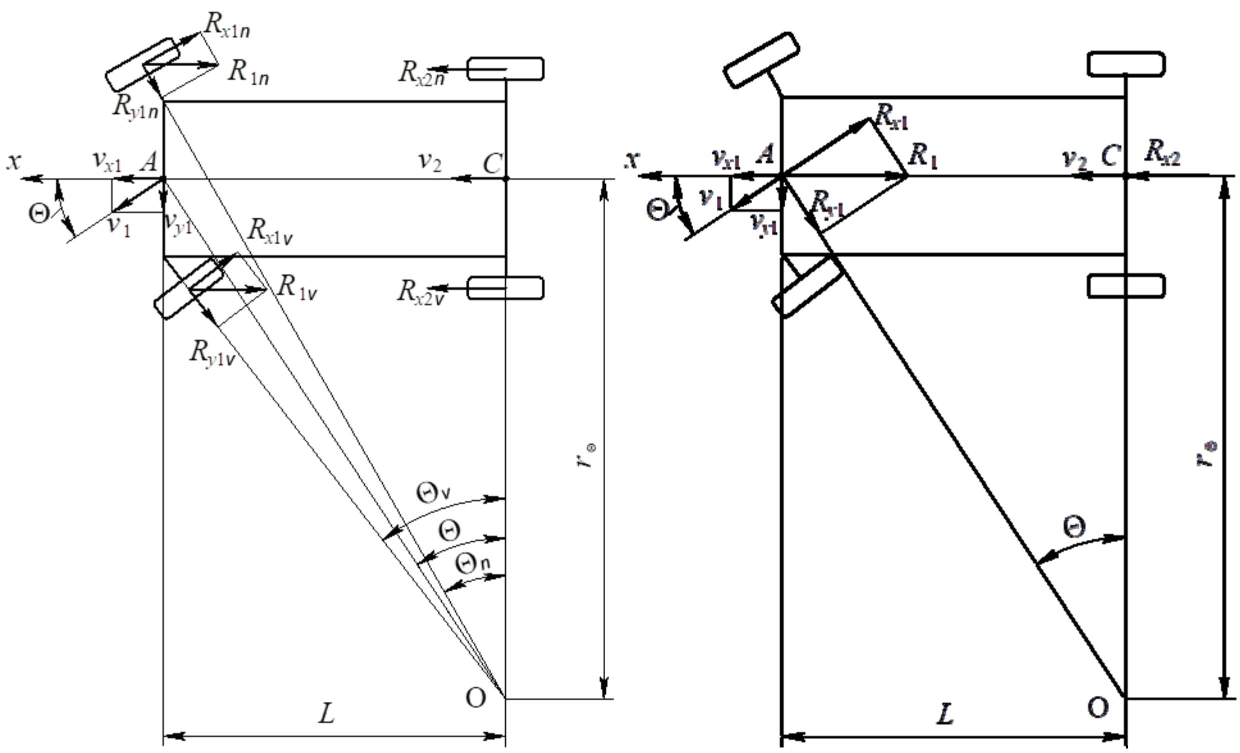


Fig. 9.1. **The scheme of turning a vehicle with rigid wheels:** a – with reactions on each wheel; b - simplified scheme (with total reactions on the axes)

When analyzing the kinematics of the turn, we will assume that the speed vector of the front axle v_1 when turning the steered wheels deviates from the longitudinal axis x by the average angle Θ, which is equal to the average value of the turning angles of the steered wheels Θ_n and Θ_v.

At the same time, the velocity vector of the rear axle v_2 coincides with the longitudinal axis x . The center of rotation of the vehicle O is located at the intersection of the perpendiculars drawn to the velocity vectors.

When the torque is applied to the driving rear wheels, a traction force R_k is formed on them , which determines the movement of the vehicle. As a result, longitudinal reactions R_{x2n} and R_{x2v} occur in the spots of contact of the driving wheels . At the same time, reactions R_{1n} and R_{1v} occur in the contact spots of the turned steered wheels , directed against the reactions R_{x2n} and R_{x2v} .

Each of the reactions R_{1n} and R_{1v} can be decomposed into two components: in the longitudinal plane of the wheel R_{x1n} and R_{x1v} and transverse - R_{y1n} and R_{y1v} .

If the sum of the reactions R_{x2n} and R_{x2v} exceeds the force of resistance to movement, then the steered wheels roll along the corresponding trajectories and the vehicle makes a turn. At the same time, longitudinal reactions R_{x1n} and R_{x1v} act in the contact zone of the steered wheels, the sum of which is equal to the rolling resistance force of the wheels of the front axle R_{x1} .

To analyze the process of turning a vehicle, let's simplify its scheme by replacing the reactions on the wheels with reactions on the corresponding axes (Fig. 9.1b).

The turning radius of a vehicle with rigid wheels r_Θ is the distance from the center of the turn to the middle plane of the vehicle, which is determined from the *OAS triangle* (Fig. 9.1a).

$$tg\Theta = \frac{L}{r_\Theta} \Rightarrow r_\Theta = \frac{L}{tg\Theta}. \tag{9.1}$$

For small angles, equation (9.1) can be written in the form

$$r_\Theta = \frac{L}{\Theta}. \tag{9.2}$$

At the same time, the angular velocity of the vehicle is ω_a , s^{-1} when moving in a circle with a radius r_Θ

$$\omega_a = \frac{v}{r_\Theta} = \frac{v}{L} \cdot \text{tg}\Theta \text{ or } \omega_a = \frac{v_a}{3.6 \cdot L} \cdot \text{tg}\Theta, \tag{9.3}$$

where v , v_a - the linear speed of the vehicle in m/s and km/h, respectively.

The linear speed of the vehicle when turning corresponds to the linear speed of the point of intersection of its longitudinal axis and the turning radius. For a vehicle with rigid wheels, this speed is equal to v_2.

The movement of the vehicle along the trajectory that corresponds to the position of the steered wheels is possible provided that they do not slip laterally. The condition of being able to turn without sideslip determines the ratio

$$R_1 \leq P_{\varphi 1}. \tag{9.4}$$

where R_1 is the total reaction on the wheels of the front axle;

$P_{\varphi 1}$ is the force of adhesion of the driven wheels of the front axle to the supporting surface.

After revealing the values of the components of (9.4), we get

$$\sqrt{R_{x1}^2 + R_{y1}^2} \leq R_{z1} \cdot \varphi_{x1}. \tag{9.5}$$

For the case of movement at a low speed, it can be assumed that the lateral reaction $R_{y1} = P_{y1} = R_{x1} \cdot \mathrm{tg}\,\Theta$. Let's transform (9.5) by solving with respect to R_{y1} and then determining R_{x1} and R_{y1} in terms of R_{z1}

$$R_{y1} \leq \sqrt{R_{z1}^2 \cdot \varphi_{x1}^2 - R_{z1}^2 \cdot f_{x1}^2}.$$

$$R_{y1} = R_{x1} \cdot tg\Theta = R_{z1} \cdot f_1 \cdot tg\Theta \leq \sqrt{R_{z1}^2 \cdot \varphi_{x1}^2 - R_{z1}^2 \cdot f_1^2}$$

or

$$f_1 \cdot tg\Theta \leq \sqrt{\varphi_{x1}^2 - f_1^2} \tag{9.6}$$

From the equation (9.6), we obtain the condition for the possibility of turning in the absence of lateral slip of the steered wheels. This condition limits the maximum turning angle of the steered wheels.

$$\Theta \leq arctg \frac{\sqrt{\varphi_{x1}^2 - f_1^2}}{f_1}. \tag{9.7}$$

If the condition in the equation (9.7) is not fulfilled, then the steered wheels do not roll in a circle, but slide in the longitudinal direction. It should be noted that, taking into account the variety of operational values of the coefficients φ and f, the limit angle of rotation Θ steered wheels according to (9.7) determines the interval of 78 ... 88 degrees. That is, when maneuvering the vehicle at low speed, the vehicle always turns without lateral sliding of the steering wheels, since the maximum angle of their rotation usually does not exceed 45 ... 55 degrees.

For the case of the vehicle moving along a curved trajectory at high speed, the condition (9.4) may not be fulfilled, and the movement occurs with lateral slip of the steered wheels. This is due to the fact that in this case the lateral reaction R_{y1} is formed by the lateral component P_{y1} of the pushing force and the centrifugal force P_{tsb1}, which falls on the driven wheels of the front axle.

$$R_{y1} = P_{y1} + P_{\text{tsb1}}, \tag{9.8}$$

where $P_{\text{tsb1}} = \dfrac{G_1}{g} \cdot \dfrac{v_1^2}{r_\Theta \cdot \cos\Theta}$ is the centrifugal force applied to the front axle.

The speed of the vehicle on a curved path, at which the lateral sliding of the steered wheels begins, is called *the critical speed under the conditions of steerability* $v_{\varphi\text{y}}$. Taking into account (9.8), revealing the centrifugal force P_{tsb1}, we transform the equation (9.6) into the form

$$R_{y1} = R_{z1} \cdot f_1 \cdot tg\Theta + \frac{G_1}{g} \cdot \frac{v_1^2}{r_\Theta \cdot \cos\Theta} \le \sqrt{R_{z1}^2 \cdot \varphi_{x1}^2 - R_{z1}^2 \cdot f_1^2}\,. \tag{9.9}$$

Taking into account (9.1) and assuming that $R_{z1} \approx G_1$, equation (9.9) for the critical case will take the form

$$f_1 \cdot tg\Theta + \frac{v_{\varphi\text{y}}^2 \cdot tg\Theta}{g \cdot L \cdot \cos\Theta} = \sqrt{\varphi_{x1}^2 - f_1^2}\,. \tag{9.10}$$

The value of the critical speed $v_{\varphi\text{y}}$ under the controllability conditions is determined from (9.10) according to this dependence

$$v_{\varphi\text{y}} = \sqrt{\left(\frac{\sqrt{\varphi_{x1}^2 - f_1^2}}{tg\Theta} - f_1^2\right) \cdot g \cdot L \cdot \cos\Theta}\,. \tag{9.11}$$

The analysis of equation (9.11) shows that the critical speed under controllability conditions $v_{\varphi\text{y}}$ depends on the base of the vehicle L, the average angle of rotation of the controlled wheels and on the conditions of interaction of the wheels with the road, coefficients φ_{x1} and f_1.

If the speed of the vehicle is greater than $v_{\varphi y}$, then the steered wheels slip in the transverse direction when turning. The greater the angle of rotation of the steered wheels, the lower the speed of the vehicle should be. If the condition $\varphi_{x1} \le f_1/\cos\Theta$ is fulfilled, for example, on a hard slippery road, the root equation (9.11) becomes zero or a negative value and, accordingly, the speed $v_{\varphi y}$ becomes an imaginary value, and the vehicle becomes uncontrollable.

9.3. Kinematics of turning a vehicle with elastic wheels

Figure 9.2 shows the turning scheme of a vehicle with elastic wheels. The steered wheels are turned to the average angle Θ, but due to the lateral deviation of the tires, the average velocity vector of the front axle v_1 is deviated by the lateral deviation angle δ_1. The velocity vector of the rear axle v_2 is deviated from the longitudinal axis of the vehicle by the lateral diversioṇ angle δ_2. At the same time, the center of rotation of the vehicle O_δ is located at the intersection of the perpendiculars to these vectors. The turning radius r_δ is defined as the distance from the turning center O_δ to the longitudinal axis of the vehicle .

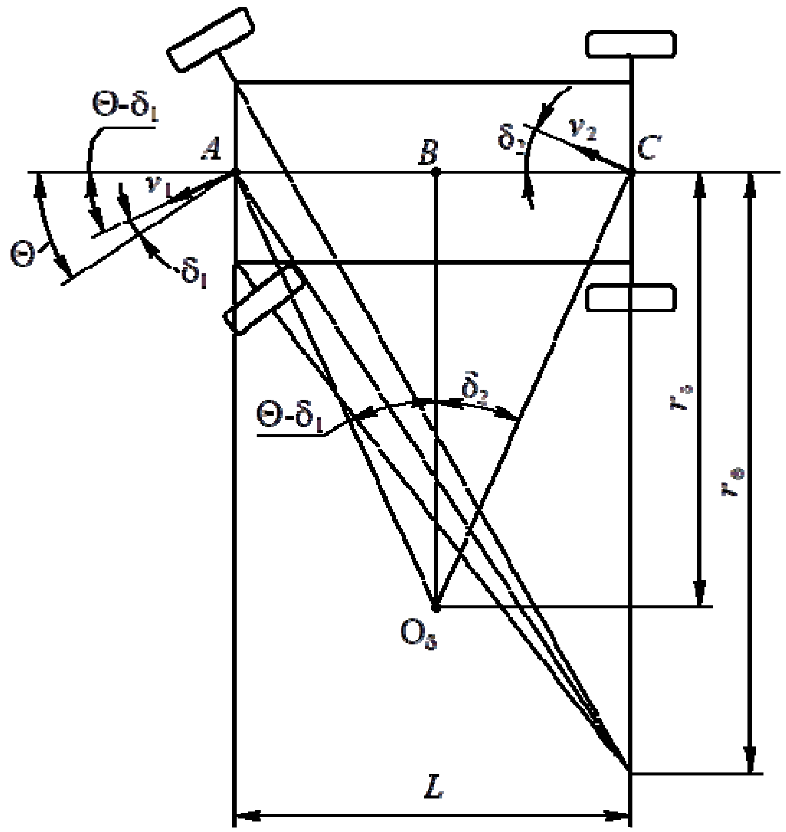


Fig. 9.2. **Scheme of turning a vehicle with elastic wheels**

It follows from Figure 9.2 that $\angle CO_\delta B = \delta_2$ and $\angle AO_\delta B = \Theta - \delta_1$, so we can write

$$\left.\begin{aligned} tg\delta_2 &= \frac{BC}{BO_\delta} \Rightarrow BC = tg\delta_2 \cdot r_\delta; \\ tg(\Theta - \delta_1) &= \frac{AB}{BO_\delta} \Rightarrow AB = tg(\Theta - \delta_1) \cdot r_\delta. \end{aligned}\right\}, \qquad (9.12)$$

where $BO_\delta = r_\delta$ is the turning radius of a vehicle with elastic wheels.

As can be seen from Figure 9.2, the base of the vehicle is equal to the sum of the legs (AB) and (BC) and taking into account (9.12) we get

$$L = AB + BC = tg(\Theta - \delta_1) \cdot r_\delta + tg\delta_2 \cdot r_\delta. \qquad (9.13)$$

Solving equation (9.13), we determine the turning radius of a vehicle with elastic tires

$$r_\delta = \frac{L}{tg\delta_2 + tg(\Theta - \delta_1)}. \qquad (9.14)$$

For small angles $tg\delta_2 \approx \delta_2$, $tg(\Theta - \delta_1) \approx (\Theta - \delta_1)$, in this case equation (9.14) can be written in the form

$$r_\delta = \frac{L}{\delta_2 + \Theta - \delta_1} \text{ or } r_\delta = \frac{L}{\Theta + \delta_2 - \delta_1}. \qquad (9.15)$$

At the same time, the angular velocity of the vehicle ω_a, when moving in a circle with a radius r_δ

$$\omega_a = \frac{v}{r_\delta} = \frac{v}{L} \cdot \left[\text{tg}(\Theta - \delta_1) + \text{tg}\delta_2 \right] \text{ or } \omega_a = \frac{v_a}{3.6 \cdot L} \cdot \left[\text{tg}(\Theta - \delta_1) + \text{tg}\delta_2 \right], \quad (9.16)$$

$$\omega_a \approx \frac{v}{L} \cdot [\Theta + \delta_2 - \delta_1] \text{ or } \omega_a \approx \frac{v_a}{3.6 \cdot L} \cdot [\Theta + \delta_2 - \delta_1]. \qquad (9.17)$$

9.4. Vehicle turning

The property of the vehicle to change the kinematic parameters of the turn under the action of external lateral forces at a fixed value of the angle of rotation of the steered wheels Θ is called *the turning of the* *vehicle.* At the same time, the vehicle can turn around its vertical axis and deviate from the trajectory set by the driver.

Depending on the ratio of the lateral deflection angles δ_1 and δ_2, three types of vehicle steering are distinguished: neutral, insufficient and excessive.

Neutral turning is observed when the condition is fulfilled $\delta_1 = \delta_2$. In this case, when the vehicle is moving in a neutral position of the steered wheels, a lateral force (for example, a gust of side wind) causes a deviation of the velocity vectors v_1 and v_2 from the longitudinal axis by the same angles. At the same time, the vehicle acquires plane-parallel motion and deviates from the trajectory set by the driver (see Fig. 9.3).

Insufficient turning is observed when condition is met $\delta_1 > \delta_2$. In this case, when the vehicle is moving in a neutral position of the steered wheels, the lateral force causes the velocity vectors v_1 and v_2 to deviate from the longitudinal axis to different angles. At the same time, the vehicle acquires a circular motion around the instantaneous center O_δ and deviates from the trajectory set by the driver with a change in the course angle (Fig. 9.3). As a result of the vehicle circular motion, a centrifugal force P_{tsb} arises , its component P_y is directed against the lateral disturbing force P_{side}, and compensates for its action.

Excessive turning is observed when the condition is met $\delta_1 < \delta_2$. In this case, when the vehicle is moving in a neutral position of the steered wheels, the lateral force causes the velocity vectors v_1 and v_2 to deviate from the longitudinal axis to different angles. At the same time, the vehicle acquires a circular motion around the instantaneous center O_δ and deviates from the trajectory set by the driver with a change in the course angle (Fig. 9.3c). As a result of the circular motion of the vehicle, a centrifugal force P_{tsb} occurs , its component P_y is directed in the same direction as the lateral disturbing force P_{side}, and therefore increases its disturbing influence.

Schemes of movement of vehicle with a fixed value of the angle of rotation of the steered wheels Θ with different types of steering are presented in Figure 9.4. The kinematic parameters of the vehicle turning under the action of external lateral forces change depending on the type of its turning.

In vehicle with neutral and insufficient steering, their turning radius r_δ exceeds the turning radius of a vehicle with rigid wheels r_Θ.

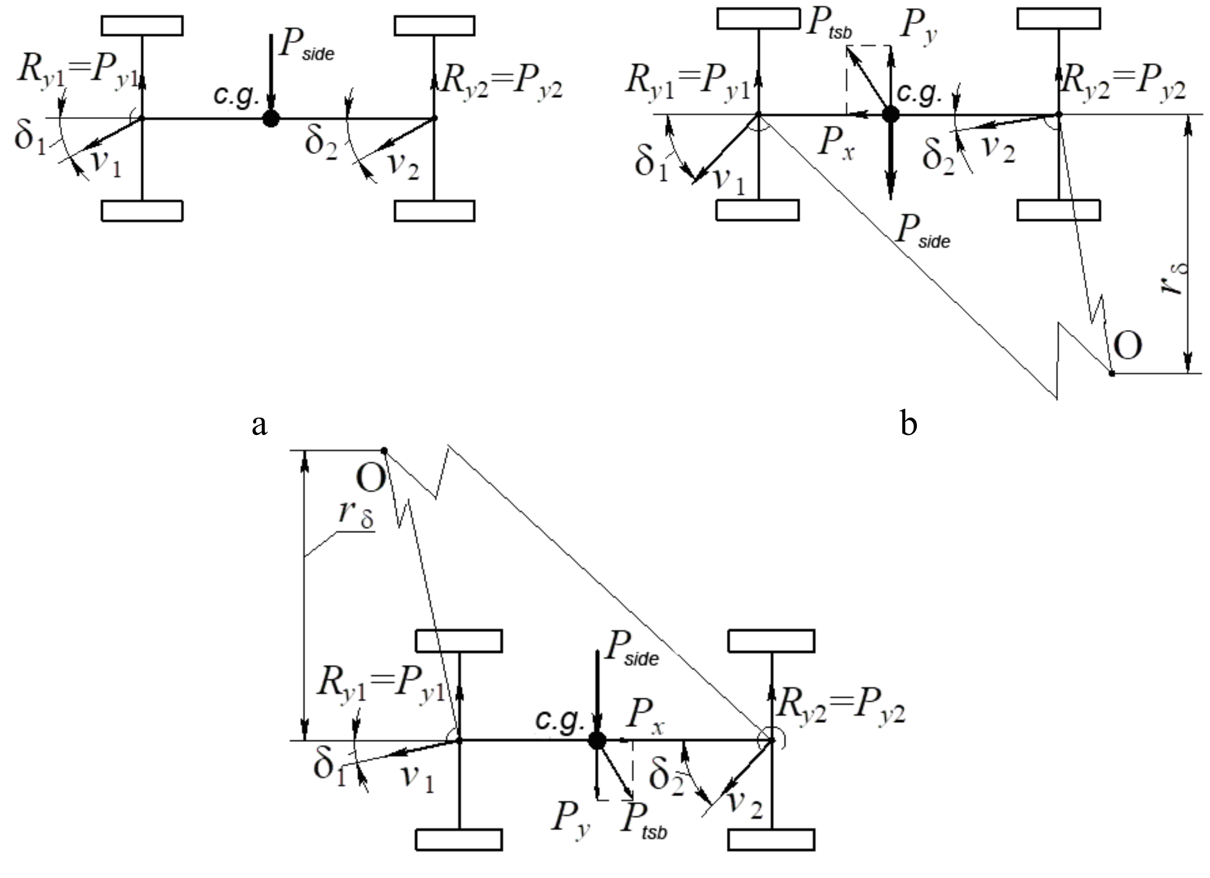


Fig. 9.3. **Scheme of movement of a vehicle with elastic wheels under the action of a lateral force:** a – at neutral steering ($\delta_1 = \delta_2$); b – with insufficient turning ($\delta_1 > \delta_2$); c – with excessive turning ($\delta_1 < \delta_2$)

At the same time, according to equation (9.9), the angular speed of the vehicle decreases. To move along the desired trajectory, the driver needs to additionally turn the steered wheels, but the vehicle movement in this case is stable. In the case of small turning angles of the steered wheels Θ of a vehicle with neutral steering, it can be assumed that $r_\delta = r_\Theta$. If $\Theta = \delta_1 = \delta_2$, then according to equation (9.7) the equality $r_\delta = r_\Theta$ is valid for any values of the angle of rotation of the steered wheels Θ .

In a vehicle with oversteer, the turning radius r_δ less than the turning radius of a vehicle with rigid wheels $r_{\Theta,}$ and at the same time, its angular speed increases accordingly. Since the turning radius r_δ of the vehicle decreases under the action of the force P_y, its longitudinal axis turns as a result of turning the wheels inside the turn. At the same time, the centrifugal force increases and causes even more deflection and, as a result, an even greater turn of the vehicle, that is, the process progresses.

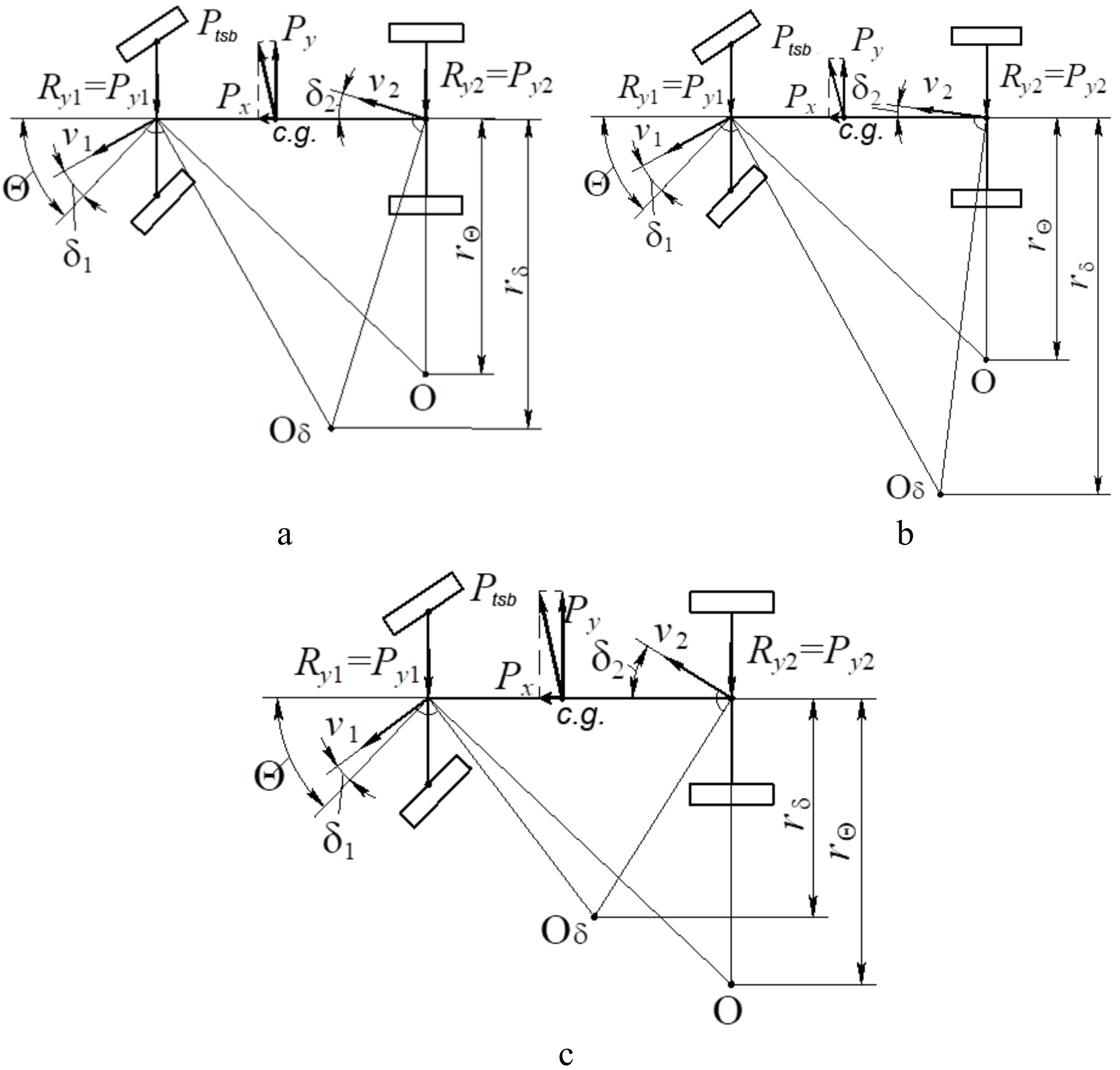


Fig. 9.4. **Scheme of movement of a vehicle with elastic wheels with steered wheels turned:** a – at neutral steering ($\delta_1 = \delta_2$); b – with insufficient turning ($\delta_1 > \delta_2$); c – with excessive turning ($\delta_1 < \delta_2$)

To extinguish the progressive turn of the vehicle, the driver must reduce the turn of the steered wheels. In this case, the movement of the vehicle is unstable. Neutral steering in operating conditions can become excessive due to a decrease in pressure in the rear axle tires and (or) an increase in the load on the rear axle. At the same time, during the design of the vehicle, insufficient steering is set to ensure a margin of stability of movement under the action of lateral forces .

9.5. The critical speed of the vehicle on the lateral deviation

The effect of a lateral force on a moving vehicle leads to a violation of the stability of the trajectory control and directional control. Moreover, even in the case of straight-line movement of the vehicle, the lateral disturbing force can cause curvilinear movement due to the lateral deviation of the tires. This leads to the appearance of a centrifugal force, the magnitude of which depends on the linear speed of the vehicle. It is obvious that at a certain value of the vehicle speed, the vehicle's movement in the direction of diversion may begin, not controlled by the driver. Such a speed is called the *critical speed of the vehicle on the lateral deviation.*

We determine the value of the critical speed of the vehicle on the lateral deviation from the equation (9.15), which, in the absence of rotation of the steered wheels, will take the form

$$r_\delta = \frac{L}{\delta_2 - \delta_1}. \tag{9.18}$$

When condition in equation (9.18) is fulfilled, in the case of $\delta_1 \neq \delta_2$ the vehicle will start moving along a curved path with a radius r_δ without turning the steered wheels.

Question: find the speed of movement of the vehicle along this trajectory, at which such a centrifugal force occurs that causes a progressive uncontrolled deflection of the wheels.

Let's determine from (9.18) the difference of the angles of the lateral deviation and reveal their values

$$\delta_2 - \delta_1 = \frac{L}{r_\delta} \Rightarrow \frac{P_{y2}}{k_{\delta 2}} - \frac{P_{y1}}{k_{\delta 1}} = \frac{L}{r_\delta}, \tag{9.19}$$

Considering that the lateral forces P_{y1} and P_{y2} are formed only by centrifugal forces falling on the rear and front suspensions, then $P_{y2} = m_2 \cdot v^2 / r_\delta$ and $P_{y1} = m_1 \cdot v^2 / r_\delta$. Substitute these values in equation (9.19) and get

$$\frac{m_2 \cdot v^2}{k_{\delta 2} \cdot r_\delta} - \frac{m_1 \cdot v^2}{k_{\delta 1} \cdot r_\delta} = \frac{L}{r_\delta} \Rightarrow \frac{m_2 \cdot v^2}{k_{\delta 2}} - \frac{m_1 \cdot v^2}{k_{\delta 1}} = L, \tag{9.20}$$

where m_2, m_1 – the mass of the vehicle, corresponding to the rear and front axles, respectively;

$k_{\delta 2}$, $k_{\delta 1}$ – coefficient of resistance to the lateral deviation of the wheels, respectively, of the rear and front axles.

Assuming that the speed v is the critical speed for the lateral lead v_δ, m/s, we determine from equation (9.20)

$$v_\delta^2 = \frac{L}{\frac{m_2}{k_{\delta 2}} - \frac{m_1}{k_{\delta 1}}} \Rightarrow v_\delta = \sqrt{\frac{L}{\frac{m_2}{k_{\delta 2}} - \frac{m_1}{k_{\delta 1}}}} \quad \text{or} \quad v_\delta = \sqrt{\frac{g \cdot L}{\frac{G_2}{k_{\delta 2}} - \frac{G_1}{k_{\delta 1}}}}. \tag{9.21}$$

The critical speed on the lateral lead $v_{a\delta}$, km/h

$$v_{a\delta} = 3.6\sqrt{\frac{L}{\frac{m_2}{k_{\delta 2}} - \frac{m_1}{k_{\delta 1}}}} \quad \text{or} \quad v_{a\delta} = 3.6\sqrt{\frac{g \cdot L}{\frac{G_2}{k_{\delta 2}} - \frac{G_1}{k_{\delta 1}}}}. \tag{9.22}$$

The critical speed at the lateral lead according to equations (9.21), (9.22) has a real value in the case of a positive value of the denominator of the underlying equation. This corresponds to an oversteer vehicle. When designing the vehicle, they try to ensure insufficient turning. If, for a number of reasons, the vehicle has an excess turn, then the critical speed for the lateral deviation should be much greater than the maximum speed of the vehicle, i.e. $v_{a\,\delta} >> v_{amax}$. Because when the vehicle reaches v and $_\delta$ there is a complete loss of controllability, even the slightest side impact causes a progressive reversal.

For a vehicle with insufficient steering, there is no critical speed on the lateral lead, which can be seen from equations (9.21), (9.22), because with a negative value of the denominator of the root equation, its value becomes an imaginary value. The turning movement of a vehicle with insufficient turning due to centrifugal force takes place at an increased radius. To maintain the desired turning radius, the driver needs to increase the turning angle of the steered wheels. To assess the controllability of vehicle with insufficient steering, the characteristic speed is determined. This is the speed at which the radius of the trajectory with elastic wheels is doubled compared to a vehicle with rigid wheels, if the angle of return of the steered wheels is the same in both cases.

Determine the angle of rotation of the steered wheels, at which the turning radius of the vehicle with elastic wheels will increase by two times

$$r_\delta = 2r_\Theta \Rightarrow \Theta = \frac{L}{r_\Theta} = 2 \cdot \frac{L}{r_\delta}. \tag{9.23}$$

Let's transform the equation (9.15) into the form $\Theta + \delta_2 - \delta_1 = L/r_\delta$ and taking equation (9.23) into account, we obtain

$$2 \cdot \frac{L}{r_\delta} + \delta_2 - \delta_1 = \frac{L}{r_\delta}. \qquad (9.24)$$

After simple transformations, we will get a dependency

$$\delta_2 - \delta_1 = -\frac{L}{r_\delta} \Rightarrow \delta_1 - \delta_2 = \frac{L}{r_\delta}. \qquad (9.25)$$

After transformations of equation (9.25), we obtain the equation that determines the characteristic speed according to the lateral lead, $v_{a\delta x}$, km/h

$$v_{a\delta x} = 3.6\sqrt{\frac{L}{\frac{m_1}{k_{\delta 1}} - \frac{m_2}{k_{\delta 2}}}} \quad \text{or} \quad v_{a\delta x} = 3.6\sqrt{\frac{g \cdot L}{\frac{G_1}{k_{\delta 1}} - \frac{G_2}{k_{\delta 2}}}}. \qquad (9.26)$$

9.6. The movement of the vehicle along a curvilinear trajectory

When a vehicle moves along a curved path, its center of gravity makes a relative movement around the instantaneous center O $_\delta$ (Fig. 9.5).

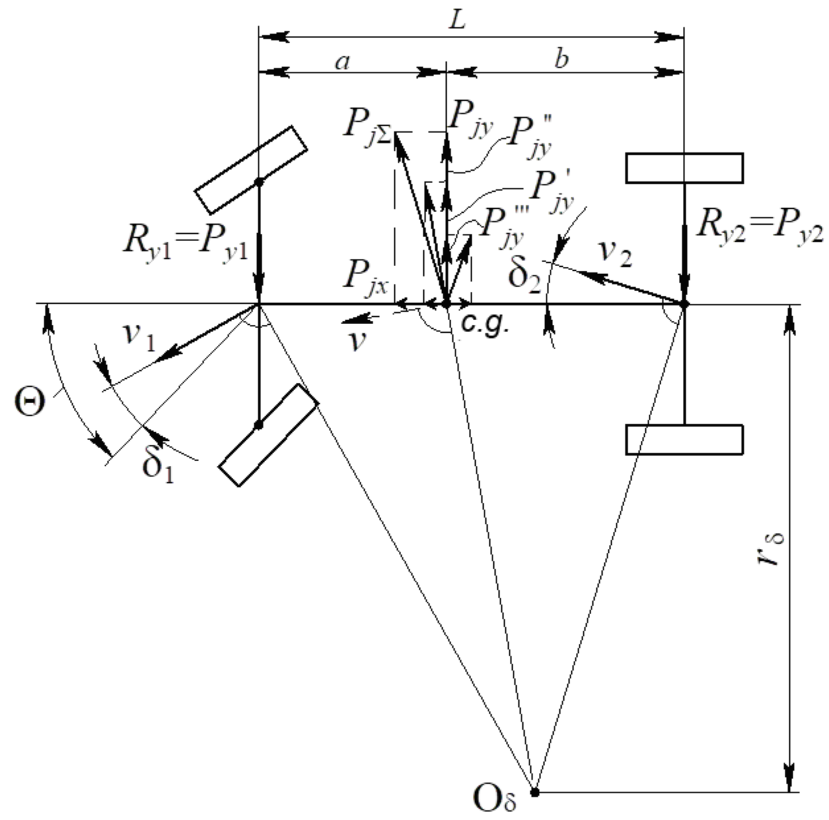


Fig. 9.5. **Scheme of vehicle movement along a curved trajectory when accelerating**

At the same time, the vehicle makes a portable movement around the center of gravity. As a result, the inertial force $P_{\Sigma j}$ acts in the center of gravity of the vehicle , the projections of which on the longitudinal and transverse axes of the vehicle are determined by the equation

$$P_{jx} = m_{\text{a}} \cdot \left[\dot{j}_{\text{a}} - v \cdot \omega_{\text{a}} \cdot \frac{b \cdot (\Theta - \delta_1) - a \cdot \delta_2}{L} \right]; \tag{9.27}$$

$$P_{jy} = m_{\text{a}} \cdot \left[v \cdot \omega_{\text{a}} + v \cdot \frac{b \cdot (\dot{\Theta} - \dot{\delta}_1) - a \cdot \dot{\delta}_2}{L} + \dot{j}_{\text{a}} \cdot \frac{b \cdot (\Theta - \delta_1) - a \cdot \delta_2}{L} \right]. \tag{9.28}$$

The lateral component of the inertial force P_{jy} is the sum of forces caused by various factors.

$$P_{jy} = P'_{jy} + P''_{jy} + P'''_{jy}, \tag{9.29}$$

where $P'_{jy} = m_{\text{a}} \cdot v \cdot \omega_{\text{a}} = \frac{m_{\text{a}} \cdot v^2}{r_\delta}$;

$$P''_{jy} = m_{\text{a}} \cdot v \cdot \frac{b \cdot (\dot{\Theta} - \dot{\delta}_1) - a \cdot \dot{\delta}_2}{L};$$

$$P'''_{jy} = m_{\text{a}} \cdot \dot{j}_{\text{a}} \cdot \frac{b \cdot (\Theta - \delta_1) - a \cdot \delta_2}{L}.$$

The first term P'_{jy} is the centrifugal force. Taking into account (9.15), we obtain the dependence of this component on the vehicle base L , the turning angle Θ and the angles of the lateral departure of the wheels δ_1 and δ_2:

$$P'_{jy} = \frac{m_{\text{a}} \cdot v^2}{r_\delta} \Rightarrow P'_{jy} = \frac{m_{\text{a}} \cdot v^2}{L} \cdot (\Theta + \delta_2 - \delta_1). \tag{9.30}$$

The second term P''_{jy} is the lateral force that appears when the steered wheels turn. Moreover, the higher the speed of the vehicle v , the turning of the steered wheels and the greater the speed of their turning $\dot{\Theta}$, the greater this component. It should be noted that it P''_{jy} depends on the base of the vehicle L and the distribution of the load between the axles.

The third component P'''_{jy} is a lateral force that appears when the speed of the vehicle changes. In the absence of lead, P'''_{jy} it acquires a positive value during accelerated movement and a negative value during deceleration.

At small turning angles of the steered wheels Θ and assuming that $\delta_1 = \delta_2 = \dot{\delta}_1 = \dot{\delta}_2 = 0$, i.e. without taking into account the lateral deviation of the wheels, the entry of the lateral components of the inertia force is simplified

$$P'_{jy} = \frac{m_a \cdot v^2 \cdot \Theta}{L}; \tag{9.31}$$

$$P''_{jy} = \frac{m_a \cdot v \cdot b \cdot \dot{\Theta}}{L}; \tag{9.32}$$

$$P'''_{jy} = \frac{m_a \cdot j_a \cdot b \cdot \Theta}{L}. \tag{9.33}$$

With uniform movement in a circle, the components (9.32) and (9.33) become zero.

9.7. The stability of the movement of the steered wheels of the vehicle

When the vehicle is moving, the steering wheels of the vehicle are affected by:

– destabilizing factors (that is, factors that disrupt the stability of the movement of the steered wheels and cause them to wobble);

– stabilizing factors (that is, factors that increase the stability of the movement of the steered wheels).

9.7.1. Destabilizing factors

1) *The gyroscopic moment of the steered wheels* arises as a result of a change in the plane of their rotation, for example, when hitting a bump in the road (Fig. 9.6). The resulting gyroscopic moment rotates the steered wheel in the horizontal plane around its axis of rotation (pivot), which causes a change in the trajectory of its movement.

The magnitude of the gyroscopic moment in the horizontal plane M_{girg} is determined by the equation (9.34).

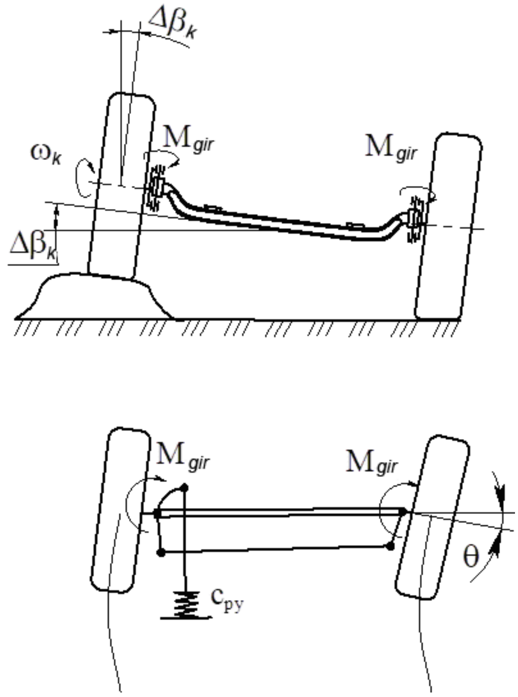


Fig. 9.6. **Influence of the gyroscopic moment of steered wheels on the trajectory of their movement** : $\Delta\beta_k$ – the angle of change in the plane of rotation of the wheel; c_{ry} - stiffness of the steering drive

$$M_{\text{girg}} = J_{\text{k}} \cdot \omega_{\text{k}} \cdot \frac{d\beta_{\text{k}}}{dt}, \qquad (9.34)$$

where $\frac{d\beta_k}{dt}$ is the angular rate of change in the plane of rotation of the steered wheels, rad/s.

With a stationary steering wheel held by the driver, the action of the gyroscopic moment causes compression by the steering drive element. When driving off a bumpy road, the angle of inclination of the wheel decreases, and the angular speed of change in the plane of rotation of the steered wheels changes its sign. This leads to a change in the direction of the gyroscopic moment, and its action consists of the elastic force of the actuator. If the frequency of the change in the direction of the gyroscopic moment coincides with the natural frequency of the wheels, resonance of oscillations occurs, which is unacceptable.

The rotation of the steered wheels in the horizontal plane around their axis of rotation causes a gyroscopic moment acting in the vertical plane M_{girv} . This leads to the rotation of the bridge in the vertical plane with dependent suspension, and in the case of independent suspension to the tilt of the wheel in the vertical plane. Thus, the vibrations of the wheels in the horizontal plane cause their vibrations in the vertical plane.

The magnitude of the gyroscopic moment of the steered wheels depends on the parameters of the wheel and the movement of the vehicle, as well as the suspension design. The moment of inertia of the wheel J_k is determined by its design. The angular speed of the wheel ω_k depends on the radius of the wheel and the speed of the vehicle. The angle of change in the plane of rotation of the wheel $\Delta\beta_k$ is determined by the height of the unevenness of the road and the design of the wheel suspension. The speed of change in the plane of rotation of the wheel depends on the type and design of the suspension. The largest gyroscopic moment is formed with dependent suspensions, since in this case both wheels lean in the same direction at the same angle $\Delta\beta_k$. At the same time, the gyroscopic moments of both wheels add up, and the swaying of the wheels increases.

b) *Imbalance of steered wheels.* Wheel imbalance occurs when its center of gravity deviates from its axis of rotation as a result of the technological features of tire manufacturing, uneven wear and other reasons. Deviation of the center of gravity of the wheel from the axis of its rotation is called *eccentricity* . Wheel imbalance is assessed by the amount of imbalance. The numerical value of the imbalance is determined by the equation

$$\Delta_k = m_k \cdot e_{st}, \tag{9.35}$$

where Δ_k is wheel imbalance, kg·m;

e_{st} – wheel eccentricity module, m.

The presence of an imbalance when the wheel rotates causes the emergence of a centrifugal force acting on the wheel

$$P_{tsb} = \Delta_k \cdot \omega_k^2. \tag{9.36}$$

Centrifugal force is always directed away from the center of rotation. When the wheel rotates, its direction constantly changes. Figure 9.7 shows that the centrifugal force creates a destabilizing moment, the direction of which changes when the wheel rotates.

The magnitude of the destabilizing moment depends on the centrifugal force and the arm of its action.

$$M_d = P_{tsb} \cdot l_{tsb}, \tag{9.37}$$

where l_{tsb} is the arm of action of the centrifugal force of the wheel, m.

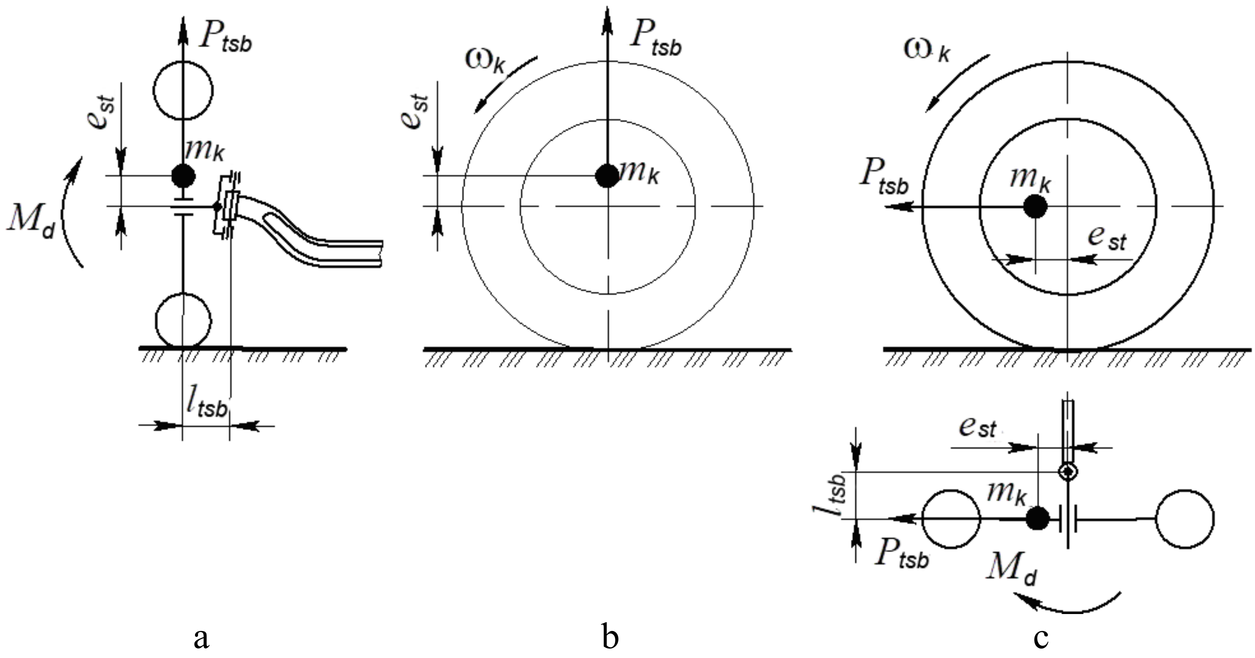


Fig. 9.7. **Scheme for determining exposure unbalance of steered wheels on their stability** : • – center of gravity of the wheel; a, b – center of gravity of the wheel located in the vertical plane; c – center of gravity of the wheel, located in the horizontal plane

The arm of the centrifugal force is the distance between the plane of rotation of the center of gravity of the wheel and the point of intersection of the axis of rotation of the wheel with the axis of its rotation.

The action of the destabilizing moment when the wheel rotates causes it to wobble. To eliminate this phenomenon, wheel balancing is carried out - the installation of loads of the appropriate mass on the wheel rim from the reverse side of the imbalance. The mass of the balancing load, necessary to eliminate the imbalance, is determined by the equation

$$m_{\text{bg}} = m_{\text{k}} \cdot \frac{e_{\text{st}}}{r_{\text{bg}}}, \tag{9.38}$$

where m_{bg} is the weight of the balancing load, kg;

r_{bg} – the distance from the wheel axis to the point of attachment (on the rim) of the balancing load, m.

c) *Inconsistency in the kinematics of the suspension and the steering drive* causes the steered wheels to turn in one direction and the other when the deformation of the suspension changes, which is due to the presence of a double connection between them and the vehicle body.

Steering wheels are installed on suspension elements connected to the body. At the same time, they have a mechanical connection with the elements of the steering drive, which are also located on the body. The front end of the spring 2 (Fig. 9.8) is connected to the frame by the hinge 1, and the rear end by the earring 3. When the spring 2 bends, the bridge beam 9 together with the rotary lever 6 and the hinge 5 moves along the arc B, and its swing axis is located close to hinge 1.

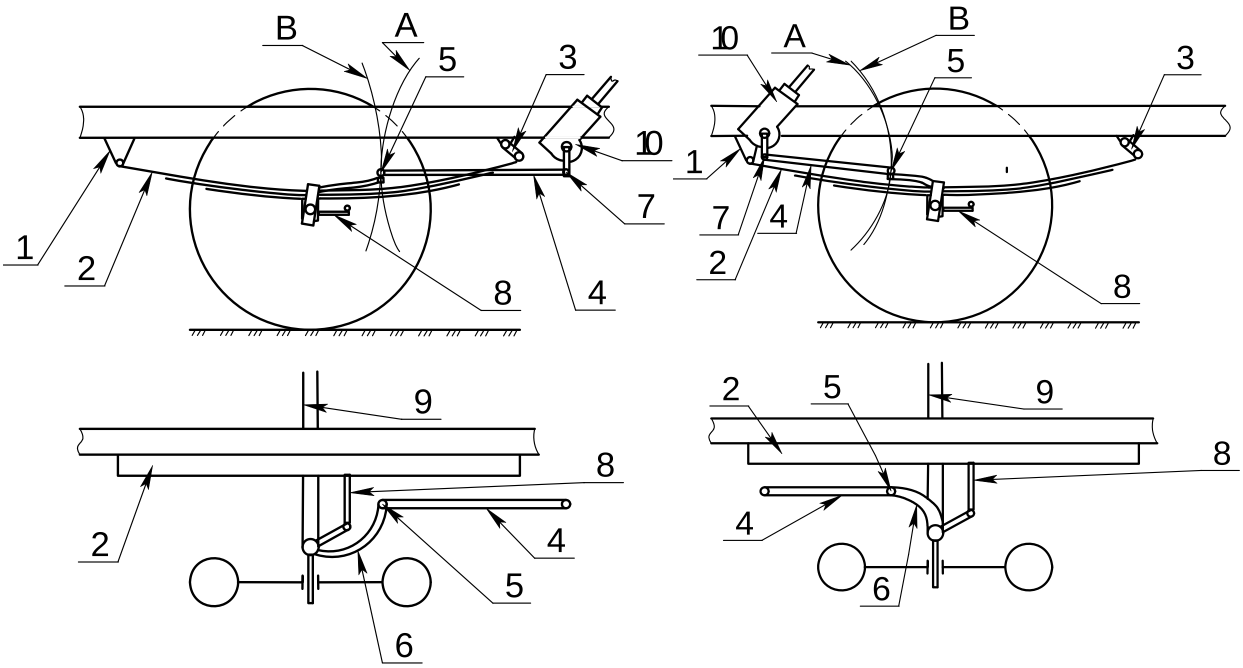


Fig. 9.8. **Schemes for determining inconsistency suspension and steering kinematics** : a – bad ; b – improved correspondence of suspension kinematics and steering drive; 1 – spring hinge; 2 – spring; 3 – an earring; 4 - longitudinal traction steering drive; 5 – hinge of the rotary lever; 6 – rotary lever; 7 – steering bipod finger; 8 – steering drive trapezoid; 9 – bridge beam; 10 – steering mechanism

The front end of the longitudinal steering rod 4 is connected by a hinge 5 to the rotary lever 6, and the rear end is connected to the bipod of the steering mechanism installed on the frame. When the deformation of the spring changes, the hinge 5 swings relative to the finger of the steering bipod 7 and describes an arc A. Curves A and B diverge, so the vertical movements of the wheels are accompanied by turning them to the right relative to the pivots. The turning (wobble) of the controlled wheels during vertical oscillations on the elastic suspension device worsens the vehicle's controllability. At the same time, the rate of driver fatigue increases.

To reduce wheel sway, it is necessary to bring the trajectories of the front axle B and the front end of the longitudinal steering rod A closer together. For this purpose, in some designs, the steering mechanism is placed in front of the front axle (Fig. 9.8 b) .

9.7.2. Stabilization of steered wheels

Stabilization of steered wheels refers to the ability to maintain a neutral position (occupied by them during straight-line movement) and automatically return to it after the cessation of external forces. Stabilization of steered wheels is provided by the following factors:

– tire stabilizing moment;

– weight stabilizing moment;

– high-speed stabilizing moment.

Weight and speed stabilizing moments are provided:

– installation of a pivot with an inclination;

– installation of controlled wheels with camber and ascent.

1) *Stabilizing torque of the tire M* $_{ssh}$ occurs when a lateral force acts on the wheel as a result of its elastic properties (Fig. 9.9).

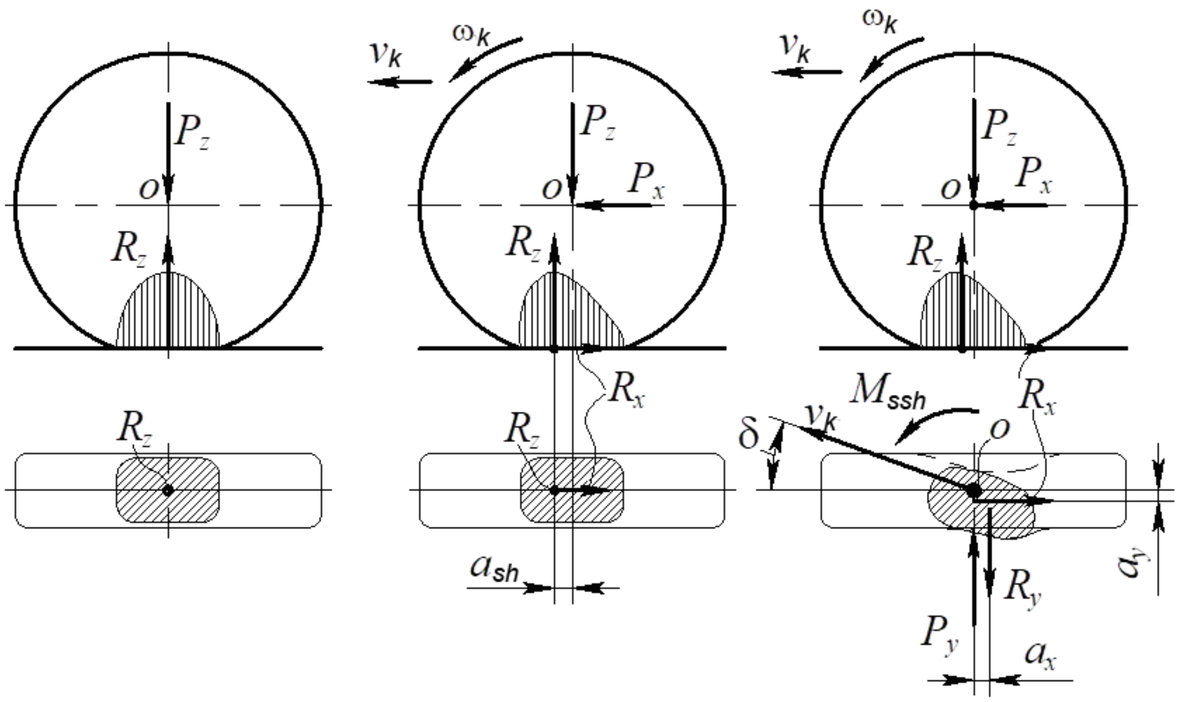


Fig. 9.9. **Scheme of forces and moments acting on the wheel in different load modes** : a – stationary wheel; b – driven mode of rectilinear movement; c – driven mode of motion under the action of a lateral force

Figure 9.9 shows that in the case of rolling of the wheel under the action of the lateral force *P*, the velocity vector v_k deviates by the lateral deviation angle δ due to the elastic deformation of the tire . At the same time, in the contact zone, in addition to the longitudinal reaction R_x, there is a lateral reaction R_y. Twisting of the tire is prevented by the elastic moment of the tire M_{ssh} . Due to the elastic properties of the tire, the reaction R_y acquires demolition in the longitudinal direction a_x, and the reaction R_x in the transverse direction - a_y. The elastic moment of the tire is equal to the sum of the reaction moments R_x and R_y

$$M_{\text{ssh}} = R_y \cdot a_x \mp R_x \cdot a_y. \qquad (9.39)$$

The upper sign in equation (9.39) corresponds to the driven and braking modes of wheel rolling, and the lower one to the driving mode.

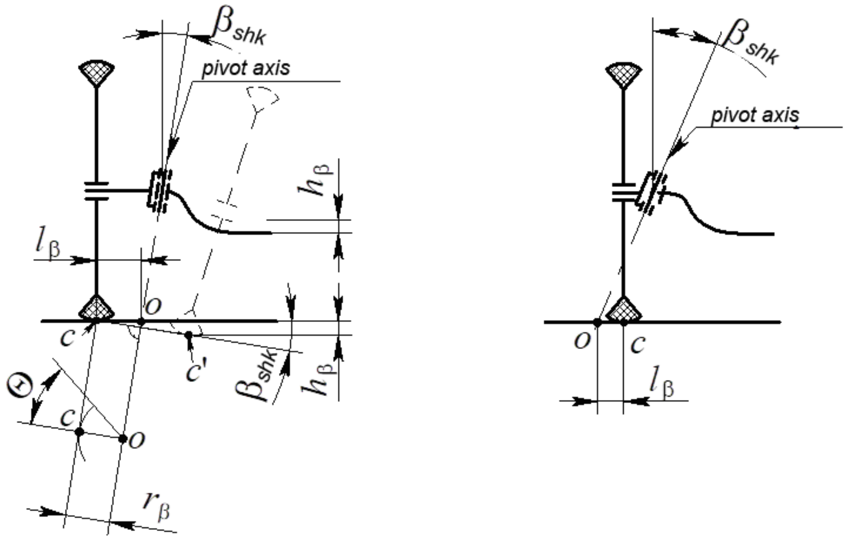
It should be noted that the moment M_{ssh} is not always sufficient to stabilize the steered wheels. It significantly decreases with an increase in the longitudinal forces acting on the wheel and on a slippery support surface. Increasing the stabilizing moment on steered wheels is achieved by tilting the axis of the pivots and installing wheels with camber and camber.

The pivot axis is conventionally called the axis around which the steered wheel rotates, regardless of the suspension design. Usually, the pivot axis is tilted in the transverse and longitudinal planes .

2) *The transverse inclination of the pivot* forms the weight stabilizing moment M_β. As a result of the transverse inclination of the pivot at an angle β_{shk} when the wheel is turned, the center of the tire imprint *s* describes a circle with a radius r_β in a plane inclined at the same angle (Fig. 9.10). Usually, the maximum turning angles of steered vehicle wheels rarely exceed 45°. To explain the occurrence of the weight stabilizing moment M_β , let's conditionally consider turning the wheel by 180°. In this case, the center of the imprint *c* will move to point *c* ', which , as can be seen from the figure , is located below the level of the support surface. Since on a solid surface the wheel cannot lower below its level, it rolls in a circle with a radius of l_β, which is called the break-in *shoulder* (Fig. 9.10). At the same time, the front part of the vehicle is raised to a height h_β. The steered wheels tend to return to their original position under the influence of the weight falling on them. Thus, there is a weight stabilizing moment M_β and stabilization of the steered wheels is ensured. The weight stabilizing moment M_β is less than the stabilizing moment of

the tire M_{sh}, but it does not depend either on the speed of movement or on the adhesion of the wheels to the supporting surface.

The distance from the center of the footprint *c* to the point of intersection of the axis of rotation with the support surface *o* is called the *break-in shoulder of the steered wheels* l_β. At the same time, if the point of intersection of the pivot axis with the support surface is inside the track of the steered wheels, then the shoulder of the break-in is considered *positive* , if it is outside the track, then it is considered *negative* .



with a positive shoulder break-in   with a negative shoulder break-in

Fig. 9.10. **Scheme of installation of the pivot of the steered wheel in the transverse plane** : β_{shk} – angle of transverse tilt of the pin; l_β – break-in shoulder; r_β is the turning radius of the controlled wheel; h_β is the lift height of the front part of the vehicle

The weight stabilizing moment on the wheels of the front steered axle can be approximately determined by the equation

$$M_\beta = G_1 \cdot l_\beta \cdot \sin\beta_{shk} \cdot \sin\Theta. \quad (9.40)$$

The weight stabilizing moment M_β increases with an increase in the angle of rotation of the steered wheels Θ, the angle of inclination of the pivot β_{shk} and the running-in arm l_β.

At the same time, the stabilization of the steered wheels increases, but the work of the driver when turning the steering wheel becomes more difficult. To facilitate the turning of the steered wheels, the break-in shoulder should be as small as possible, which is ensured by the installation of the wheel with an inclination and departure. And ensuring the stabilization of steered wheels is achieved by increasing the transverse inclination of the pivot.

The angle of the transverse inclination of the pin usually has the following value:

β_{shk} = 7 ° ÷10 ° – for trucks;

β_{shk} = 4 ° ÷6 ° – in passenger vehicle.

3) *Longitudinal inclination of the pivot* at an angle α_{shk} forms a high-speed stabilizing moment M_{α} due to the creation of the arm l_{α} of the lateral reaction R_y (Fig. 9.11a). In the case of low elasticity of the tire, it can be considered that this shoulder is equal to the distance from the center of the impression to the axis of the pin. With elastic tires, significant wear a_x of the lateral reaction R_y occurs , and in this case the arm of the lateral reaction is $l_{\alpha+\delta}$.

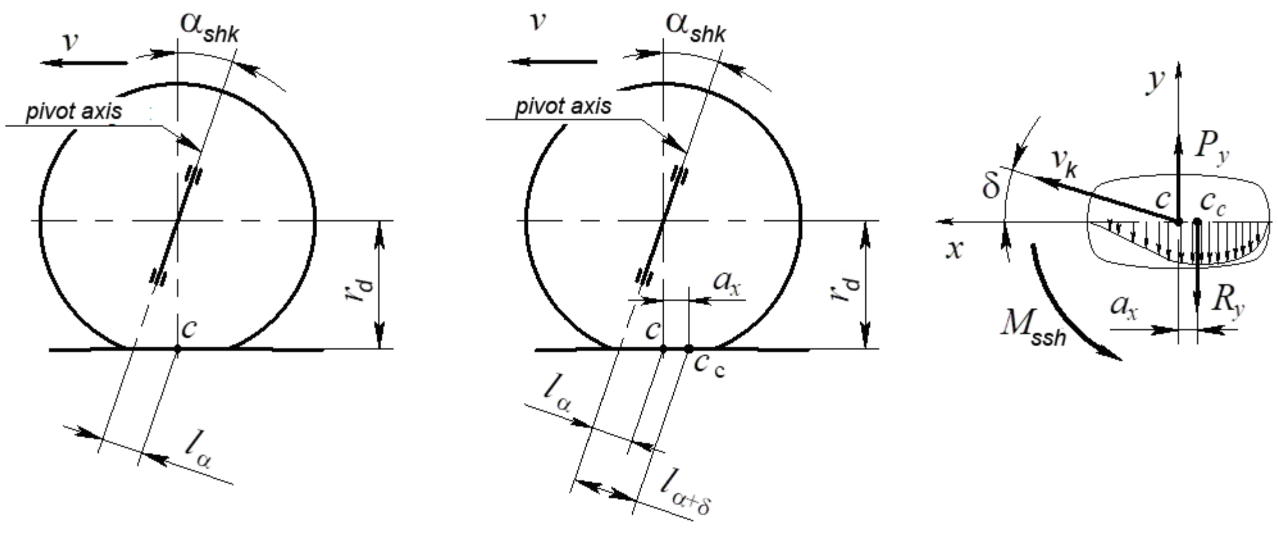


Fig. 9.11. **Scheme of formation of the stabilizing moment M_{α} when the pivot of the steered wheel is tilted in the longitudinal plane:**
a, b – respectively without taking into account and taking into account the stabilizer tire moment M_{ssh}; c – diagram of the formation of the stabilizer tire moment M_{ssh}

The speed stabilizing moment defines the equation

$$M_{\alpha} = R_y \cdot l_{\alpha+\delta} = R_y \cdot \left(r_d \cdot \sin\alpha_{shk} + a_x \cdot \cos\alpha_{shk} \right). \qquad (9.41)$$

where $l_{\alpha+\delta}$ is the arm of the lateral reaction, taking into account the elasticity of the tire.

Figure 9.11c shows the diagram of elementary lateral reactions in the zone of contact of the tire with the road when the wheel is rolling with a lateral force P_y . At the same time, the total reaction R_y acquires the demolition of a_x . The lateral force P_y and the reaction R_y form on the shoulder a_x the turning moment of the tire, which turns the wheel around the vertical axis z . As a result of the elastic properties of the tire on the wheel, an elastic moment of the tire M_{ssh} is formed , which counteracts the turning moment of the wheel and therefore stabilizes its position.

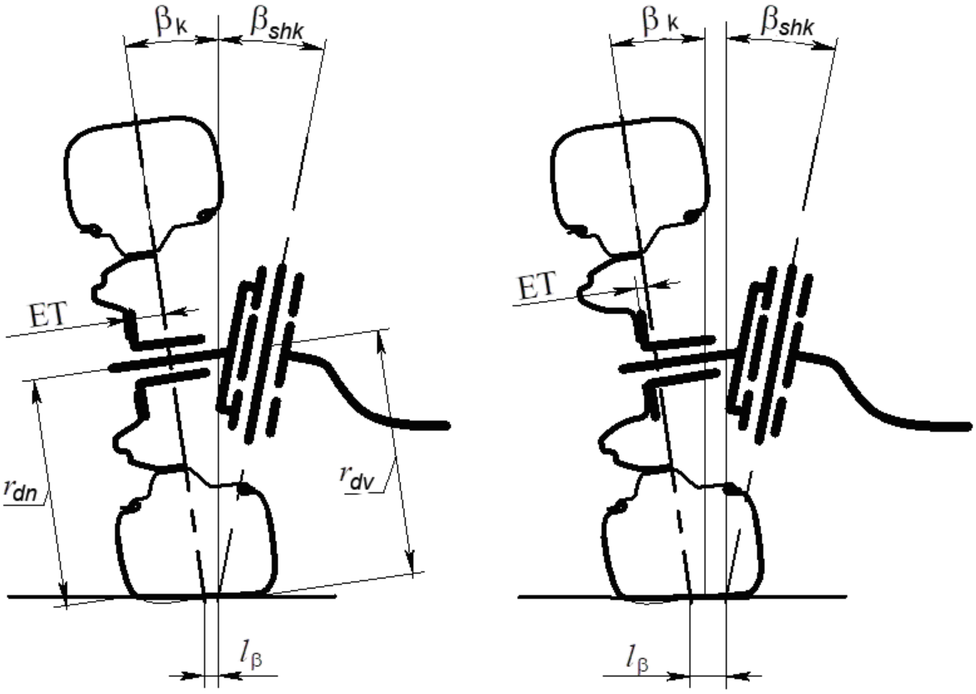
The lateral reaction R_y caused by the centrifugal force is a function of the vehicle speed v

$$R_y = P_{\text{tsb}} = \frac{G_1}{g} \cdot \frac{v^2}{r_\delta}. \tag{9.42}$$

The greater the speed v, the greater R_y, and, accordingly, the stabilizing moment M_α . Therefore, M_α is called *the high-speed stabilizing moment.* Usually, the angle of the longitudinal inclination of the pin is $\alpha_{\text{shk}} = 2 \div 4$ ° . On passenger vehicle with highly elastic tires, at high speed, the moment M_α becomes important and makes driving difficult. To ensure the comfort of control in this case, the angle of longitudinal inclination of the pin α_{shk} is reduced , and sometimes even to zero. At the same time, the high-speed stabilizing moment is completely formed by the stabilizing moment of the tire. It should be noted that at low speeds, the stabilizing moment of the tire does not provide reliable stabilization of the steered wheels.

To increase the reliability of the stabilization of the steered wheels, create the least resistance to movement, reduce tire wear and reduce fuel consumption, they must roll in vertical planes parallel to the longitudinal axis of the vehicle. To achieve this, they are installed with camber in the vertical plane and with ascent in the horizontal plane .

4) *Camber of steered wheels* is their installation with a deviation from the vertical plane. Installation of a cambered wheel is ensured by tilting the axis of the trunnion in the vertical plane. The angle between the plane of the wheel and the vertical plane is called *the camber angle* β_k (Fig. 9.12). If the wheel is tilted outward from the vehicle, then the camber is considered positive, and when it is tilted inward, it is considered negative.



a b

Fig. 9.12. **Installing a cambered wheel:**
a, b – installation of wheels with different ET clearance

The camber of the steered wheels is necessary so that when selecting the gaps in the bearings and when the parts of the steered bridge are deformed, they occupy a vertical position. When the wheels are cambered, the running-in arm is reduced, which makes it easier to control, and a force appears that presses the wheel to the center of the trunnion and the external bearing is unloaded. Usually, the camber angle is $\beta_k = 0 \div 1°$. In order not to tilt the wheel to a greater angle, to obtain the necessary shoulder, the wheel is run-in with different ET deviations (Fig. 9.12).

Wheels installed with a camber tend to roll along divergent trajectories, since $r_{dn} < r_{dv}$ (Fig. 9.12), which contributes to the compression of all elements of the steering drive and the elimination of clearances.

In order for the steered wheels to roll in planes parallel to the longitudinal axis of the vehicle, they are installed with an ascent in the horizontal plane.

5) *Ascent of steered wheels* is their installation with a deviation from the longitudinal plane. Angles of convergence of steered wheels are angles δ_k between the plane of rotation of the wheel and the longitudinal plane parallel to the longitudinal axis of the vehicle.

The toe-in of the steered wheels can be estimated as the difference in distances Δ_{uk} between the wheels measured from the front and back at the same height (Fig. 9.13).

$$\Delta_{uk} = B - A. \tag{9.43}$$

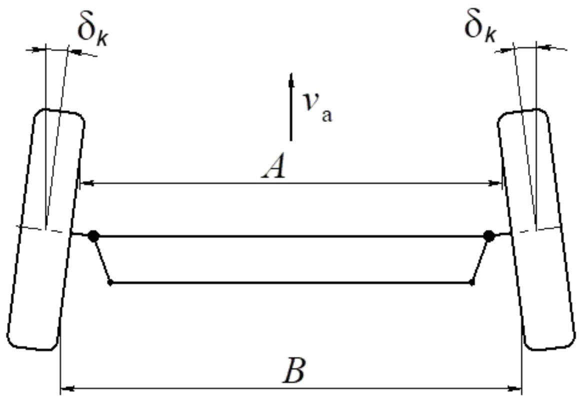


Fig. 9.13. **Scheme of installation of controlled wheels with climbing**

Angles of convergence of steered wheels δ_k in vehicle are set within 0°20 ′ ...1° depending on the design of the steering drive. The greater the number of hinges in the steering trapezium, which perceive the longitudinal reactions of the steered wheels, the greater the angle δ_k. The toe-in of the steered wheels Δ_{uk} corresponds to the angle δ_k and, depending on the radius of the wheels, is 2 ... 12 mm.

The installation of controlled wheels with camber and toe-in ensures their straight rolling on the road without lateral sliding. At the same time, in the case of the correct ratio between the angles of camber and toe-in, the rolling of the steered wheels is ensured with a small lateral deviation, which forms a stabilizing moment of the tire, in which the resistance to movement, fuel consumption and tire wear will be minimal. In fig. 9.14 shows the experimentally obtained plots of elementary lateral reactions on the wheel during its rolling with camber and ascent. The results were obtained with an increase in the ascent angle with other parameters unchanged. As the angle of descent from zero increases (left diagram), the plots of elementary lateral forces in contact decrease, reaching a minimum at a certain angle, after which they begin to increase again, but in the opposite direction. The third diagram from the left corresponds to the correct ratio between the camber and ascent angles. But it should be noted that when the air pressure in the tires or the weight that falls on the steered axle changes, elementary side reactions in contact change.

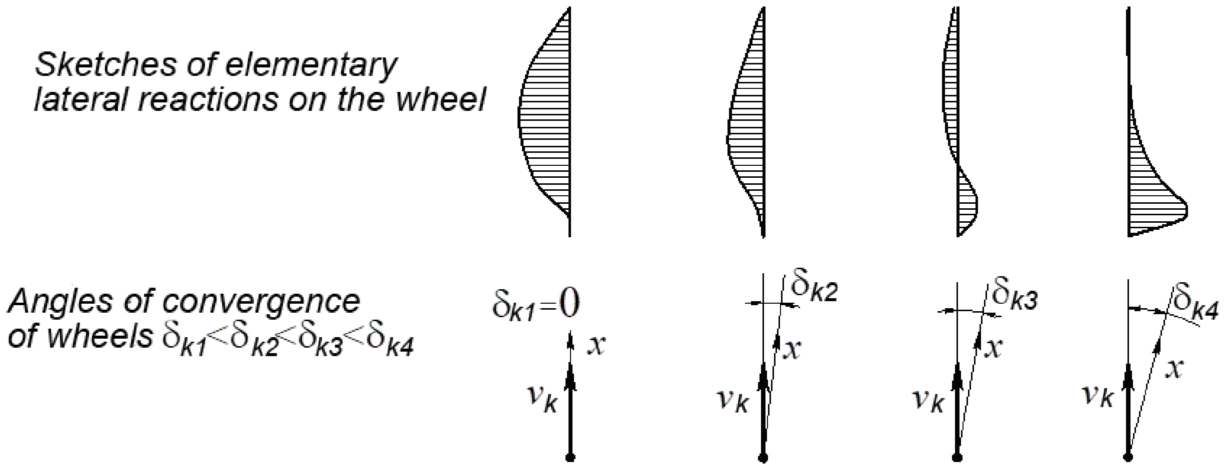


Fig. 9.14. **Plots of side reactions on the steered wheel:**
x is the plane of rotation of the wheel

Therefore, the correct ratio between camber and camber angles is determined for a vehicle with a full load, which ensures a small lateral deviation of the steered wheels precisely at maximum loads.

Control questions

1. What is vehicle handling and name the estimated handling indicators?
2. Draw a turning diagram of a vehicle with rigid wheels.
3. How is the turning radius of a vehicle with rigid wheels and its angular velocity determined?
4. Draw a turning diagram of a vehicle with elastic wheels.
5. How is the turning radius of a vehicle with elastic wheels and its angular velocity determined?
6. What is vehicle turning and name its types?
7. Draw the diagrams of the movement of the vehicle with different types of turning under the action of a lateral force if the steered wheels are not turned.
8. Draw the diagrams of the movement of the vehicle with different types of turning if the steered wheels are turned.
9. What is the critical speed of the vehicle on the lateral lead and how to determine it?
10. Write the equation of motion of a vehicle along a curved path.
11. Name the destabilizing factors of the motion of steered wheels.
12. Name the stabilizing factors of the motion of steered wheels.

TOPIC 10

INCREASED RESISTANCE TO MOVEMENT AND VEHICLE PASSABILITY

10.1. Classification of vehicle by passability and traffic obstacles

The vehicle *passability* is an operational property that determines the possibility of its movement in poor conditions, off-road and when overcoming various obstacles at the maximum possible speed.

Depending on the purpose, vehicle have different passability and in this connection they are divided into:

– vehicle of normal passability, intended for driving on roads of five categories. Such vehicle have a 4x2 or 2x4 wheel formula, a non-locking differential in the transmission, regular profile tires with unregulated pressure;

- off-road vehicles intended for driving on roads of five categories and off-road terrain. Such vehicle have a wheel formula of 4x4 or 6x6, the transmission has a differential that is partially or completely locked, wide-profile tires with adjustable pressure;

- high-passability vehicle are designed for driving on roads and off-road terrain with increased resistance and obstacles (mountainous and swampy terrain, water obstacles, etc.). Such vehicle have a special layout, additional equipment and equipment that increase their passability.

Depending on the nature of the impact that can cause a loss of vehicle passability, traffic obstacles are divided into three types:

- those that create great resistance to movement: broken roads, steep climbs, threshold obstacles (stumps, stones), swampy terrain, loose sand, loose snow, fords, ditches;

– capable of causing the vehicle to overturn: steep descents, hills, slippery roads;

- capable of causing flooding: water obstacles, burning swamps.

Loss of vehicle passability may be complete or partial. *The complete loss of passability* is the stopping of the vehicle movement due to the impossibility of overcoming any obstacles. *Partial loss of passability* characterizes a decrease in the efficiency of using the vehicle in these road conditions.

10.2. Vehicle passability parameters

The vehicle's passability parameters are influenced by various factors that characterize both the vehicle and the soil. Factors that characterize a vehicle can be divided into three groups:

– traction-dynamic;

– geometric;

– constructive.

The specified factors are interrelated, therefore such a distribution is conditional - for the convenience of a comparative assessment of the passability of vehicle. At the same time, a distinction is made between the *reference and profile passability of the vehicle* .

10.2.1. Bearing capacity

Cross-country ability characterizes the ability of the vehicle to move on soft roads, on crossings (for example, pontoons, ice crossings) and on steep ascents and descents.

Mass of the vehicle . A vehicle with a lower mass has a higher passability on soft soils, on terrain with an unstable surface, on icy and artificial crossings. The weight of the vehicle also affects other parameters of passability: towing weight, specific power, etc.

Towed weight of the vehicle m_φ is part of its mass, which creates normal loads of the driving wheels.

Coefficient of adhesion utilized mass of the vehicle k_φ is the ratio of the adhesion utilized mass to the total mass of the vehicle. In the general case for a vehicle on a sloped surface

$$k_\varphi = \frac{m_\varphi}{m_a} = \frac{m_\varphi \cdot g \cdot \cos\alpha}{m_a \cdot g \cdot \cos\alpha} = \frac{G_\varphi}{G} = \frac{G_\varphi}{G_a \cdot \cos\alpha}, \tag{10.1}$$

where G_φ is the trailer weight of the vehicle;

G is the weight of the vehicle.

Vehicle curb weight G_φ – is the load on the driving wheels.

The condition of the vehicle's ability to move is determined by the clutch of the drive wheels

$$P_\varphi \geq P_\psi \text{ or } G_\varphi \cdot \varphi_x \geq G_a \cdot \psi, \tag{10.2}$$

whence follows

$$\frac{G_{\varphi}}{G_{a}} \geq \frac{\psi}{\varphi_x} \Rightarrow k_{\varphi} \geq \frac{f \cdot \cos\alpha + \sin\alpha}{\varphi_x}. \quad (10.3)$$

The greater k_{φ}, the higher the traction force behind the clutch. For trucks with wheel formulas 4x2, 6x4, the adhesion utilized mass coefficient depends on the number of driving axles, the design of the chassis, the layout of the chassis and is 0.65 ... 0.7, for vehicle with all driving wheels $k_{\varphi} = 1$. For main roads road trains, the minimum permissible value of the adhesion utilized mass coefficient $k_{\varphi} = 0.31$ is established, which corresponds to the worst possible road conditions of their operation ($\varphi_x = 0.2$; $f = 0.12$; $i_{max} = 0.06$).

Specific power of the vehicle, kW/t - the ratio of the nominal power of the engine to the total weight of the vehicle

$$N_{ud} = \frac{N_{e\max}}{m_a}. \quad (10.4)$$

The specific power characterizes the ability to overcome protracted climbs at an average speed of movement and short (concentrated) obstacles using the kinetic energy of the vehicle. The greater the specific power, the higher the throughput of the same type of rolling stock. Many concentrated obstacles (a short but steep climb, a swampy area, etc.) can be overcome on the move using the vehicle's kinetic energy. But the possibility of intense acceleration of the vehicle depends on the specific power. Overcoming protracted climbs occurs at a constant speed, which also depends on the specific power. The specific power of trucks with normal cross-country ability is N_{ud} = 6...12 kW/t, and vehicles with increased cross-country ability N_{ud} = 10...20 kW/t .

Rolling resistance power N_f is the sum of the rolling resistance capacities of all wheels of the vehicle.

Power of movement resistance is the sum of the power of rolling resistance and the power spent on overcoming friction in the transmission and resistance to lift, inertia, air (and the trailer).

The rutting power of the vehicle - part of the power of the rolling resistance of the vehicle, which is spent on the deformation of the support surface by the wheels of the vehicle.

Full traction power of the vehicle $P_{traction}$ is the sum of the total traction force of all driving wheels.

Free traction force of the vehicle R_{sv} is the traction force of the vehicle, which is equal to the difference between the total traction force of the vehicle, which moves uniformly on a horizontal support surface, the sum of the air resistance force and the sum of the wheel resistance forces.

Coefficient of free traction force of the vehicle k_t is the ratio of the free traction force to the gravity force of the full mass vehicle

$$k_т = \frac{P_k - P_v - P_{f2} - P_{f1}}{G_a} = \frac{P_k - P_v}{G_a} - \frac{P_f}{G_a} = D - f. \tag{10.5}$$

The coefficient of free traction force k_t characterizes the traction properties of the vehicle when overcoming difficult sections of the road with a large coefficient of movement resistance.

The pressure of the wheels on the support surface is estimated by the average pressure in the contact zone of the wheel r_k and the average pressure of the tread protrusions in the contact spot of the wheel r_{kpr}.

$$p_k = \frac{G_k}{F_k};\ p_{kpr} = \frac{G_k}{F_{kpr}}, \tag{10.6}$$

where G_k is the part of the vehicle's weight falling on the wheel, N;

F_k – the area of the spot of contact of the wheel with the road, m ^{2};

F_{kpr} – the area of the tread protrusions in the spot of contact of the wheel with the road, m ^{2}.

The pressure of the wheel on the support surface characterizes the passability on roads with a soft surface (snow, sand, loose soil, etc.). The low pressure of the wheels on the soft supporting surface reduces the depth of the traces, which reduces the resistance to movement and improves the passability of the vehicle. On roads with a surface that is deacidified, but with a solid base, the best passability is achieved with high tire pressure, as dirt is pushed out of the contact zone and the tread interacts with the solid base. For road vehicle on paved roads, the maximum pressure is regulated: r_k = 0.6 MPa, r_{kpr} = 0.85 MPa .

The coefficient of adhesion of the wheels to the road characterizes the passability of the vehicle on wet soils and slippery (frozen) roads. An increase in the coefficient of adhesion increases the passability on such roads.

The following indicators are used for the comparative evaluation of road train passability:

– traction force on the hook – the force applied to the vehicle from the side of the trailer;

– the specific traction force on the vehicle hook - the traction force on the vehicle hook, related to the total weight of the vehicle ;

– traction power on the hook of the vehicle – power, which is equal to the product of the traction force on the vehicle hook by the speed of the vehicle;

– specific traction power on the hook of the vehicle - power equal to the ratio of the traction power on the hook of the vehicle to the total weight of the vehicle.

10.2.2. Profile permeability

Cross-country ability characterizes the vehicle's ability to drive over uneven road surfaces and its ability to fit into the road dimensions. These are the geometric parameters of the vehicle, which depend on its design and layout (Fig. 10.1).

Ground clearance is the distance between the lowest point of the vehicle and the road. It characterizes the possibility of movement without touching concentrated obstacles (stones, stumps, bushes, etc.) and the possibility of movement on soft soils. The value of road clearances is performed within the following limits:

– road trucks - from 245 mm to 290 mm;
– off-road trucks - from 315 mm to 400 mm;
– passenger vehicle - from 120 mm to 190 mm.

Increasing the ground clearance allows you to increase the passability, which can be achieved by increasing the diameter of the wheels and reducing the dimensions of the main gear.

Front and rear overhang of the vehicle is the distance from the extreme point of the contour of the front (rear) protruding part of the vehicle along the length to the plane of the perpendicular support surface and passing through the centers of the front (rear) wheels of the vehicle.

Angles of front α_1 *and rear overhang* α_2 are the angles formed by the plane of the road and the planes tangent to the front and rear wheels and to the protruding lower points of the front and rear parts of the vehicle. They characterize passability on uneven roads during entry and exit from an obstacle.

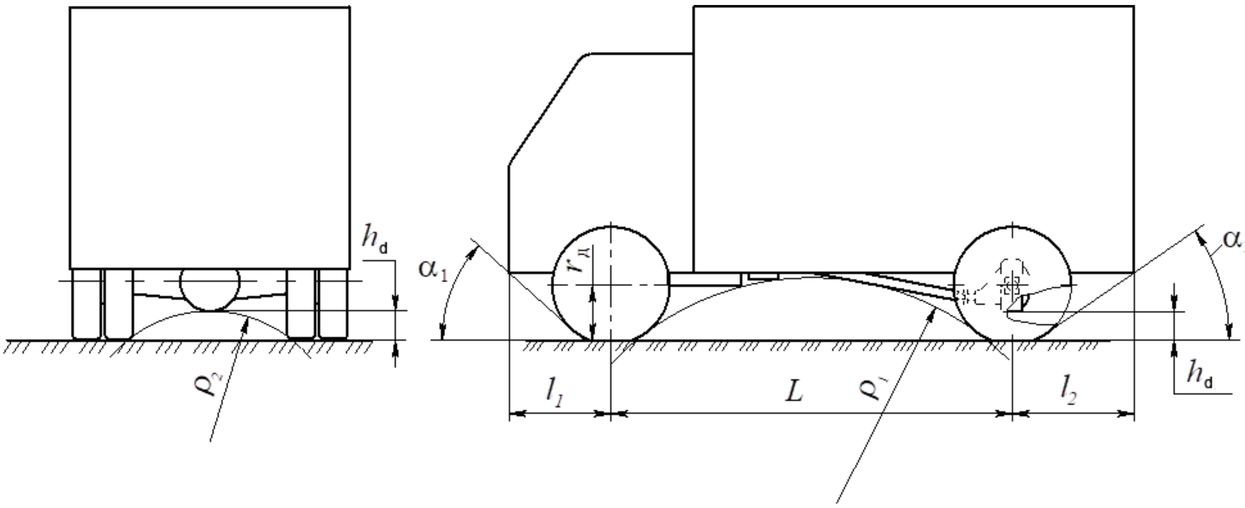


Fig. 10.1. **Profile parameters of the vehicle**

The values of the angles of the front and rear overhangs are performed within the following limits:

– road trucks - $\alpha_1 = 25°...42°$ and $\alpha_2 = 18°...38°$;
– heavy duty trucks - $\alpha_1 = 35°...55°$, and $\alpha_2 = 32°...42°$;
– heavy-duty trucks - $\alpha_1 > 60°...70°$ and $\alpha_2 > 60°...70°$;
– passenger cars - $\alpha_1 = 22°...35°$ and $\alpha_2 = 15°...30°$;
– buses - $\alpha_1 > 8°$ and $\alpha_2 > 8°$.

Longitudinal ρ_1 *and transverse* ρ_2 *traffic radii* – the radii of the cylinders tangent to the wheels and lower points of the vehicle, respectively, in the longitudinal and transverse planes. These radii define the contours of obstacles that the vehicle can overcome without hitting them. The smaller the radii, the higher the passability. The values of the longitudinal and transverse radii of patency are performed within the following limits: road trucks – ρ_1 from 2.7 m to 5.5 m; heavy duty trucks – ρ_1 from 2.0 m to 3.5 m.

Angles of flexibility in vertical β_v *and horizontal* β_g *planes* – angles of possible deviation of the axis of the coupling loop of the trailer from the axis of the towing hook (Fig. 10.2). The angle of vertical flexibility of the road train characterizes its passability over road irregularities, and the angle of horizontal flexibility is the ability to fit into the road dimensions, that is, maneuverability. The values of the angles of flexibility in the vertical β_v and horizontal β_g planes are performed within the following limits: road trucks with two-axle trailers – $\beta_v > \pm 62°$, $\beta_g > \pm 55°$; road trucks with semi-trailers – $\beta_v > \pm 8°$, $\beta_g > \pm 90°$.

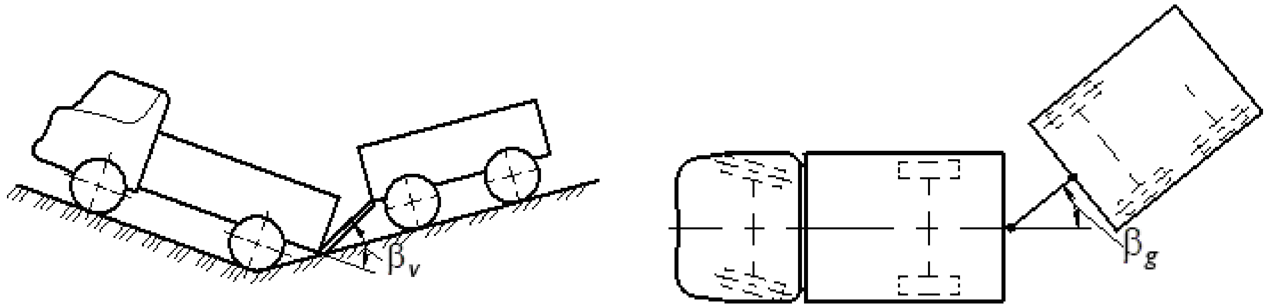


Fig. 10.2. **Angles of flexibility of road trains**

The inner r_{vn} and outer r_n are the turning radii - the distance from the center of rotation according to the nearest and most distant points of the vehicle at the maximum turn of the steered wheels (Fig. 10.3).

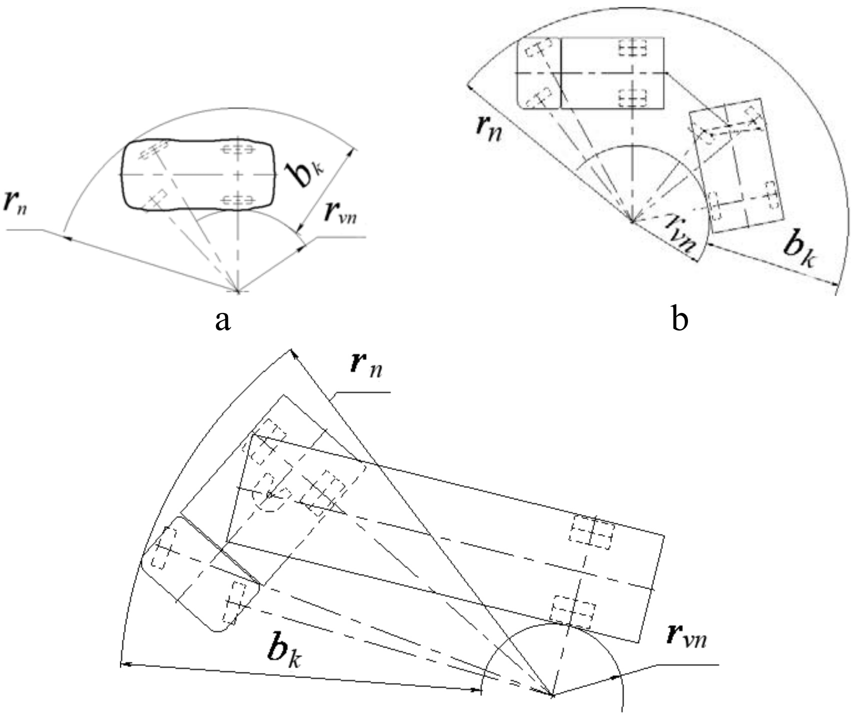


Fig. 10.3. **Turning radii and turning width of vehicle and road trains:**
a – car; b, c – road trains

The turning width of the vehicle b_k is the difference between the outer and inner turning radii (Fig. 10.3). The turning radii and turning width of the corridor characterize the maneuverability of rolling stock.

Bridge skew angle β_m is the sum of the angles of rotation of the front and rear axles relative to the longitudinal axis of the vehicle (Fig. 10.4). The larger the angle β_m, the better the vehicle wheels adapt to the unevenness of the support surface and the better their contact with the road is preserved.

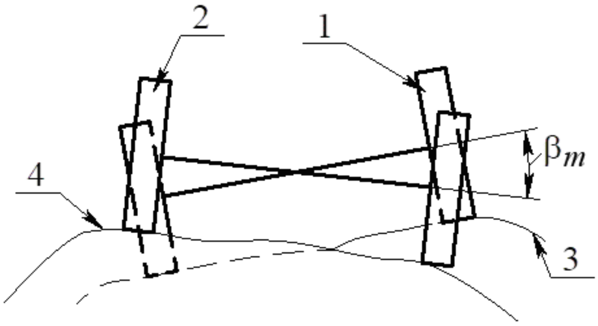


Fig. 10.4. **Angle of misalignment of the bridges (view of the vehicle from the rear):** 1, 2 – front and rear bridges; 3, 4 – support surface under the wheels of the front and rear axles, respectively

With small skew angles of bridges on rough terrain, a significant redistribution of loads on the wheels and even "hanging out" of one of them can occur. The magnitude of the skew angles of the bridges is not regulated by the standards, but is determined by the purpose of the vehicle.

Coefficient of coincidence of traces of front and rear wheels $\eta_s = b_{sp}/b_{sz}$ (Fig. 10.5). The closer the value of the coefficient η_s is to unity, the less the rolling resistance of the wheels on deformable soils. An exception is movement on marshy ground .

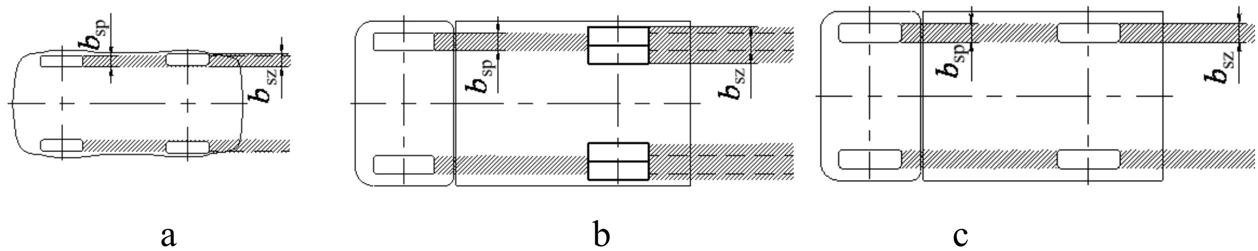


Fig. 10.5. **Traces of front and rear wheels of** vehicle**:**
a, b – traces that do not coincide; c – traces that coincide;
b_{sp}, b_{sz} – the width of the track of the front and rear wheels, respectively

10.2.3. Complex permeability factor

The comprehensive passability factor characterizes the efficiency of using the vehicle during its operation on difficult roads and off-road. It takes into account the decrease in productivity (due to a decrease in the average speed of traffic and the weight of the transported cargo) and the deterioration of fuel economy (due to an increase in fuel consumption) in these operating conditions compared to highway roads.

This factor is equal to

$$\Pi_k = \frac{G_{gr} \cdot V_a \cdot q_{ssh}}{G_{grsh} \cdot V_{ash} \cdot q_s}, \qquad (10.7)$$

where G_{gr}, G_{grsh} – payload of the vehicle when moving on off-road and highway, respectively;

V_a, V_{ash} – the average speed of the vehicle on off-road and highway, respectively;

q_s, q_{ssh} – fuel consumption of the vehicle when driving on off-road and highway, respectively.

10.3. Overcoming concentrated obstacles

10.3.1. Maximum angle of elevation

The maximum angle of elevation α_{max}, which can be overcome by the vehicle, is limited by the traction capabilities and traction of the driving wheels. Overcoming the climb by the vehicle with the maximum angle of climb α_{max} occurs when using the maximum possible traction force on the drive wheels. In this case, the condition is fair

$$D_{max} = \psi_{max} = f \cdot \cos\alpha_{max} + \sin\alpha_{max}. \qquad (10.8)$$

Let's transform the equation (10.8) so that the angle α_{max} was determined by one trigonometric function. For this, we will take out the parentheses $\sqrt{f^2+1}$

$$D_{max} = \sqrt{f^2+1} \cdot \left(\frac{f}{\sqrt{f^2+1}} \cdot \cos\alpha_{max} + \frac{1}{\sqrt{f^2+1}} \sin\alpha_{max} \right). \qquad (10.9)$$

To determine the coefficients in front of the functions $\cos\alpha_{max}$ and $\sin\alpha_{max}$, we use an auxiliary triangle with legs equal to f and 1. Accordingly, the hypotenuse is equal to $\sqrt{f^2+1}$ (Fig. 10.6).

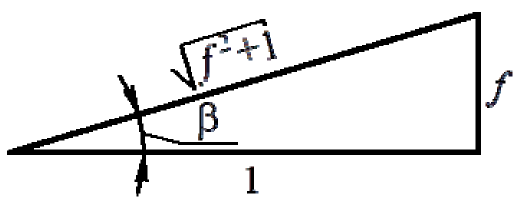


Fig. 10.6. **Scheme for determining coefficients in equation (10.9)**

As can be seen from Figure 10.6, the ratio $f \cdot \left(\sqrt{f^2+1}\right)^{-1} = \sin\beta$, and $\left(\sqrt{f^2+1}\right)^{-1} = \cos\beta$. Let's substitute the values of the coefficients in the equation (10.9) and convert it to the form

$$D_{\max} = \sqrt{f^2+1} \cdot \left(\sin\beta \cdot \cos\alpha_{\max} + \cos\beta \cdot \sin\alpha_{\max}\right).$$

or

$$D_{\max} = \left(\sqrt{f^2+1}\right) \cdot \sin\left(\alpha_{\max} + \beta\right). \qquad (10.10)$$

From which we will get

$$\alpha_{\max} + \beta = \arcsin\left(\frac{D_{\max}}{\sqrt{f^2+1}}\right) \Rightarrow \alpha_{\max} = \arcsin\left(\frac{D_{\max}}{\sqrt{f^2+1}}\right) - \beta. \qquad (10.11)$$

Taking into account (see Figure 10.6) that $tg\ \beta = f$ finally the equation for determining the maximum angle of elevation that the vehicle can overcome under the condition of traction will take the form

$$\alpha_{\max} = \arcsin\left(\frac{D_{\max}}{\sqrt{f^2+1}}\right) - \mathrm{arctg}\, f\ . \qquad (10.12)$$

Overcoming the climb by the vehicle with the maximum angle of climb $\alpha_{\max}$ possible when using the maximum possible clutch force on the drive wheels. In this case, the condition is fair

$$D_{\varphi} = \psi_{\max} = f \cdot \cos\alpha_{\max} + \sin\alpha_{\max}. \qquad (10.13)$$

$$\alpha_{\max} = \arcsin\left(\frac{D_{\varphi}}{\sqrt{f^2+1}}\right) - \mathrm{arctg}\, f\ . \qquad (10.14)$$

If the dynamic factor is determined through the adhesion utilized mass coefficient, then the condition for the possibility of movement (10.13) will take the form

$$k_{\varphi} \cdot \varphi_x \cdot \cos\alpha_{\max} = \psi_{\max} = f \cdot \cos\alpha_{\max} + \sin\alpha_{\max}. \qquad (10.15)$$

After dividing equation (10.15) by cos α_{max} and transformations, we will get an equation for determining the maximum angle of elevation that the vehicle can overcome, provided that the driving wheels are engaged

$$\alpha_{max} = \operatorname{arctg}\left(k_{\varphi} \cdot \varphi_{x} - f\right). \tag{10.16}$$

To determine the maximum angle of elevation α_{max}, which can be overcome by the vehicle, the values calculated by equation (10.12) and equation (10.14) or (10.16) are compared, and the smaller value is chosen.

Checking the vehicle for stability against overturning while moving at a uniform speed on a rise with an angle of α_{max} (Fig. 10.7).

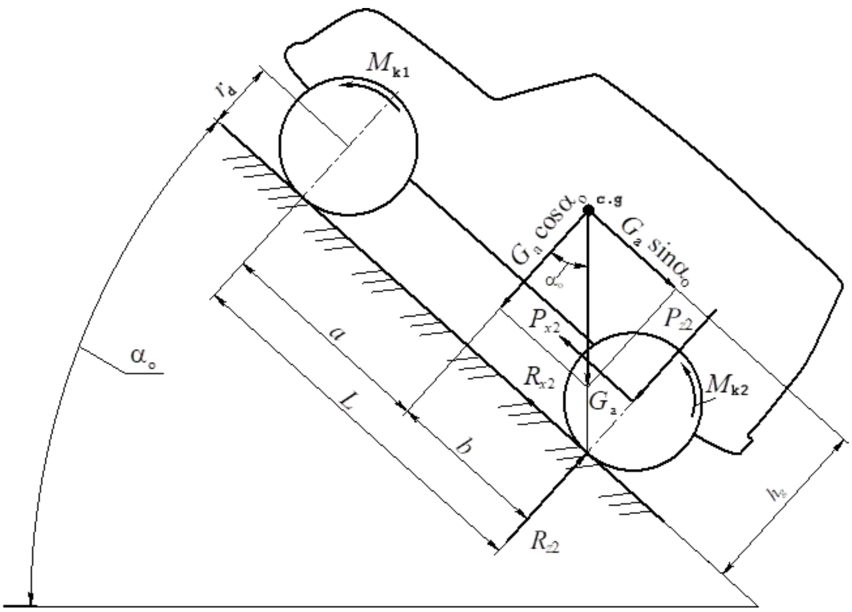


Fig. 10.7. **The scheme of determining the stability of the vehicle when moving uphill**

Overturning the vehicle when the vehicle moves uphill is carried out around the center of the wheel tire impression. Figure 10.7 shows the critical position at which the gravity vector crosses the center of the wheel footprint. A further increase in the angle of elevation causes the vehicle to overturn. If the gravity vector crosses the axis of the rear wheels, the body roll around it begins.

The condition of the vehicle's stability when moving uphill, which limits its passability

$$\alpha_{max} \le \alpha_{o} = \operatorname{arctg}\left(\frac{b}{h_g}\right), \tag{10.17}$$

where α_o is the critical angle for longitudinal overturning.

When designing a vehicle, condition (10.17) is ensured by the design, but in operation, b and h_g can change significantly.

10. 3.2. Maximum descent angle

The maximum angle of descent α_{max} , overcome by the vehicle, is limited by the possibilities of wheel adhesion utilized (Fig. 10.8). At the same time, the violation of the vehicle's passability is limited in the form of uncontrolled longitudinal sliding and the inability to control the speed of descent or overturning in the event of hitting a concentrated obstacle (stump, stone).

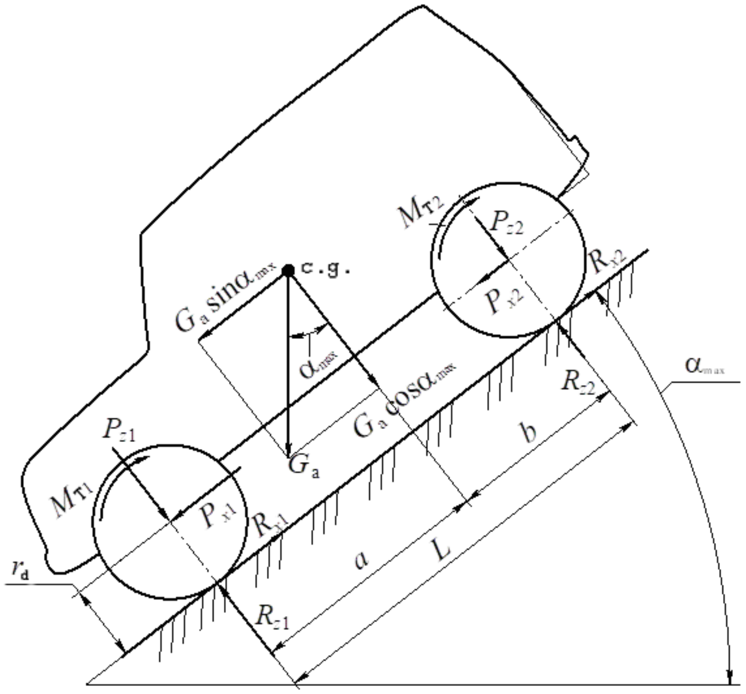


Fig . 10. 8. **Scheme of the vehicle when moving downhill**

The condition of maintaining a controlled speed on the descent

$$\alpha_{max} = \alpha_{\varphi} = \operatorname{arctg} \varphi_x, \qquad (10.18)$$

where α_{φ} – critical angle for longitudinal slip.

10. 3. 3. The maximum height of the overcome threshold

Overcoming a threshold obstacle by a vehicle is the successive rolling of the wheels of the front and rear axles through this obstacle. The nature of the wheel rolling over a threshold obstacle depends on its type: driven or leading. Figure 10.9 shows a scheme for determining the maximum height of the threshold that a vehicle with one driving axle can overcome if the driven wheels hit the threshold. Here we consider the possibility of overcoming the threshold without using the vehicle's inertia force, i.e. only due to the traction force of the driving wheels. We consider the wheel and threshold absolutely rigid. Air resistance and rolling forces are not taken into account, as they are insignificant in magnitude.

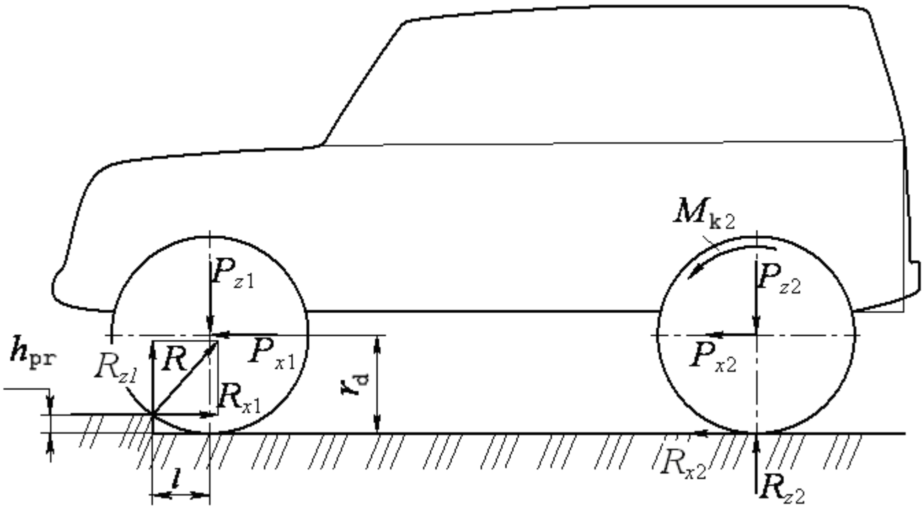


Fig. 10.9. **Diagram of the forces acting on the vehicle when moving threshold with driven wheels:**
h_{pr} – threshold height; l is the action arm of the reaction R_{z1}

The torque M_{k2} forms a longitudinal reaction R_{x2} and a free force P_{x2} on the wheels of the rear axle, which is transmitted to the axis of the front wheels o . When hitting a threshold, a reaction R *occurs on the wheel*, which can be represented in the form of components R_{z1} and R_{x1} . At the same time, the rolling of the wheel over the threshold will cause its detachment from the surface in front of the threshold. Relative to the point of contact of the wheels with the threshold, the force P_{x1} creates a rolling moment of the wheels, and the normal load on the driven wheels P_z is a moment of resistance . The maximum height of the threshold overcome by the driven wheel is determined from the equation

$$P_{x1} \cdot \left(r_d - h_{pr}\right) - P_{z1} \cdot l = 0, \qquad (10.19)$$

Taking into account that $l = \sqrt{r_d^2 - \left(r_d - h_{pr}\right)^2}$, let's transform (10.19) into the form

$$P_{x1} \cdot \left(r_d - h_{pr}\right) - P_{z1} \cdot \sqrt{r_d^2 - \left(r_d - h_{pr}\right)^2} = 0. \qquad (10.20)$$

Let's make a square and open the brackets

$$P_{x1}^2 \cdot \left(r_\text{d} - h_\text{pr}\right)^2 - P_{z1}^2 \cdot \left[r_\text{d}^2 - \left(r_\text{d} - h_\text{pr}\right)^2 \right] = P_{x1}^2 \cdot \left(r_\text{d} - h_\text{pr}\right)^2 - \\ -P_{z1}^2 \cdot r_\text{d}^2 + P_{z1}^2 \cdot \left(r_\text{d} - h_\text{pr}\right)^2 = \left(r_\text{d} - h_\text{pr}\right)^2 \cdot \left(P_{x1}^2 + P_{z1}^2\right) - P_{z1}^2 \cdot r_\text{d}^2 = 0. \quad (10.21)$$

Let's solve equation (10.21) with respect to $\left(r_\text{d} - h_\text{pr}\right)^2$ and open the brackets:

$$\left(r_\text{d} - h_\text{pr}\right)^2 - \frac{P_{z1}^2 \cdot r_\text{d}^2}{\left(P_{x1}^2 + P_{z1}^2\right)} = r_\text{d}^2 - 2 \cdot r_\text{d} \cdot h_\text{pr} + h_\text{pr}^2 - \frac{P_{z1}^2 \cdot r_\text{d}^2}{\left(P_{x1}^2 + P_{z1}^2\right)} = 0. \quad (10.22)$$

Let's rewrite (10.22) in the form

$$h_\text{pr}^2 - 2 \cdot r_\text{d} \cdot h_\text{pr} + \left(r_\text{d}^2 - \frac{P_{z1}^2 \cdot r_\text{d}^2}{\left(P_{x1}^2 + P_{z1}^2\right)} \right) = 0. \quad (10.23)$$

Equation (10.23) is the given quadratic equation, the root of which

$$h_\text{pr} = r_\text{d} \pm \sqrt{r_\text{d}^2 - \left(r_\text{d}^2 - \frac{P_{z1}^2 \cdot r_\text{d}^2}{P_{x1}^2 + P_{z1}^2} \right)} = r_\text{d} \pm r_\text{d} \sqrt{\frac{P_{z1}^2}{P_{x1}^2 + P_{z1}^2}}. \quad (10.24)$$

The physical meaning of the sign before the second term in the equation (10.24) has only a minus sign, so it is possible to write

$$h_\text{pr} = r_\text{d} \left(1 - \sqrt{\frac{P_{z1}^2}{P_{x1}^2 + P_{z1}^2}} \right). \quad (10.25)$$

When hitting the threshold of the driving wheel, there is a reaction R from the force R_x and a reaction R' from the torque M_k1 (Fig. 10.10). To determine the height of the threshold, which can be overcome by the driving wheel of a vehicle with four driving wheels, we will compile the equation of moments relative to the point of contact with the threshold

$$M_\text{k1} + P_{x1} \cdot \left(r_\text{d} - h_\text{pr}\right) - P_{z1} \cdot l = 0, \quad (10.26)$$

After transformations similar to the transformations carried out above, it is possible to write

$$h_\text{pr} = r_\text{d} - \sqrt{\frac{P_{z1}^2 \cdot r_\text{d}^2 - M_\text{k1}^2}{P_{x1}^2 + P_{z1}^2}}. \quad (10.27)$$

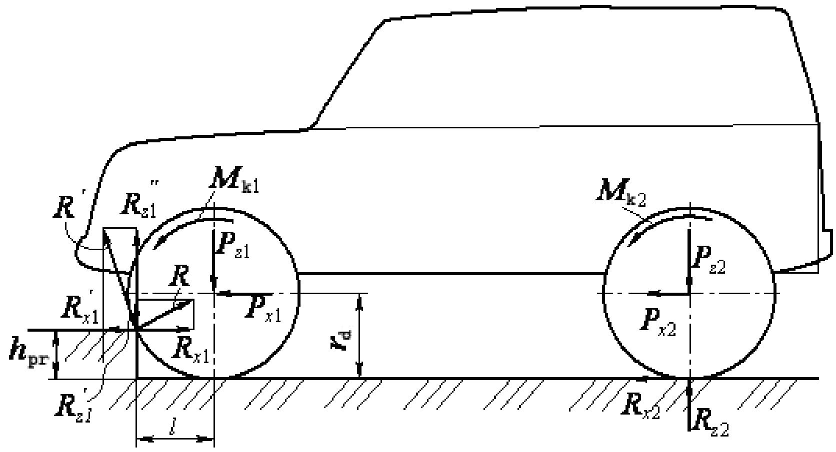
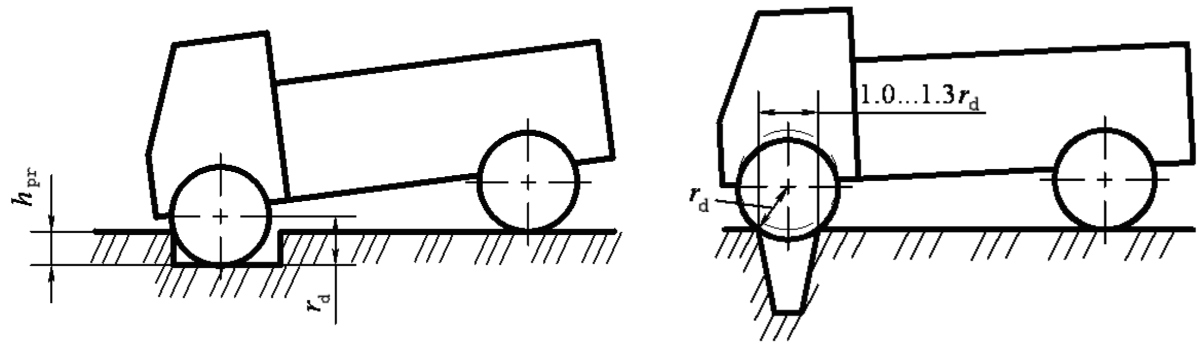


Fig. 10.10. **Diagram of the forces acting on the vehicle when moving threshold from the place with the driving wheels**

It can be seen from equation (10.27) that the greater the moment M_k applied to the wheel, the smaller the value under the root, which means the greater the height of the overcome threshold. Overcoming the threshold of the maximum height from a place is limited by the clutch of the driving wheels. With the coefficient of adhesion $\varphi_x = 0.6$ for a vehicle with the same loads on the axles, the maximum height of the threshold overcome by the driven wheels according to (10.25) is $h_{pr} = 0.15\ r_d$, and by the driving wheels according to (10.27) $h_{pr} = 0.53\ r_d$. Research has established that for driven wheels, even with the use of inertia, the maximum height of the overcome threshold does not exceed 2/3 r_d.

10. 3.4. The maximum width of the overcome ditch

If the depth of the ditch does not exceed the radius of the wheel, then the possibility of overcoming it is determined by the size and type of wheels. In this case, overcoming the ditch consists of successive overcoming of the ledge and threshold (Fig. 10.11a).



a b

Fig. 10.11. **Schemes for overcoming a ditch with wheeled vehicles by the 4x4 formula:** a – a shallow ditch; b - a deep ditch

The ability to overcome a deep ditch is determined by the size and type of wheels, the number and location of the driving axles, the position of the center of gravity along the length of the vehicle. So, for vehicle with a 4x4 wheel formula, as well as 6x4 and 6x6, if the center of gravity is not located above the middle axis, the width of the overcome ditch (with strong edges) does not exceed 1.0 ... 1.3 of the wheel radius (Fig. 10.11b).

For high-passability vehicle with an 8x8 wheel formula, the width of the overcome ditch depends on the location of the wheel axes and the center of gravity (Fig. 10.12). The lowest passability is for vehicle with two front and two rear axles close together (Fig. 10.12a), and the best - with three rear axles close together (Fig. 10.12d).

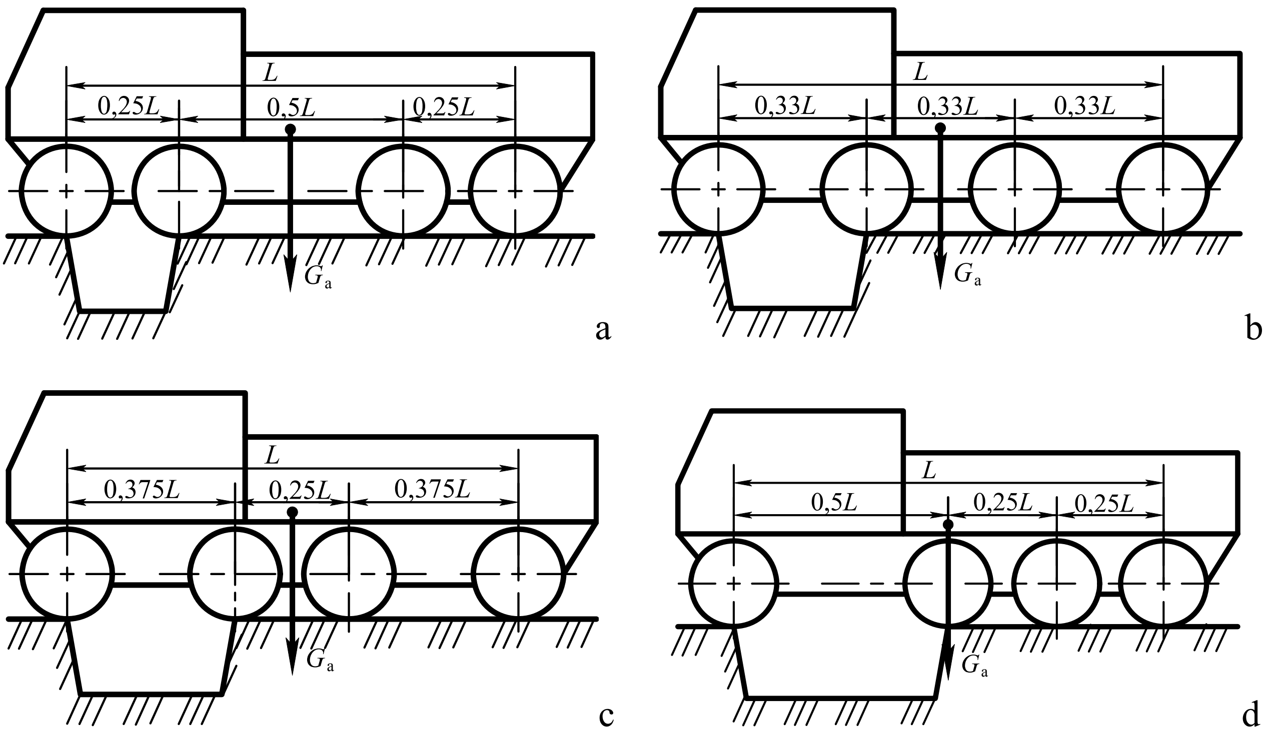


Fig. 10.12. **Schemes for overcoming a ditch with multi-axle vehicles**

At the same time, the suspension design should limit the lowering of the wheels and the center of gravity of the vehicle should be located between the middle axles.

10.3.5. The maximum depth of the ford overcome

The depth of the ford overcome depends on the design of the vehicle, the density of the bottom surface and the qualifications of the driver. Figure 10.13 explains the influence of the vehicle design on the depth of the ford overcome.

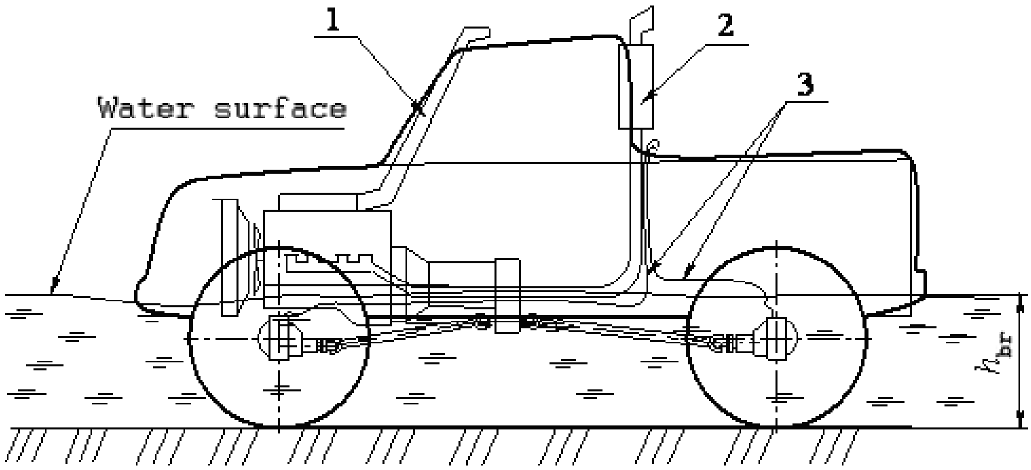


Fig. 10.13. **Scheme for overcoming the ford by vehicle:**
1 – air intake; 2 – exhaust gas release system; 3 – tubes for ventilation of unit crankcases; h_{br} is the depth of the ford

Design factors of the vehicle affecting the depth of the ford overcome by it:

– the number of driving wheels (with an increase in the number of driving wheels, the depth of the ford overcome increases);

– engine type:

– the diesel engine is better adapted to overcome the ford, the height of the location of the air intake (air cleaner), generator, crankcase ventilation holes and exhaust gas discharge pipe affects it;

– the engine with spark ignition is less adapted to overcome the ford. The height of the location of the generator, the ignition coil and other components of the ignition system, the air intake of the crankcase ventilation holes and the exhaust gas discharge pipe are affected;

– the design of the ventilation elements of the crankcases of the transmission units and the height of the ventilation opening;

- the design of the vehicle body can give the vehicle buoyancy, which worsens the contact of the wheels with the bottom surface and reduces the traction force of the driving wheels with the bottom surface.

10. 4. Design factors affecting the passability of the vehicle

A number of structural factors affect the vehicle's passability:

– engine design (type and power);

– type of wheels (leading or driven);

– wheel construction (dimensions, tread pattern);

– track of front and rear wheels;

– type of wheel suspension (dependent, dependent balancing, independent);

– construction of the main gear (double central, double spaced);

– type of differential (low friction, high friction, presence of blocking);

– regulation of tire pressure;

– hydromechanical transmission, reduced transmission in the transfer box;

– additional devices that increase patency (anti-slip chains, winches, etc.)

Engine design. It is easier to give a vehicle with a diesel engine increased cross-country ability than a vehicle with a spark ignition engine. This is explained by the fact that with the same power, a diesel engine has a higher torque and a lower rotation frequency.

The type and design of the wheels affect the vehicle's ability to overcome areas with soft, wet soil, with a hard, slippery surface. With an increase in the wheel radius, passability through rapids, ditches and the ability to overcome fords increases.

The track of the front and rear wheels . As mentioned in subsection 10.2.2, the coincidence of the tracks of the front and rear wheels reduces the rolling resistance when driving on soft soils, which contributes to increased patency.

Wheel suspension type. When driving on rough terrain, the passability of vehicle depends significantly on the contact of the driving wheels with the supporting surface. The best adaptation to surface irregularities and reliable contact of the driving wheels with the ground are provided by independent wheel suspensions. Balancer suspension of the axle group of multi-axle vehicle with a large angle of misalignment of the axles also reduces the probability of wheel separation from the ground. The use of such suspensions increases the passability of the vehicle.

Design of the main gear. To increase the passability of vehicle, a double central transmission is used, which allows you to reduce the size of the crankcase and increase the vehicle's ground clearance. An even greater increase in ground clearance is achieved by using a double spaced transmission. In this case, the crankcase of the central gearbox is not only reduced, but also the height of the drive axis of its wheels is increased.

Differential type. The differential is a transmission unit that allows the wheels of one axle to rotate at different angular velocities, which increases the vehicle's handling. At the same time, it significantly reduces the passability of the vehicle on slippery roads.

When one of the driving wheels slips, its angular speed and the parts connected to it (semi-axle and semi-axle gear) increase compared to the angular speed of the known gear and the differential case. Let's denote it as the angular velocity of the semi-axis running into ω_{zab} (see Fig. 10.14). At the same time , the semi- axis connected by the non-slipping drive wheel rotates more slowly with an angular velocity ω_{ot} (in a separate case, it does not rotate at all $\omega_{ot} = 0$). Since during the rotation of the semi-axial gears there is friction between their occipital surface and the differential housing, a moment of friction occurs directed against their rotation (Fig. 10.14). Taking this into account, it is possible to determine the torque transmitted by the differential to the drive wheels:

$$M_{ob} = 0,5(M_0 - M_{tr}); \tag{10.28}$$

$$M_{ot} = 0,5(M_0 + M_{tr}), \tag{10.29}$$

where M_{ob} , M_{ot} , M_{tr} are the torque on the overtaking (slipping) wheel, on the trailing wheel, and the total friction moment in the differential, respectively.

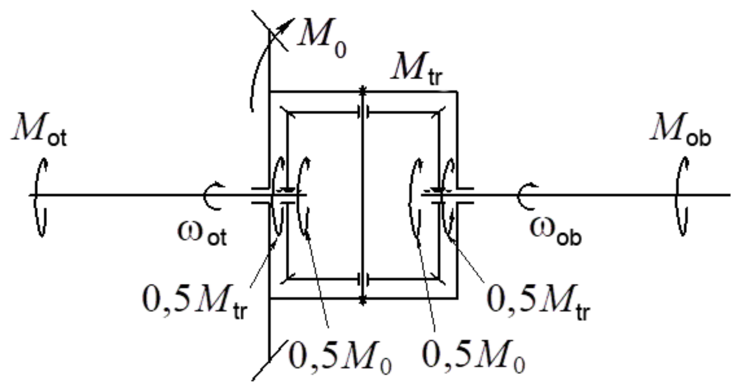


Fig . 10.14 **. Torque distribution scheme differential when one wheel slips**

When one wheel skids (overtaking wheel), the differential, due to the moment of friction, increases the transmission of torque to the wheel that does not skid (lagging wheel). As a result, the total traction force increases, and the vehicle's passability increases.

The total traction force on the driving wheels at different moments on the half-axes is determined by the equation

$$P_k = \frac{M_{ob} + M_{ot}}{r_d}. \tag{10.30}$$

The "ideal" differential changes the magnitude of the friction moment depending on the driving conditions and distributes the torque between the half-axles so that both driving wheels fully realize the traction properties with the supporting surface. At the same time, torques are formed on the drive wheels, which are equal to their moments of adhesion with the supporting surface, which are determined by the corresponding coefficients of adhesion

$$M_{\text{ob}} = M_{\varphi\text{ob}} = 0{,}5 \cdot R_z \cdot \varphi_{x\min} \cdot r_{\text{d}}, \tag{10.31}$$

$$M_{\text{ot}} = M_{\varphi\text{ot}} = 0{,}5 \cdot R_z \cdot \varphi_{x\max} \cdot r_{\text{d}}, \tag{10.32}$$

where $M_{\varphi \text{ ob}}$, $M_{\varphi \text{ ot}}$ are the clutch moments of the overtaking and lagging wheel, respectively;

R_z – normal reaction on the driving axis;

$\varphi_{x \min}$, $\varphi_{x \max}$ – adhesion utilized coefficients of the wheel that rotates faster, i.e. overtakes, and the wheel that lags behind, respectively.

In this case, the total traction force acquires the value of the traction force of the driving wheels with the supporting surface (the maximum possible value in the given traction conditions)

$$P_{\text{k}} = P_{\varphi} = \frac{M_{\varphi\text{ob}} + M_{\varphi\text{ot}}}{r_{\text{d}}} = R_z \frac{\varphi_{x\min} + \varphi_{x\max}}{2}. \tag{10.33}$$

In the absence of friction in the differential, the same moments are distributed to both wheels. At the same time , if one of the wheels, which is located on a surface with a low coefficient $\varphi_{x \min}$, slips, then the moment of the slipping wheel's clutch is transmitted to the second wheel, which is located on a surface with a higher coefficient of adhesion $\varphi_{x \max}$. That is, in this case, the moments on the driving wheels are the same. At the same time, the trailing wheel is on the surface with the coefficient of adhesion $\varphi_{x \max}$, but does not use its potential for adhesion

$$M_{\text{ob}} = M_{\varphi\text{ob}} = 0.5 \cdot R_z \cdot \varphi_{x\min} \cdot r_{\text{d}}, \tag{10.34}$$

$$M_{\text{ot}} = M_{\varphi\text{ot}} = 0.5 \cdot R_z \cdot \varphi_{x\min} \cdot r_{\text{d}}. \tag{10.35}$$

In this case, the total traction force is limited by the adhesion utilized coefficient $\varphi_{x \min}$ and is determined by the equation (10.36).

$$P_{\text{k}} = P_{\varphi} = \frac{M_{\varphi\text{ob}} + M_{\varphi\text{ot}}}{r_{\text{d}}} = R_z \cdot \varphi_{x\min}. \qquad (10.36)$$

As can be seen from (10.33), friction in the differential improves the passability of the vehicle on slippery roads. This is due to the fact that the moment of friction, which is formed in the differential, is transmitted to the wheel that does not slip, on which additional traction force is formed. To improve the passability of the vehicle on slippery roads, high-friction differentials are used.

Friction in the differential creates resistance to the rotation of its gears or , as it is customary to say , blocks their rotation. The greater the friction in the differential, the greater the level of locking the gears. The differential locking level is estimated by the locking coefficient k_{δ}. In the theory of the vehicle, three *methods of determining the coefficient of differential lock are used:*

- the first way

$k_{\delta} = \frac{M_{\text{tr}}}{M_0}$; k_{δ} is determined by the interval 0 – 1;

- the second way

$k_{\delta} = \frac{M_{\text{ot}}}{M_0}$; k_{δ} is determined by the interval 0.5 – 1;

- the third way

$k_{\delta} = \frac{M_{\text{ot}}}{M_{\text{ob}}}$; k_{δ} is determined by the interval $1 - \infty$.

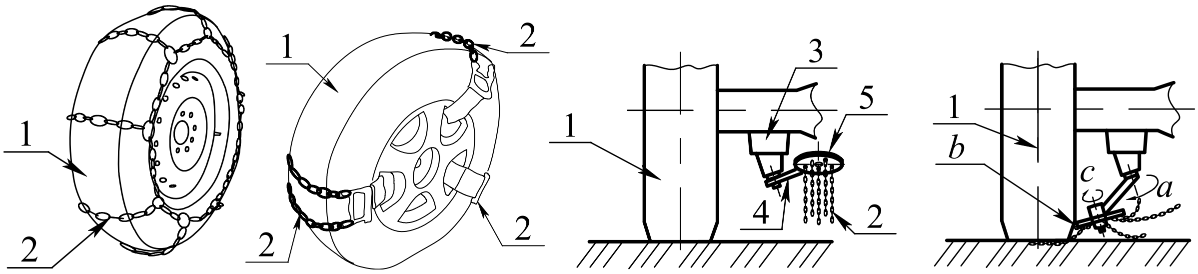
In this tutorial, the first method of determining the differential lock coefficient is used. So, for road vehicle, low-friction differentials are used, which have a locking factor k_{δ} = 0.1...0.15. High-friction differentials with a locking coefficient k_{δ} = 0.35...0.6 are installed on vehicles with increased cross-country ability .

Downshift in the transfer case significantly increases the vehicle's passability on slippery roads and especially when driving on soft and wet soils. Due to the reduction gear, it is possible to move at a minimum speed (0.5...1.5 km/h) and the possibility of its smooth change. A particularly smooth start of movement and a smooth change in the speed of the vehicle is achieved by the use of hydro-mechanical night gears. This provides the possibility of continuous and smooth movement on difficult areas and, as a result, improves patency.

Regulation of air pressure in tires. The use of tire pressure regulation systems allows you to change it within 0.05...0.35 MPa, depending on the road conditions. Therefore, the passability of a vehicle equipped with tire pressure regulation systems significantly increases and in some cases approaches the passability of tracked vehicles.

The configuration, dimensions and saturation of the tire tread pattern significantly affect traction and rolling resistance. Wide-profile and arched tires allow to reduce the pressure on the ground, which increases the passability .

Anti-slip chains (Fig. 10.15). Anti-slip chains are installed on the wheels in the form of bandages or bracelets that cover the tire (Fig. 10.15a, b). When anti-skid chains are installed on the driving wheels of the vehicle, their links are pressed into the support surface and the surface area of the wheels' contact with the road increases, which contributes to increasing the traction force and increasing the passability on soft, loose g runts, on snow cover.



a b c

Fig. 1 0 . 15. **Anti-slip chains** : a – with a "bandage" type installation on a wheel; b – with a "bracelet" wheel installation; c - with automated cause for action; 1 – wheel; 2 – anti-slip chain; 3 – bracket; 4 – lever; 5 - disc

Anti-skid chains are used only to temporarily increase the passability of the vehicle on difficult sections of the road. Anti-slip chains are installed on the wheels in the form of bandages or bracelets that cover the tire (Fig. 10.15a, b). When driving on hard roads, they must be removed. When driving on hard roads with frequently repeated snow drifts or areas with a soft surface, installing and removing chains greatly reduces the average speed of the vehicle, which negatively affects the complex factor of passability. Anti-skid chains with an automated drive to action significantly improve the passability of the vehicle on such roads (Fig. 10.15c). When the vehicle is moving on a hard road, the chains do not have contact with the wheel and the supporting surface. On soft links, the driver turns on the drive, which turns lever 4 and brings disk 5 into contact with the tire of wheel 1. Wheel 1, which skids, causes disk 5 with chains 2 to rotate. Chains 2 coming into contact between the tire and the supporting surface contributes to increasing the traction force and increasing the passability.

Devices for self-extraction. The use of self-extraction shaft devices allows you to significantly increase the passability of the vehicle when overcoming particularly difficult sections of the road. Such devices include winches with a drive from the power take-off box. They also include self-extraction winches mounted on the hub of the drive wheel.

Control questions

1. What is included in the concept of vehicle passability?
2. Name the types of vehicle passability.
3. What are traction and support-adhesion utilized parameters of patency determined?
4. List the geometric parameters of the vehicle that determine its overall passability.
5. What does the complex factor of the vehicle's passability depend on?
6. How can the vehicle's passability be increased?
7. What is the relationship between the pressure in contact between the tires and the road and the internal pressure of the air in the tire for different roads?
8. What determines the maximum angle of ascent and descent that a vehicle can overcome?
9. What determines the maximum height of the threshold that the vehicle can overcome when moving?
10. What determines the maximum width of the ditch and the depth of the ford that the vehicle can overcome?
11. Name the types of structural factors affecting the passability of a vehicle.
12. How do the type and design of the engine, wheels and their suspensions affect the vehicle's passability?
13. Explain why and how the friction in the conventional gear differential of the driving wheels affects the passability of the vehicle?
14. What is the differential lock coefficient and what are its approximate values for different types of differentials?
15. Name the means of increasing the passability of the vehicle.

TOPIC 11

THE SWAYING OF THE VEHICLE AND THE SMOOTHNESS OF ITS MOVEMENT

11.1. General concepts and vibration meters

During the operation of vehicle, free and forced oscillations of its body and units occur. The main sources of forced vibrations are the interaction of the wheels with the support surface, the imbalance of the wheels and other rotating masses. Free oscillations of the body and suspension occur when the vehicle moves on a smooth road after crossing a single unevenness and the absence of other disturbing factors. Forced high-frequency oscillations cause external noise and noise in the cabin, as well as vibration of vehicle components. Noise and vibration are low-amplitude, high-frequency oscillations.

Suspension and tires are the main devices that reduce dynamic loads and vibration load on the vehicle. The driver and passengers are additionally protected from these effects by elastic seats.

The vehicle's ability to provide the possibility of long-term driving on various roads without fatigue or burdensome feelings for passengers and the driver, while ensuring high speeds of movement and preservation of cargo, is evaluated by its operational property, which is called *smoothness of movement* .

The smoothness of movement depends on:

– ride comfort and cargo storage;
– average speed of movement;
– vehicle performance;
– fuel consumption;
– mileage between repairs.

When analyzing the smoothness of movement, two main parts of the vehicle are distinguished as oscillating systems: the sprung mass m_p and the unsprung mass m_n .

Estimated indicators of the smoothness of the vehicle:

– frequency of self-oscillations of the body ν, 1/s;
– technical frequency of oscillations n , number/min;
– amplitude of oscillations, $z_{max} = A$, m.

Levels of vibration load of the driver, passengers, body:
– rms vibration velocity $\dot{z}$, m/s;
– rms vibration acceleration $\ddot{z}$, m/s ^{2} .

11.1.1. The simplest model of oscillations of a sprung mass

Various models are used to study oscillations. The simplest model is a sprung mass on a spring (Fig. 11.1). If the sprung mass is brought out of equilibrium, then free oscillations will occur without damping, while assuming the absence of frictional forces in the spring and between the sprung mass and air. Oscillations occur as a result of the action of three forces: the force of gravity G_p and the force of inertia F_i , the sprung mass, the elastic force of the spring (the mass of the spring is small and is not taken into account).

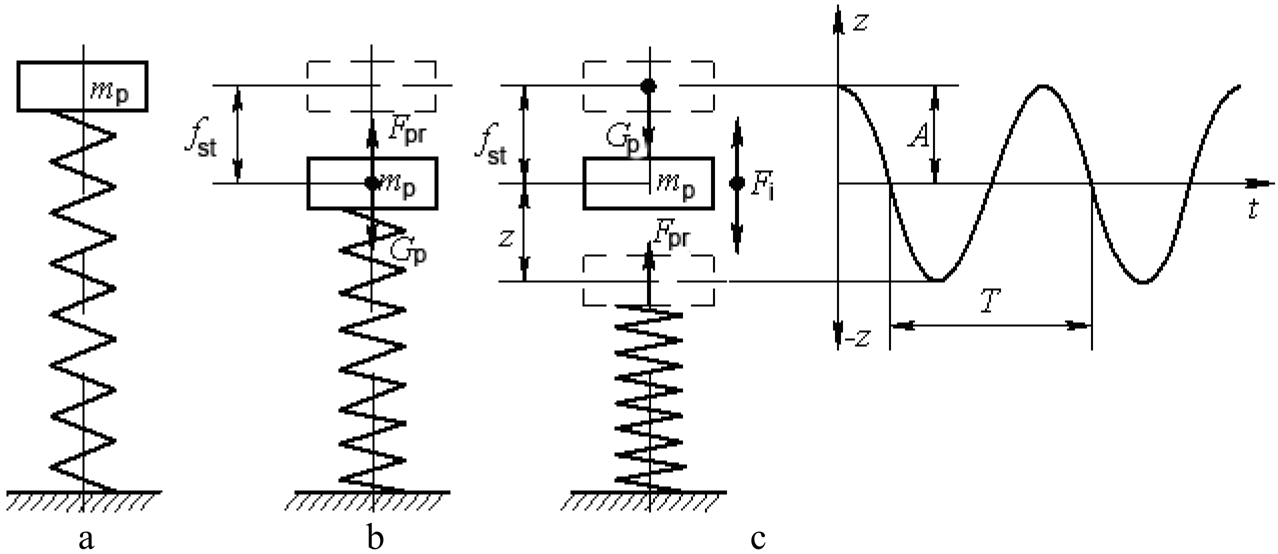


Fig . 11.1. **Scheme of a single-mass oscillating system without fading and its characteristics:** a – in the free state of the suspension (spring); b – in the static state of the system; c – in the oscillatory state of the system and its characteristics

The oscillation of the simplest one-mass oscillating system (Fig. 11.1) is described by the differential equation

$$m_p \cdot \ddot{z} + \left(f_{st} + z\right) \cdot c_p - G_p = 0 \ , \qquad (11.1)$$

where f_{st} is the static deformation of the suspension (spring) (deformation at a static position of the mass);

z , $\ddot{z}$ – vertical movement and acceleration of movement of the position of the sprung mass from the static position;

c_p – stiffness coefficient of the suspension (spring).

In equation (11.1), the first component characterizes the force of inertia of the sprung mass, and the second - the force of the spring when its deformation changes.

Taking into account that the gravitational force of the sprung mass G_p is a constant value and in a static state is balanced by the force of the spring (Fig. 11.1a), i.e. $G_p = f_{st} \cdot c_p$, let's transform equation (11.1) into the form

$$\ddot{z} + \frac{c_p}{m_p} z = 0. \tag{11.2}$$

Let's enter the notation $\omega = \sqrt{\frac{c_p}{m_p}}$ is the angular frequency of oscillations and reduce equation (11.2) to the canonical form

$$\ddot{z} + \omega^2 \cdot z = 0. \tag{11.3}$$

The solution of this differential equation of free undamped oscillations is known and has the form

$$z = z_{max} \cdot \sin(\omega \cdot t), \tag{11.4}$$

where z_{max} – amplitude of oscillations (maximum displacement from a static position);

t is the time of oscillations.

Equation (11.4) shows that the oscillations of the sprung mass have a sinusoidal character.

11.1.2. The frequency of oscillations of the sprung mass and its dependence on the suspension parameters

When analyzing fluctuations, the following definitions are used:

– the linear frequency of natural (free) oscillations is the reciprocal of the period *T* of one oscillation

$$\nu = \frac{1}{T}; \tag{11.5}$$

– the angular frequency of natural (free) oscillations

$$\omega = 2\pi\nu = \frac{2\pi}{T}; \tag{11.6}$$

– the technical frequency of natural (free) oscillations

$$n = \frac{60}{T} = \frac{60}{2\pi} \cdot \omega . \qquad (11.7)$$

The smoothness of the vehicle's movement mainly depends on the frequency of oscillations, so let's determine its dependence on the parameters of the spring (elastic element). Let's rewrite equation (11.5) taking into account (11.6)

$$\nu = \frac{1}{T} = \frac{\omega}{2\pi} . \qquad (11.8)$$

Given that $\omega = \sqrt{\frac{c_{\text{p}}}{m_{\text{p}}}}$, and let's $c_{\text{p}} = \frac{G}{f_{\text{st}}}; m_{\text{p}} = \frac{G}{g}$ transform equation (11.8) into the form

$$\nu = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{c_{\text{p}}}{m_{\text{p}}}} = \frac{1}{2\pi}\sqrt{\frac{G \cdot g}{f_{\text{st}} \cdot G}} = \frac{1}{2\pi}\sqrt{\frac{g}{f_{\text{st}}}} . \qquad (11.9)$$

Taking into account equation (11.7), it is not difficult to express the dependence of the technical frequency of oscillations on the parameter of the vehicle - the static travel of the suspension

$$n = \frac{60}{2\pi}\omega = \frac{30}{\pi}\omega = \frac{30}{\pi}\sqrt{\frac{c_{\text{p}}}{m_{\text{p}}}} = \frac{30}{\pi}\sqrt{\frac{G \cdot g}{f_{\text{st}} \cdot G}} = \frac{30}{\pi}\sqrt{\frac{g}{f_{\text{st}}}} . \qquad (11.10)$$

It can be seen from equations (11.9) and (11.10) that the frequency of oscillations of the sprung mass clearly depends on the static deformation of the elastic element of the suspension f_{st}.

11.2. Oscillating system of the vehicle

The vehicle is a multi-mass oscillating system with multiple degrees of freedom. The scheme of the plane oscillating system of the vehicle is shown in Fig. 11.2.

Each mass has six degrees of freedom - three linear and three angular.

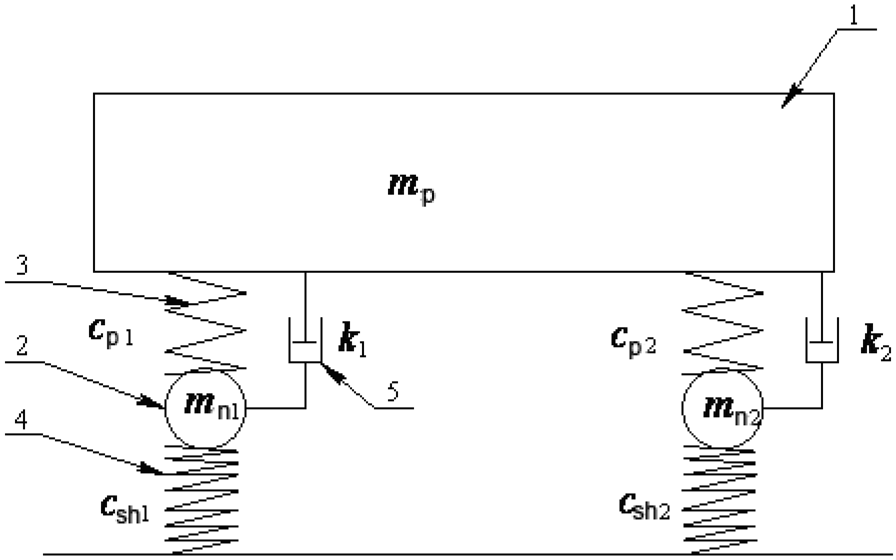


Fig. 11.2. **Scheme of a flat model of the vehicle's oscillating system:**
1 – sprung mass of the vehicle m_p ; 2 – unsprung masses m_{p1} and m_{p2} ; 3 – front and rear suspensions with stiffness c_{p1} and c_{p2} ; 4 – tires with stiffness c_{sh1} and c_{sh2} ; 5 – shock absorbers with coefficients of inelastic resistance k_1 and k_2 .

To study the oscillations of the vehicle, we will introduce the following assumptions:

– damping of oscillations is not taken into account (we exclude from the scheme in Fig. 11.2 damping devices-shock absorbers);

– we neglect the influence of unsprung masses m_{n1} and m_{n2} on the fluctuations of the sprung mass m_p (exclude the masses m_{n1} and m_{n2} from the diagram in Fig. 11.2);

– the stiffness of the elastic elements of the suspension and tires is replaced by the reduced stiffness of the suspension.

Taking into account the accepted assumptions, the diagram of the model of the oscillating system of the vehicle will take the form shown in Fig. 11.3.

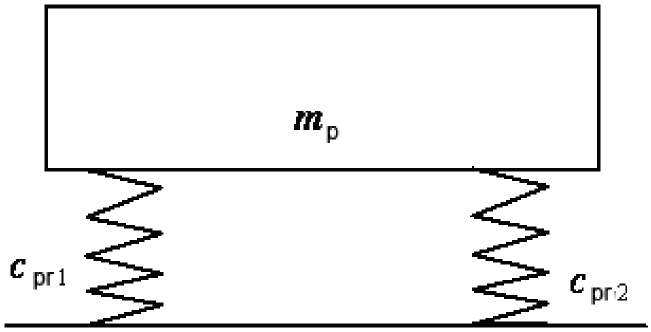


Fig . 11.3. **Scheme of a simplified model of a one-mass oscillator vehicle systems:** c_{pr1} , c_{pr2} – reduced stiffness of the front and rear suspensions

11.3. The stiffness of the suspension is reduced

The resulting stiffness of the suspension is the stiffness of a conditional elastic element, which, under a given load, forms a deformation equal to the sum of the deformations of the elastic element of the suspension and the tire.

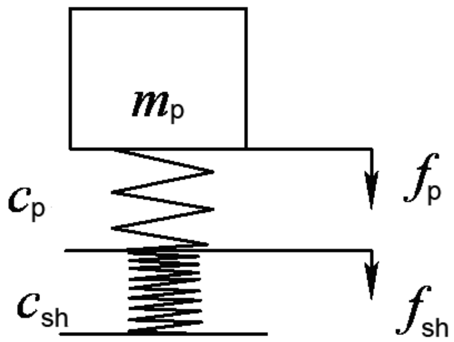


Fig. 11.4. **The scheme for determining the reduced stiffness of the suspension:**
f_p – static deformation of the elastic element of the suspension;
f_{sh} – static deformation of the tire

The (total) deformation of the suspension is given

$$f_{st} = f_p + f_{sh}. \quad (11.11)$$

Having expressed in equation (11.11) each deformation due to weight and the corresponding stiffness, after the transformations we determine the reduced stiffness of the suspension

$$\frac{G_p}{c_{pr}} = \frac{G_p}{c_p} + \frac{G_p}{c_{sh}}; \rightarrow \frac{1}{c_{pr}} = \frac{1}{c_p} + \frac{1}{c_{sh}} \rightarrow c_{pr} = \frac{c_p \cdot c_{sh}}{c_p + c_{sh}}, \quad (11.12)$$

where G_p is the weight of the sprung mass m_p;
c_{pr} – coefficient of reduced stiffness of the suspension.

11.4. Free oscillations of the vehicle

This is the oscillation of the vehicle when moving on a flat road after crossing a single unevenness. To study the free oscillations of the vehicle in the longitudinal vertical plane, flat models of its oscillatory system are used. Depending on the tasks of vibration study, different models are used. To simulate the oscillations of the sprung mass of the vehicle when building the model (Fig. 11.5), we will make the following assumptions:

– the sprung mass is replaced by three reduced masses M_1, M_2, M_3 connected to each other by a rigid rod;
– the effect of damping and the effect of unsprung masses on the oscillations of the sprung mass are not taken into account.

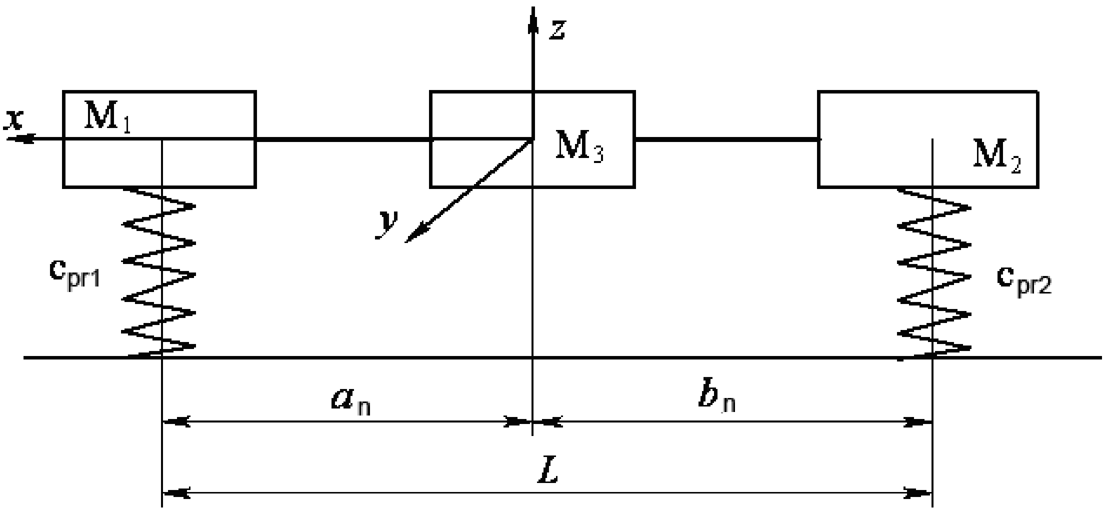


Fig. 11.5. **Scheme of the three-mass model of the oscillating system of the vehicle:** a_n, b_n are the longitudinal coordinates of the center of gravity sprung mass of the vehicle

In order for this model of the three-mass oscillating system to dynamically correspond to the actual sprung mass of the vehicle, the conditions must be met:

1. The sum of all three reduced sprung masses is equal to the sprung mass of the vehicle

$$M_1 + M_2 + M_3 = m_p. \quad (11.13)$$

2. The center of gravity of the three-mass oscillating system must coincide with the center of gravity of the sprung mass of the vehicle

$$M_1 \cdot a_n = M_2 \cdot b_n. \quad (11.14)$$

3. The moments of inertia of the three-mass oscillating system and the sprung mass of the vehicle relative to the Y axis passing through the center of gravity must be equal

$$M_1 \cdot a_n^2 + M_2 \cdot b_n^2 = m_p \cdot \rho_y^2, \quad (11.15)$$

where ρ_y is the radius of the moment of inertia of the sprung mass of the vehicle relative to the y axis.

Let's solve these three equations together and find the value of the reduced (concentrated) masses M_1, M_2, M_3. To do this, we express the masses M_1, M_2 from equation (11.14) and alternately substitute them into equation (11.15), from which we find the values of M_1, M_2. And then, substituting these values into equation (11.15), we determine M_3.

From equation (11.14) we get

$$M_1 = M_2 \frac{b_n}{a_n}; \rightarrow \text{ in (11.15) } M_2 \frac{b_n}{a_n} a_n^2 + M_2 \cdot b_n^2 = m_p \cdot \rho_y^2,$$

after grouping and transformation we get

$$M_2 \cdot b_n \cdot (a_n + b_n) = m_p \cdot \rho_y^2,$$

given that $(a_n + b_n) = L$, the equation can be rewritten in the form

$$M_2 \cdot b_n \cdot L = m_p \cdot \rho_y^2.$$

From which we will get

$$M_2 = m_p \cdot \frac{\rho_y^2}{b_n \cdot L}. \quad (11.16)$$

Analogous transformations allow writing

$$M_1 = m_p \cdot \frac{\rho_y^2}{a_n \cdot L}. \quad (11.17)$$

Masses M_1 and M_2 are parts of the sprung mass of the vehicle, given according to the front and rear suspensions. Substitute the obtained values of M_1 and M_2 into equation (11.13)

$$m_p \cdot \frac{\rho_y^2}{a_n \cdot L} + m_p \cdot \frac{\rho_y^2}{b_n \cdot L} + M_3 = m_p;$$

$$M_3 = m_p \left[1 - \frac{\rho_y^2}{L} \cdot \left(\frac{1}{a_n} + \frac{1}{b_n} \right) \right];$$

$$M_3 = m_p \left(1 - \frac{\rho_y^2}{a_n \cdot b_n} \right). \quad (11.18)$$

Mass M_3 is a part of the sprung mass concentrated in the center of the sprung mass m_p of the vehicle.

Let's write down the system of equations of free oscillations of the front and rear suspensions for the three-mass oscillating model of the vehicle

$$M_1 \cdot \ddot{z}_1 + c_{pr1} \cdot z_1 + M_3 \cdot \ddot{z}_2 = 0; \quad (11.19)$$

$$M_2 \cdot \ddot{z}_2 + c_{pr2} \cdot z_2 + M_3 \cdot \ddot{z}_1 = 0. \quad (11.20)$$

Let's divide equation (11.19) by M_1, and equation (11.20) by M_2

$$\ddot{z}_1 + \frac{c_{pr1}}{M_1} \cdot z_1 + \frac{M_3}{M_1} \cdot \ddot{z}_2 = 0; \quad (11.21)$$

$$\ddot{z}_2 + \frac{c_{pr2}}{M_2} \cdot z_2 + \frac{M_3}{M_2} \cdot \ddot{z}_1 = 0. \quad (11.22)$$

Let's enter the notation

$$\frac{c_{pr1}}{M_1} = \omega_1^2, \ \frac{c_{pr2}}{M_2} = \omega_2^2 \text{ and } \frac{M_3}{M_1} = \eta_{01}, \ \frac{M_3}{M_2} = \eta_{02},$$

where ω_1, ω_2 are partial frequencies of free oscillations;

η_{01}, η_{02} – coupling coefficients of free oscillations of the front and rear suspensions.

Taking into account the accepted notations, we will obtain a system of equations describing the free oscillations of the three-mass oscillating model of the vehicle

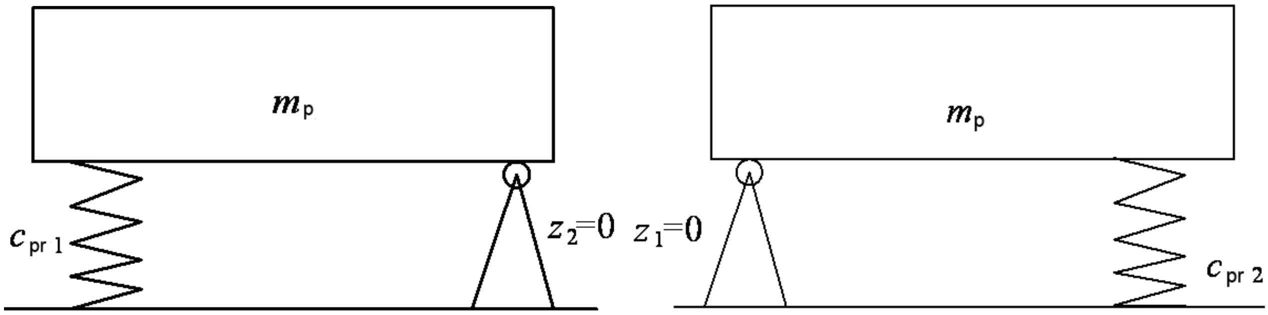
$$\ddot{z}_1 + \omega_1 \cdot z_1 + \eta_{01} \cdot \ddot{z}_2 = 0; \quad (11.23)$$

$$\ddot{z}_2 + \omega_2 \cdot z_2 + \eta_{02} \cdot \ddot{z}_1 = 0. \quad (11.24)$$

The system of equations (11.23), (11.24) is connected, that is, the frequencies of free oscillations of the front and rear suspensions are dependent on each other. Equations of oscillations of the front suspension (11.23) contain the component $\eta_{01} \cdot \ddot{z}_2$. The greater the coupling coefficient η_{01}, the greater the influence on the vibrations of the front suspension is exerted by the acceleration of the vibrations of $\ddot{z}_2$ the rear suspension. A similar connection of oscillations reflects the equation of oscillations of the rear suspension (11.24).

11.5. Partial frequencies of free oscillations

Partial frequency is called the frequency of oscillations of the system in which all degrees of freedom, except one, are limited. Partial frequencies can be obtained for any complex oscillating system if the movement of masses is limited so that the system has one degree of freedom each time. As shown in Figure 11.6, the sprung mass m_p has only one degree of freedom.



$$\ddot{z}_1 + \omega_1 \cdot z_1 = 0 \qquad \ddot{z}_2 + \omega_2 \cdot z_2 = 0$$

a b

Fig. 11.6. **Schemes of oscillating systems with one degree of freedom:**
a – oscillations on the front suspension;
b – oscillations on the rear suspension

When limiting the oscillations of the rear suspension (Fig. 11.6a), the oscillations on the front suspension become independent, since $z_2 = 0$, and when limiting the oscillations of the front suspension (Fig. 11.6b), the oscillations on the rear suspension become independent, since $z_1 = 0$.

11.6. Two-mass oscillating system of the vehicle

When designing a vehicle to improve the smoothness of the ride, they try to ensure that there is no connection between the vibrations of the suspensions. This is achieved by the appropriate distribution of sprung masses (units, cargo, etc.) relative to the center of gravity of the sprung mass of the vehicle (body).

As can be seen from equations (11.23), (11.24), in order for the oscillations of the suspensions to be independent of each other, it is necessary that the coupling coefficients of the oscillations be equal to zero

$$\frac{M_3}{M_1} = \eta_{01} = 0; \quad \frac{M_3}{M_2} = \eta_{02} = 0;$$

The coefficients η_{01} and η_{02} are equal to zero if the sprung mass $M_3 = 0$ is given. Taking into account equation (11.18), we can write

$$M_3 = m_p \left(1 - \frac{\rho_y^2}{a_n \cdot b_n} \right) = 0. \qquad (11.25)$$

It can be seen from (11.25) that $M_3 = 0$ $\frac{\rho_y^2}{a_n \cdot b_n} = 1$ if $\rho_y^2 = a_n \cdot b_n$.

determine the value of the reduced sprung masses M_1, M_2, if $M_3 = 0$ and respectively $\rho_y^2 = a_n \cdot b_n$. Substitute the value of the radius of the moment of inertia ρ_y in (11.17) and (11.16)

$$M_1 = \frac{m_p \cdot \rho_y^2}{a_n \cdot L} = \frac{m_p \cdot a_n \cdot b_n}{a_n \cdot L} = m_p \frac{b_n}{L} = m_{p1}; \qquad (11.26)$$

$$M_2 = \frac{m_p \cdot \rho_y^2}{b_n \cdot L} = \frac{m_p \cdot a_n \cdot b_n}{b_n \cdot L} = m_p \frac{a_n}{L} = m_{p2}. \qquad (11.27)$$

Let's introduce the concept of the coefficient of distribution of sprung masses

$$\varepsilon_y = \frac{\rho_y^2}{a_n \cdot b_n}.$$

The complete lack of coupling of suspension vibrations is achieved at $\varepsilon_y = 1$. It should be noted that such a distribution of sprung masses is difficult to ensure not only in the design of the vehicle, but also during its operation. A good smoothness of the stroke is achieved with values of the coefficient of distribution of sprung masses within $\varepsilon_y = 0.8...1.1$. In this case, when studying oscillations in models of oscillating systems, the sprung mass of the vehicle m_n can be presented in the form of two reduced masses M_1 and M_2, and these reduced masses are hingedly connected to each other by a rigid weightless rod (Fig. 11.7).

For such a system, the reduced masses M_1 and M_2 have a certain physical meaning - they characterize the sprung masses m_{p1}, m_{p2} that fall on the front and rear suspensions when the vehicle is stationary. The system of equations (11.23), (11.24) breaks down into two equations that are not related to each other.

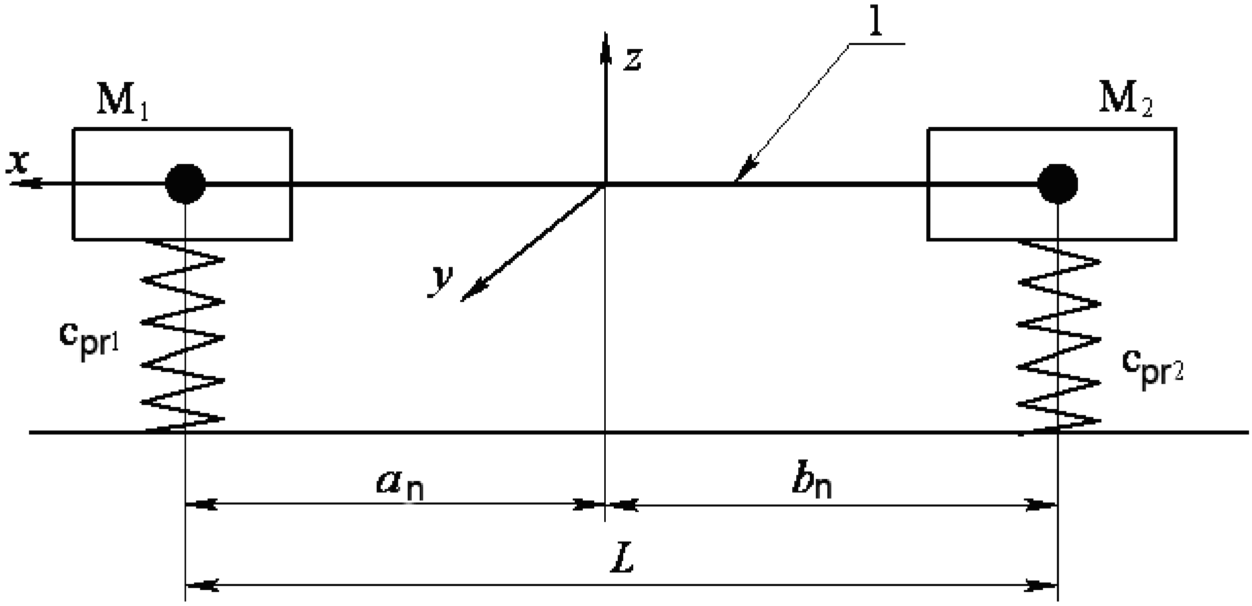


Fig. 11.7. **Scheme of the model of a two-mass oscillating system vehicle:** 1 – hinge connection with a weightless rigid rod

The mass m_{p1} oscillates with a frequency ω_1

$$\ddot{z}_1 + \omega_1^2 \cdot z_1 = 0, \qquad (11.28)$$

where $\omega_1 = \sqrt{\dfrac{c_{pr1}}{m_{p1}}}$ – partial frequency of oscillations of the front suspension.

The mass m_{p2} oscillates with the frequency ω_2

$$\ddot{z}_2 + \omega_2^2 \cdot z_2 = 0, \qquad (11.29)$$

where $\omega_2 = \sqrt{\dfrac{c_{pr2}}{m_{p2}}}$ is the partial frequency of oscillations of the rear suspension.

11.7. Free oscillations of sprung and unsprung masses of the vehicle without taking into account damping (four-mass model)

Assumption:

– there is no connection between the vibrations of the rear and front parts of the body, i.e. $\varepsilon_y = 1$;

- there are no vibration resistance forces, i.e. there is no vibration damping.

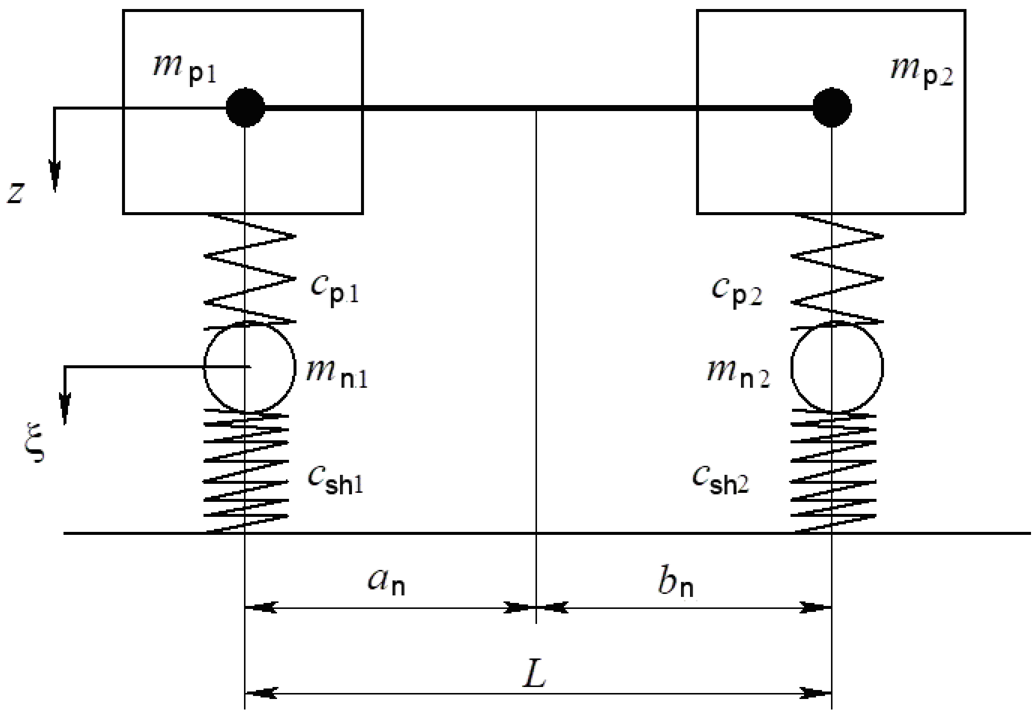


Fig . 11. 8 . **Scheme of the four-mass oscillating system of the vehicle excluding fading**

The oscillations of the front and rear suspensions are independent of each other and are described by the same equations. Therefore, we will consider the equation without indicating the index of the axis.

We have two oscillating systems on each axis:

– fluctuations of the sprung mass;

$$m_{\text{p}} \cdot \ddot{z} + c_{\text{p}} \cdot (z - \xi) = 0, \tag{11.30}$$

where ξ is tire deformation;

– fluctuations of the unsprung mass

$$m_{\text{n}} \cdot \ddot{\xi} + c_{\text{sh}} \cdot \xi - c_{\text{p}} \cdot (z - \xi) = 0. \tag{11.31}$$

After dividing equation (11.30) by m_{p}, equation (11.31) by m_{n} and simple transformations, we obtain

$$\ddot{z} + \omega_0^2 \cdot (z - \xi) = 0; \tag{11.32}$$

$$\ddot{\xi} + \omega_{\text{k0}}^2 \cdot \xi - \omega_{\text{k00}}^2 \cdot z = 0; \tag{11.33}$$

where $\omega_0 = \sqrt{\dfrac{c_{\text{p}}}{m_{\text{p}}}}$ is the partial frequency of oscillations of the sprung mass m_{p} with a stationary unsprung mass m_{n} (fig. 11.9a) ;

$\omega_{k0} = \sqrt{\dfrac{c_{\text{p}} + c_{\text{sh}}}{m_{\text{n}}}}$ – partial frequency of oscillations of the unsprung mass m_{n} with a stationary sprung mass m_{p} (Fig. 11.9b) ;

$\omega_{\text{k00}} = \sqrt{\dfrac{c_{\text{p}}}{m_{\text{n}}}}$ is the partial frequency of oscillations of the unsprung mass m_{n} with a stationary sprung mass m_{p} and at $c_{\text{sh}} = 0$ (Fig. 11.9a) .

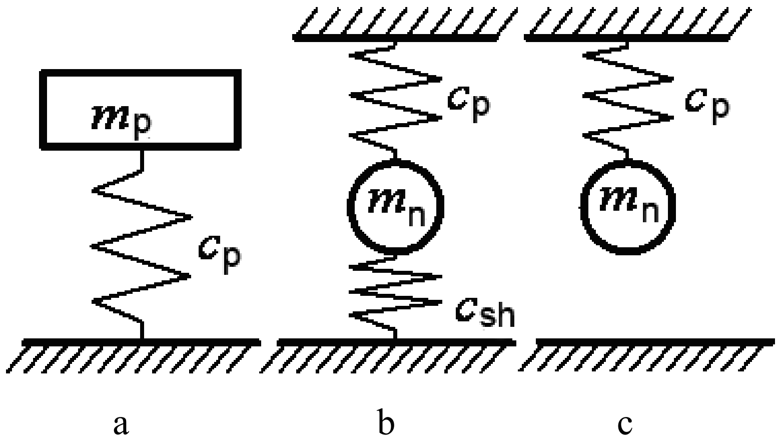


Fig. 11.9. **Scheme of oscillations with partial frequencies:**
a – sprung mass with a stationary unsprung mass;
b – unsprung mass with a stationary sprung mass;
c – an unsprung mass with a fixed sprung mass and when $c_{sh} = 0$

Conclusion from the system of equations (11.32), (11.33) – the vehicle has four frequencies of free oscillations: two low and two high. The fluctuations of the sprung mass and the unsprung mass are dependent, since both equations (11.32), (11.33) include z and ξ .

When designing a vehicle, they try to reduce the relationship between the vibrations of the sprung and unsprung masses by increasing the ratios $c_{\text{sh}}/c_{\text{p}}$ and $m_{\text{p}}/m_{\text{n}}$ (Table 11.1).

Table 11.1 – **Characteristic ratios of stiffnesses and masses**

Type of vehicle	s_{sh}/s_p	m_p/m_n
Passenger cars: small class average higher	 3...4 7...10 10...20	For both suspensions in any weight state > 4
Trucks	2.5...5	For rear suspension with full load > 4; No load in all cases < 4

If the stiffness ratio corresponds to the values indicated in table 11.1, and $m_p/m_n > 4$, then the error in calculating the vibration frequencies without taking into account the mutual influence does not exceed 1%. For trucks, if $m_p/m_n < 4$, the error can reach several percent (< 10%).

Approximate values (without taking into account mutual influence):

– low frequency oscillations (oscillations of the sprung mass – body)

$$\omega_1 = \sqrt{\frac{c_{p1}}{m_{p1}}}; \quad \omega_2 = \sqrt{\frac{c_{p2}}{m_{p2}}}; \tag{11.34}$$

– high frequency oscillations (oscillations of unsprung masses – wheels, bridges)

$$\omega_{k1} = \sqrt{\frac{c_{p1} + c_{sh2}}{m_{n1}}}; \quad \omega_{k2} = \sqrt{\frac{c_{p2} + c_{sh2}}{m_{n2}}}. \tag{11.35}$$

Usually, the oscillation frequencies correspond to the intervals:

$$\omega_1, \omega_2 = \begin{cases} 0.8\text{-}1.3\,\text{Hz}\ (50\text{-}80 \text{ oscillations/min}) \text{ - passenger cars;} \\ 1.2\text{-}1.8\,\text{Hz}\ (75\text{-}110 \text{ oscillations/min}) \text{ - trucks and buses.} \end{cases}$$

$$\omega_{k1}, \omega_{k2} = \begin{cases} 8\text{-}12 \text{ Hz}\ (500\text{-}750 \text{ oscillations/min}) \text{ - passenger cars;} \\ 6{,}5\text{-}9 \text{ Hz}\ (400\text{-}550 \text{ oscillations/min}) \text{ - trucks and buses.} \end{cases}$$

11.8. Free oscillations of the vehicle taking into account damping

11.8.1. Oscillatory process with damping

In the process of oscillations, energy is almost always dissipated, and therefore free oscillations are decaying.

Resistance forces causing damping of oscillations:
– resistance in the shock absorber (force of inelastic resistance);
– interleaf friction in springs;
– friction in bushings, hinges, etc.;
– friction in the tire.

The resistance force of the shock absorber is proportional to the speed of movement of its piston V_p. The characteristics of the shock absorber can be represented in the form of an inclined beam (Fig. 11.10).

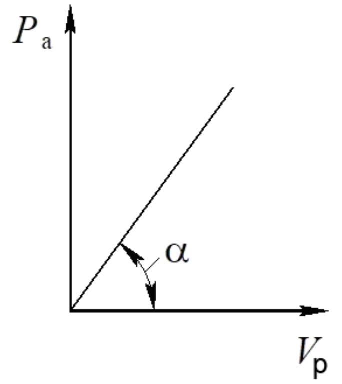


Fig. 11.10. **Characteristics of the shock absorber $P_a = f(V_p)$:**
R_a is the resistance force of the shock absorber;
V_p is the movement speed of the shock absorber piston

The dependence of the resistance force of the shock absorber on the speed of movement of its piston is characterized by the coefficient of inelastic resistance. The value of the coefficient of inelastic resistance of the shock absorber is determined as the tangent of the angle of inclination of the characteristic beam (Fig. 11.10).

$$k = \text{tg}\alpha = \frac{P_a}{V_p}. \qquad (11.36)$$

Accordingly, the resistance force of the shock absorber can be expressed as:

$$P_a = k \cdot V_p, \qquad (11.37)$$

where k is the coefficient of inelastic resistance of the shock absorber, N/(m/s) = kg/s.

The speed of movement of the shock absorber piston is related to the speed of movement of the wheel. This connection depends on the type of suspension and the installation of shock absorbers.

When analyzing the smoothness of the vehicle, we will consider the dependent wheel suspension with vertically installed shock absorbers. In this case, when the wheels hit an unevenness, the speed of its movement and the speed of movement of the shock absorber piston are equal.

The type of oscillation process of the sprung mass of the vehicle, taking into account the resistance in the suspension, is shown in Fig. 11.11.

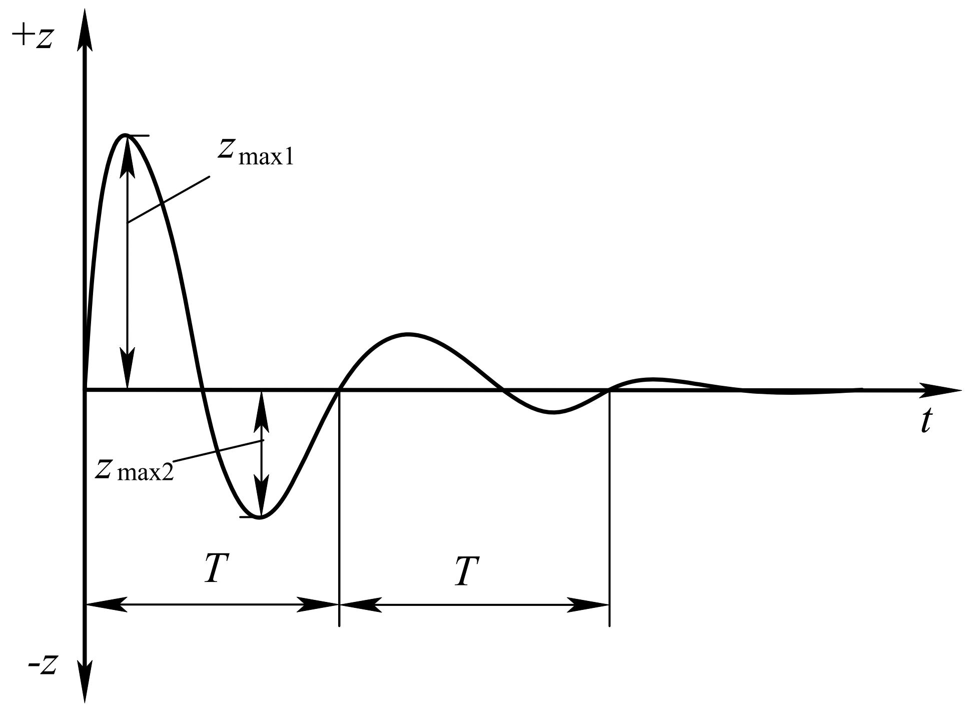


Fig . 11. 11 . **Characteristics of damping oscillations of a sprung mass**

As a result of energy dissipation, the amplitude of oscillations decreases over time. It should be noted that the oscillation period practically does not change.

11.8.2. A two-mass model of the vehicle's oscillating system with consideration of damping

When developing the model (Fig. 11.12), the following assumptions were made:

1 – there is no connection between the vibrations of the rear and front parts of the body, i.e. $\varepsilon_y = 1$;

2 – unsprung masses are not taken into account: $m_{p1} = m_{p2} = 0$;

3 – the suspension is dependent, shock absorbers are installed vertically: $V_p = \dot{z}$.

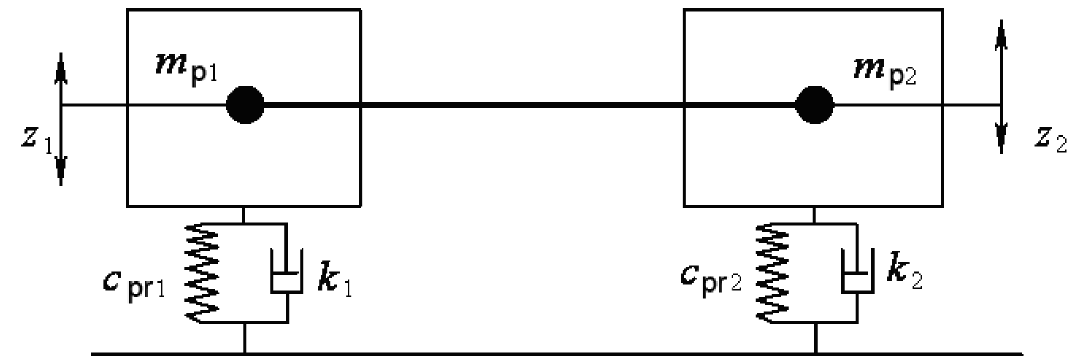


Fig. 11.12. **Scheme of the two-mass oscillating system of the vehicle taking into account fading:** c_{pr1}, c_{pr2} – reduced stiffness of the front and rear suspension, respectively; k_1, k_2 are coefficients of inelastic resistance of shock absorbers

The equation of oscillations of sprung vehicle masses

$$m_p \cdot \ddot{z} + k \cdot \dot{z} + c_{pr} \cdot z = 0 \qquad : m_p; \tag{11.38}$$

$$\ddot{z} + \frac{k}{m_p} \cdot \dot{z} + \frac{c_{pr}}{m_p} \cdot z = 0 \ . \tag{11.39}$$

Let's enter the notation $h = 0.5 \cdot k \cdot m_p^{-1}$ is the coefficient of inelastic resistance of the suspension.

In this case, the equation of oscillations of the sprung masses of the vehicle, taking into account damping, has the form

$$\ddot{z} + 2 \cdot h \cdot \dot{z} + \omega_0^2 \cdot z = 0 . \tag{11.40}$$

The coefficient of inelastic resistance of the shock absorber k characterizes its resistance force depending on the speed of the piston (the rate of deformation of the suspension), but does not give an idea about the damping of oscillations in the suspension, because it does not take into account the oscillating mass.

The coefficient of inelastic resistance of the suspension h takes into account the oscillating mass and gives a clearer idea of the damping of the oscillations in the suspension. The larger h is, the faster the oscillations decay. Therefore, this coefficient is also called *the damping coefficient or the damping coefficient.*

For a comparative evaluation of the smoothness of the movement of vehicle of different classes and types, the *relative damping coefficient of the suspension is used* ψ_q.

$$\psi = \frac{h}{\omega_0} = \frac{k}{2 \cdot m_{\text{p}} \cdot \omega_0} = \frac{k}{2 \cdot m_{\text{p}} \sqrt{\dfrac{c_{\text{pr}}}{m_{\text{p}}}}};$$

$$\psi = \frac{k}{2\sqrt{c_{\text{pr}} \cdot m_{\text{p}}}}. \tag{11.41}$$

It was established that if ψ_q = 0.15...0.3, the smoothness of the vehicle's movement is good. If ψ_q = 0.3 – better stability, and if ψ_q = 0.15, then the suspension is softer, but worse stability.

11.8.3. A four-mass model of the vehicle's oscillating system with consideration of damping

In order to take into account the influence of unsprung mass fluctuations on sprung mass fluctuations in the oscillating system model (Fig. 11.13) , it is necessary to add tire stiffness and unsprung masses.

Make assumptions:

1 – there is no connection between the vibrations of the rear and front parts of the body, i.e. $\varepsilon_y = 1$;

2 – the suspension is dependent, shock absorbers are installed vertically: $V_{\text{p}} = \dot{z}$;

3 - the force of friction in the tire is neglected.

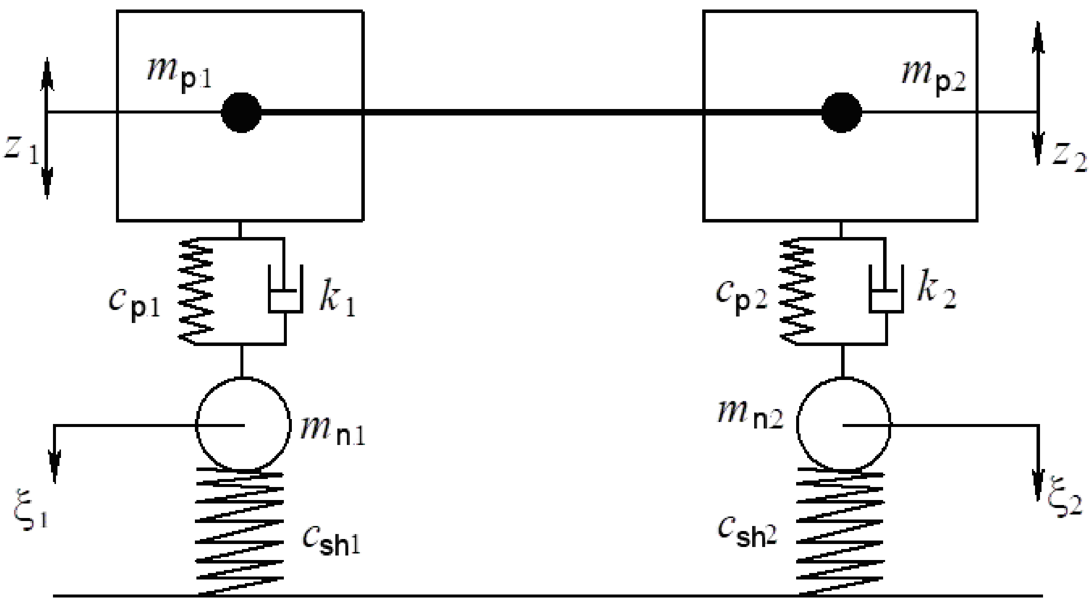


Fig . 11.1 3 . **Scheme of a four-mass oscillating system of the vehicle taking into account the fading**

Since the assumption is made about the independence of the vibrations of the front and rear suspensions, the equations of oscillations for them will have the same form. Therefore, we will write them down without specifying the axis index.

Equation of motion of a sprung mass

$$m_{\text{p}} \cdot \ddot{z} + k \cdot \left(\dot{z} - \dot{\xi}\right) + c_{\text{p}} \cdot \left(z - \xi\right) = 0. \qquad (11.42)$$

The equation of motion of an unsprung mass

$$m_{\text{n}} \cdot \ddot{\xi} - k \cdot \left(\dot{z} - \dot{\xi}\right) - c_{\text{p}} \cdot \left(z - \xi\right) + c_{\text{sh}} \cdot \xi = 0. \qquad (11.43)$$

Let's transform both equations into the form

$$\ddot{z} + \frac{k}{m_{\text{p}}} \cdot \left(\dot{z} - \dot{\xi}\right) + \frac{c_{\text{p}}}{m_{\text{p}}} \cdot \left(z - \xi\right) = 0; \qquad (11.44)$$

$$\ddot{\xi} - \frac{k}{m_{\text{n}}} \cdot \left(\dot{z} - \dot{\xi}\right) - \frac{c_{\text{p}}}{m_{\text{n}}} \cdot z + \frac{c_{\text{p}} + c_{\text{sh}}}{m_{\text{n}}} \cdot \xi = 0. \qquad (11.45)$$

Let's enter the notation:

– $h_0 = 0.5 \cdot k \cdot m_{\text{p}}^{-1}$ – partial coefficient of damping of oscillations of the sprung mass m_{p};

– $h_{\text{k}} = 0.5 \cdot k \cdot m_{\text{n}}^{-1}$ – partial damping coefficient of unsprung mass oscillations m_{n}.

Let's write the notation adopted in the equations, without taking into account the damping of oscillations (11.32), (11.33):

– $\omega_0 = \sqrt{\dfrac{c_{\text{p}}}{m_{\text{p}}}}$ – partial frequency of oscillations of the sprung mass m_{p} with a stationary unsprung mass m_{n} (Fig. 11.9a);

– $\omega_{\text{k0}} = \sqrt{\dfrac{c_{\text{p}} + c_{\text{sh}}}{m_{\text{n}}}}$ – partial frequency of oscillations of the unsprung mass m_{n} with a stationary sprung mass m_{p} (Fig. 11.9b);

– $\omega_{\text{k00}} = \sqrt{\dfrac{c_{\text{p}}}{m_{\text{n}}}}$ – partial frequency of oscillations of the unsprung mass m_{n} with a stationary sprung mass m_{p} and at $c_{\text{sh}} = 0$ (Fig. 11.9 c).

Taking into account the accepted notations, equations (11.44), (11.45) will take the form

$$\ddot{z} + 2 \cdot h_0 \cdot \left(\dot{z} - \dot{\xi}\right) + \omega_0^2 \cdot \left(z - \xi\right) = 0; \tag{11.46}$$

$$\ddot{\xi} - 2 \cdot h_{\text{к}} \cdot \left(\dot{z} - \dot{\xi}\right) + \omega_{\text{к}0}^2 \cdot \xi - \omega_{\text{к}00}^2 \cdot z = 0. \tag{11.47}$$

It is obvious from equations (11.46) and (11.47) that the oscillations of sprung and unsprung masses are interconnected. As it was shown, in undamped oscillations, the mutual influence of sprung and unsprung masses can be neglected in many cases. In this case, in equations (11.46) and (11.47) the terms determining the relationship of these equations can be excluded without significant error. That is, in equation (11.46) we can assume that the parameters ξ and $\dot{\xi}$ are equal to zero, and in equation (11.47) we can assume that they z and $\dot{z}$ are equal to zero. Taking this into account, it is possible to write down the equations of oscillations of sprung and unsprung masses

$$\ddot{z} + 2 \cdot h_0 \cdot \dot{z} + \omega_0^2 \cdot z = 0; \tag{11.48}$$

$$\ddot{\xi} + 2 \cdot h_{\text{k}} \cdot \dot{\xi} + \omega_{\text{k}0}^2 \cdot \xi = 0. \tag{11.49}$$

The solutions of equations (11.48) and (11.49) have the form

$$z = \sin\left(\omega_h \cdot t + \varphi_{\text{p}}\right) \cdot e^{-h_0 \cdot t}; \tag{11.50}$$

$$\xi = \sin\left(\omega_{\text{k}h} \cdot t + \varphi_{\text{n}}\right) \cdot e^{-h_{\text{k}} \cdot t}, \tag{11.51}$$

where $\omega_h = \sqrt{\omega_0^2 - h_0^2}$ – the frequency of oscillations of the sprung mass with damping;

$\omega_{\text{k}h} = \sqrt{\omega_{\text{k}0}^2 - h_{\text{k}}^2}$ – the frequency of oscillations of the unsprung mass with damping;

φ_{p} – the initial phase angle of oscillations of the sprung mass;

φ_{n} is the initial phase angle of oscillations of the unsprung mass;

t - oscillation time.

11.8.4. Analysis of the effect of damping in the suspension on the frequency of oscillations

Oscillations of the sprung mass with damping occur with a frequency calculated by equation (11.52).

$$\omega_h = \sqrt{\omega_0^2 - h_0^2} = \omega_0 \cdot \sqrt{1 - \frac{h_0^2}{\omega_0^2}} = \omega_0 \cdot \sqrt{1 - \psi_{q0}^2}\ . \qquad (11.52)$$

As mentioned in subsection 11.8.2, the value of the relative damping coefficient of the sprung mass $\psi_{q\,0}$ for vehicle correspond to the interval 0.15...0.3. Since the coefficient $\psi_{q\,0}$ in equation (11.52) is reduced to the second power, then even at its maximum value of 0.3, 0.09 is subtracted from the root of unity and the difference between ω_h and ω_0 does not exceed 5%.

That is, the influence of even the most intensive damping on the frequency of oscillations of the sprung mass is insignificant and it can be assumed that the oscillations occur with the frequency of the natural oscillations ω_0, which is uniquely determined by the static movement of the suspension.

Oscillations of the unsprung mass with damping occur with frequency

$$\omega_{kh} = \sqrt{\omega_{k0}^2 - h_k^2} = \omega_{k0} \cdot \sqrt{1 - \frac{h_k^2}{\omega_{k0}^2}} = \omega_{k0} \cdot \sqrt{1 - \psi_{qk}^2}\ . \qquad (11.53)$$

The value of the relative damping coefficient of the unsprung mass $\psi_{q\,k}$ of the vehicle is determined by an interval of 0.25 ... 0.45. According to equation (11.53), it can be stated that the frequency of decaying oscillations $\omega_{k\,h}$ differs from the frequency of natural oscillations ω_{k0} by no more than 11%.

11.9. Forced oscillations of the vehicle

11.9.1. Definition. Road model

During the movement of the vehicle, there are not only free oscillations, but also forced ones.

Forced oscillations are the oscillations that the vehicle carries out as a result of periodic forced oscillations caused by the undulating road surface. During movement, random disturbances occur, causing forced oscillations of a random nature. In order to identify the physical essence and basic regularities of the forced oscillations of the vehicle, we will first idealize the road, considering its microprofile as consisting of sinusoidal irregularities - waves (Fig. 11.14).

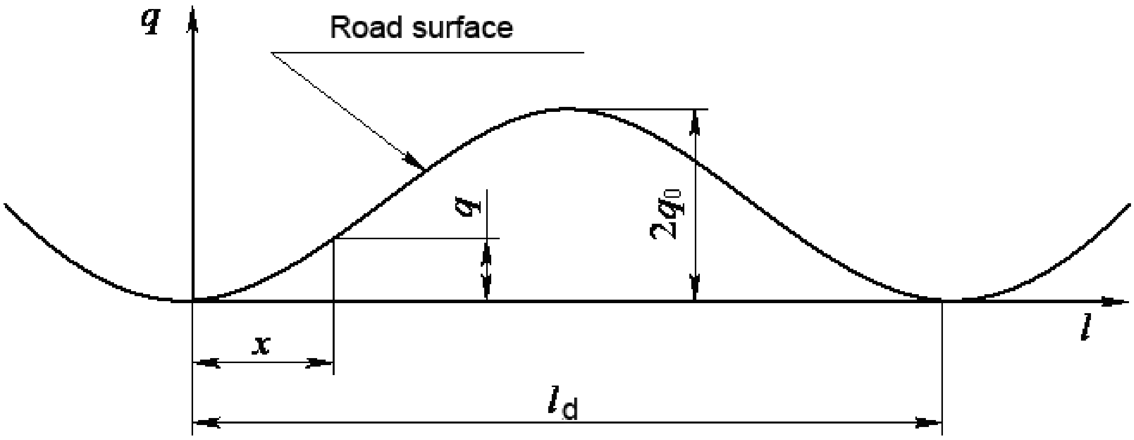


Fig. 11.14. **Road roughness model:**
q_0, l_d – respectively, the amplitude and wavelength of road irregularities;
x is the abscissa of the point for which the ordinate q is determined

If the bottom of the wave is considered as the beginning of the reference, then the current vertical coordinate q of the longitudinal section of the road is determined by the equation

$$q = q_0 \left[1 - \cos\left(\frac{2\pi \cdot x}{l_d} \right) \right]. \qquad (11.54)$$

With uniform movement of the vehicle $v = const$ the equality $x = v \cdot t$ is valid. Therefore, for this case, equation (11.54) can be written in the form

$$q = q_0 \left[1 - \cos\left(\frac{2 \cdot \pi \cdot v \cdot t}{l_d} \right) \right] = q_0 \left[1 - \cos(\omega_d \cdot t) \right], \qquad (11.55)$$

where $\omega_d = \dfrac{2 \cdot \pi \cdot v}{l_d}$ is the frequency of disturbances (the frequency of road irregularities);

t is the time of the vehicle's movement.

11.9.2. Oscillatory model of forced oscillations of the vehicle

To study the forced oscillations of a two-axle vehicle in which $\varepsilon_y = 1$, that is, there is no connection between the vibrations of the suspensions , a model has been developed (see Fig. 11.15).

At the same time, assumptions are made that there is no connection between the oscillations of the sprung and unsprung masses.

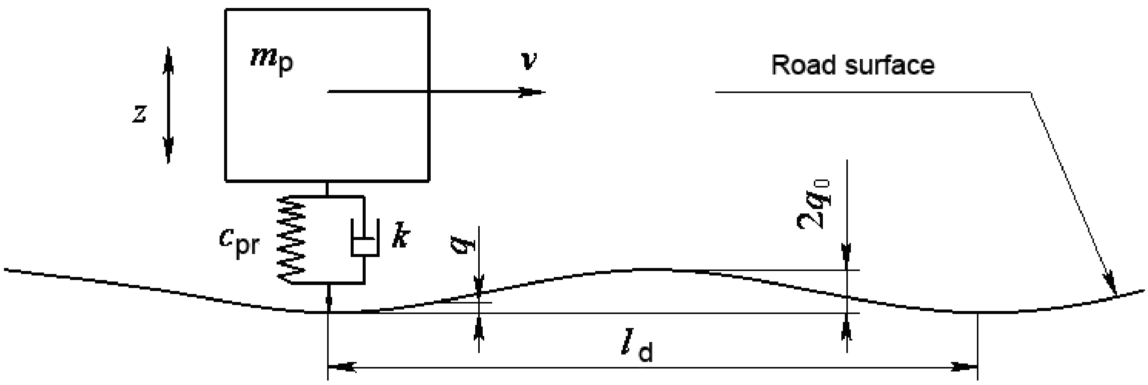


Fig . 11.1 5 . **Scheme of the oscillatory model of forced oscillations vehicle**

The equation of motion of a sprung mass during forced oscillations

$$m_p \cdot \ddot{z} + k\left(\dot{z} - \dot{q}\right) + c_{pr}\left(z - q\right) = 0, \tag{11.56}$$

substitute the value of q into equation (11.56) and open the brackets

$$m_p \cdot \ddot{z} + k \cdot \dot{z} - k\left\{q_0\left[1-\cos\left(\omega_d \cdot t\right)\right]\right\}' + c_{pr} \cdot \left(z - q_0\left[1-\cos\left(\omega_d \cdot t\right)\right]\right) = 0, \tag{11.57}$$

take the derivative of the third component, divide it by m_p and transfer the components of the equation containing ω_d to the right-hand side

$$\ddot{z} + \frac{k}{m_p} \cdot \dot{z} + \frac{c_{pr}}{m_p} \cdot z = \frac{k}{m_p} q_0 \cdot \omega_d \cdot \sin\left(\omega_d \cdot t\right) + \frac{c_{pr}}{m_p} \cdot q_0\left[1-\cos\left(\omega_d \cdot t\right)\right]. \tag{11.58}$$

In equation (11.58), we replace $\dfrac{k}{m_p} = 2h_0$ and $\dfrac{c_{pr}}{m_p} = \omega_0^2$.

$$\ddot{z} + 2h_0 \cdot \dot{z} + \omega_0 \cdot z = 2h_0 \cdot q_0 \cdot \omega_д \cdot \sin\left(\omega_d t\right) + \omega_0^2 \cdot q_0\left[1-\cos\left(\omega_d t\right)\right]. \tag{11.59}$$

In the right-hand side of equation (11.59) we put it in parentheses $\omega_0^2 \cdot q_0$

$$\ddot{z} + 2h_0 \cdot \dot{z} + \omega_0 \cdot z = \underbrace{\omega_0^2 \cdot q_0\left[\frac{2h_0 \cdot \omega_d}{\omega_0^2}\sin\left(\omega_d \cdot t\right) + 1 - \cos\left(\omega_d \cdot t\right)\right]}_{\text{acceleration of the stimulating effect}}. \tag{11.60}$$

Each component in the left-hand side of the equation and its right-hand side represent partial accelerations. The first term is the acceleration of the sprung mass due to the force of inertia, the second term is the resistance force of the shock absorber, the third term is the force of the elastic element, and the right side is the acceleration due to the disturbing force from the road.

Equation (11.60) is a linear inhomogeneous differential equation of the oscillations of the sprung mass of the vehicle under the action of a disturbing force from the side of the road during uniform movement over road irregularities. The general solution of this equation is known (11.61), which is the sum of the solution of the homogeneous equation A (for the case when the right-hand side of the equation is zero) and the partial solution B.

$$z = \underbrace{\left[c_1 \cdot \sin\left(\omega_h \cdot t\right) + c_2 \cdot \cos\left(\omega_h \cdot t\right)\right] \cdot e^{-h_0 t}}_{\text{A}} \underbrace{+ q_0 + z_\text{a} \cdot \sin\left(\omega_\text{d} \cdot t + \varphi_v\right)}_{\text{Б}}, \quad (11.61)$$

where c_1^*, c_2^* – permanent integration;

$\omega_h = \omega_0^2 \sqrt{1 - \psi_{q0}^2}$ – the frequency of damping oscillations of the sprung mass;

$z_\text{a} = q_0 \cdot \omega_0 \sqrt{\dfrac{\psi_{q0}^2 \cdot \omega_\text{d}^2 + \omega_0^2}{\left(\omega_0^2 - \omega_\text{d}^2\right)^2 + \psi_{q0}^2 \cdot \omega_0^2 \cdot \omega_\text{d}^2}}$ – amplitude of forced oscillations;

φ_v is the phase angle of forced oscillations.

In equation (11.61), the first component A is called free accompanying oscillation, the second component B is called steady oscillation. After a time interval, component A can be neglected and equation (11.61) takes the form

$$z = q_0 + z_\text{a} \cdot \sin\left(\omega_\text{d} \cdot t + \varphi_v\right). \quad (11.62)$$

Conclusions from equation (11.62):

1 – steady forced oscillations occur with the frequency of forced oscillations (they do not depend on whether there are inelastic supports in the system and what their value is);

2 – the amplitude of constant oscillations does not depend on time and initial conditions. Over time, they do not fade in the presence of inelastic resistances;

3 – at a constant value of ψ_{q0} and ω_0, the amplitude of constant forced oscillations z_a of the sprung mass depends on the ratio between the natural frequency ω_0 and the frequency of forced oscillations ω_h;

4 – for constant forced oscillations in the presence of inelastic resistances, a phase shift of displacement and forced force is characteristic.

11.9.3. Amplitude-frequency characteristic of forced oscillations of the vehicle

Amplitude-frequency response (frequency response) is the dependence of amplitude values of displacements, vibration velocities and vibration accelerations on the frequency of forced oscillations. With the help of frequency response, it is convenient to analyze the smoothness of the vehicle when it moves at different speeds on roads with different unevenness parameters.

In the amplitude-frequency characteristic (Fig. 11.16), it is more convenient to use relative values:

– relative amplitude of the forced oscillations of the sprung mass

$$\frac{z}{q_0} = 1 + \frac{z_0}{q_0} \cdot \sin\left(\omega_\text{d} \cdot t + \varphi_\text{p}\right); \tag{11.63}$$

– relative vibration speed of the sprung mass

$$\frac{\dot{z}}{q_0} = \frac{z_0}{q_0} \cdot \cos\left(\omega_\text{d} \cdot t + \varphi_\text{p}\right); \tag{11.64}$$

– relative vibration acceleration of the sprung mass

$$\frac{\ddot{z}}{q_0} = -\frac{z_0}{q_0} \cdot \omega_\text{d}^2 \cdot \sin\left(\omega_\text{d} \cdot t + \varphi_\text{p}\right). \tag{11.65}$$

The frequency response of the truck shown in Figure 11.16 contains the dependences of the relative amplitude of the forced oscillations z / q_0, the relative vibration acceleration $\ddot{z}/q_0$ of the sprung mass and the relative amplitude of ξ/q_0 the unsprung mass on the frequency of the forced oscillations ω_d. The frequency of forced oscillations ω_d is determined by the height of road irregularities q_0 and the speed of the vehicle. The numerical values of frequency response in Figure 11.16 correspond to the parameters of the truck.

Five zones can be distinguished on the amplitude-frequency characteristic of the vehicle.

The pre-resonance zone of oscillations characterizes oscillations when the vehicle is moving at low speeds on roads with long bumps. In this case, the relative amplitude of vibrations of the body and wheels is insignificant, and the relative accelerations of the body are small. The absolute values of these parameters depend on the height of road bumps.

The low-frequency resonance zone occurs when the length of road bumps decreases at a constant speed (or when the speed of the vehicle increases when driving on a road with a given length of bumps). This leads to an increase in the frequency of forced oscillations. The less the frequency of forced oscillations differs from the natural frequency of oscillations of the body, the greater the amplitude of oscillations of the body. In turn, an increase in the amplitude of vibrations of the body due to the suspension causes an increase in the amplitude of vibrations of the wheels.

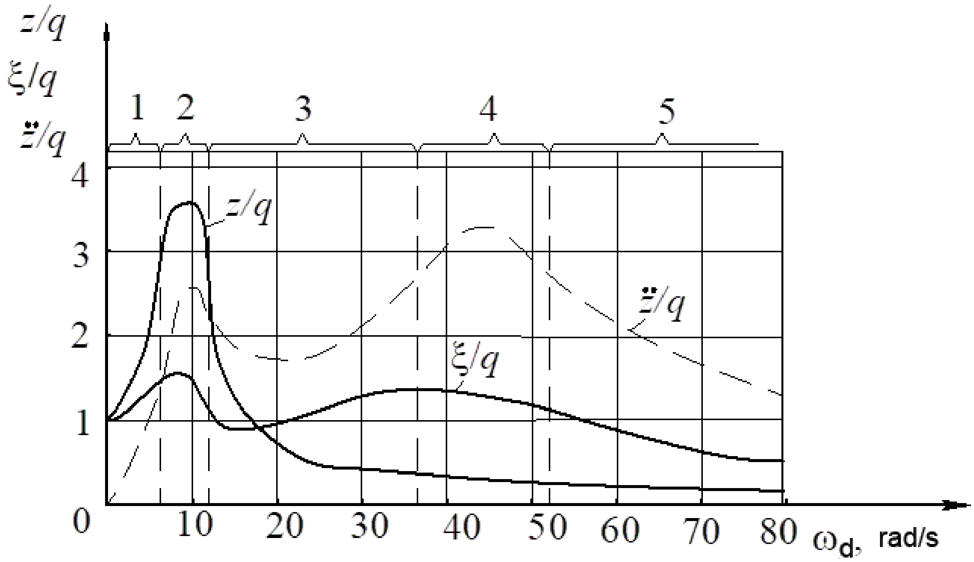


Fig. 11.16. **Amplitude-frequency characteristic of cargo vehicle:**
1 – pre- resonance zone; 2 – zone of low-frequency resonance; 3 – interresonance zone; 4 – zone of high-frequency resonance; 5 – resonant zone

The inter-resonance zone is characterized by a decrease in the amplitudes of vibrations of the body and wheels, as well as a decrease in vibration accelerations of the body compared to the region of low-frequency resonance.

The zone of high-frequency resonance occurs in the case of a significant increase in the frequency of forced oscillations. In this zone , the frequency of forced oscillations approaches the values of the natural frequency of oscillations of unsprung masses, which causes an increase in the amplitude of their oscillations. When the frequency of forced oscillations increases, the difference between it and the natural frequency of oscillations of the body increases, which causes a decrease in the amplitude of its oscillations. That is, the high-frequency resonance zone is characterized by a decrease in the amplitude of vibrations of the body and an increase in the amplitude of vibrations of the wheels. In turn, significant movements of the wheels lead to an increase in vibration accelerations of the body.

In the post-resonance zone and frequency response, there is a decrease in the amplitudes of body and wheel movements, as well as vibration accelerations of the body compared to the high-frequency resonance zone , which is caused by an increase in the difference between the forced and natural frequencies of oscillations.

As a result of the frequency response analysis, it is possible to determine the rational speed of the vehicle on the road with a given length of bumps, at which the smoothness of the vehicle will be comfortable. This is achieved when the vehicle is moving at a speed at which the vibrations of the body and wheels will not occur in the resonance zones.

Control questions

1. What is the smoothness of the vehicle?
2. Name the vehicle smoothness meters.
3. What parameters of the suspension depend on the frequency of oscillations of the sprung mass of the vehicle?
4. What frequency of oscillations of the sprung mass of the vehicle is the most comfortable for the human body? At what value of the static stroke of the suspension is this frequency ensured?
5. What is the reduced stiffness of the suspension and how to determine it?
6. Under what condition will the circuit of the three-mass model of oscillations of the sprung mass of the vehicle turn into a two-mass one?

7. What is the phenomenon of vehicle galloping and under what condition does this phenomenon not occur?

8. Why was the concept of the relative damping coefficient of the suspension introduced, what does it characterize and what are the approximate values of this coefficient?

9. What parameters of the oscillatory process are affected by the presence of an inelastic resistance (shock absorber) in the suspension?

9. What characterizes the coefficient of inelastic resistance of the shock absorber and the coefficient of inelastic resistance of the suspension?

10. Draw a diagram of the oscillating system of the vehicle taking into account the oscillations of unsprung masses and taking into account the inelastic resistance in the suspension.

11. Name the relative indicators of assessment of forced oscillations of the vehicle.

12. How to evaluate the smoothness of a vehicle's movement based on the amplitude-frequency characteristic?

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АВТОМОБІЛЬ. ТЕОРІЯ ТА ЕКСПЛУАТАЦІЙНІ ВЛАСТИВОСТІ

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