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Physical foundations of vehicle stability when moving uphill and at longitudinal roll back

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Abstract. Problem. *The determination of the stages of stability for vehicles as technical systems when moving uphill on an inclined section of the road and their possible longitudinal roll back is considered. The considered stages ensure the safe operation of vehicles in various conditions and with various combinations of the values of the parameters that are included in the proposed equations characterizing the stability of the vehicle.* ***Goal.*** *The purpose of the paper is to improve the method of researching the physical foundations of the stability of vehicles when moving uphill on an inclined plane, the formation of mathematical models, which allows to significantly improve the mathematical support of the methods of calculating the stability of two-axle vehicles (cars, trucks, tractors, special and specialized vehicles, etc.)* ***Methodology.*** *The approaches adopted in the work to achieve the set goal are based on the classical laws of the mechanics of free-standing or moving objects, namely: the principle of possible movements, where calculated dynamic and mathematical models with their solutions are able to preserve the given conditions and parameters, and characterize the state of stability of the vehicle in case of its longitudinal roll back.* ***Results.*** *It was established and confirmed by a numerical examples that the longitudinal stability of a vehicle moving uphill depends on the longitudinal road slope, the vehicle design, the elasticity of the road surface, the critical overturning angle of the vehicle and its weight parameters.* ***Originality.*** *The proposed mathematical model of the longitudinal stability of the vehicle when moving uphill showed that its solution significantly depends on the values of the geometric parameters of the uphill and the vehicle, and also depends on the position of the center of gravity of the vehicle relative to the surface of the road surface.* ***Practical value****. The obtained equations for determining the values of the stability parameters of the vehicle when moving uphill on an inclined road section make it possible to specify the parameters for ensuring the logistics of vehicles in the conditions of their operation on uphill.*

Key words: *stability, longitudinal stability, moving uphill, critical overturning angle, longitudinally overturning, center of gravity..*

Introduction

In the physical sense, the stability of the vehicle when moving uphill is the ability of the system (vehicle) to return to its previous position (maintain the direction of movement, resist overturning) after it (he) has been removed from this state by any internal or external acting forces or factors. The main sign of loss of stability is overturning of the vehicle, for example when driving on a steep incline, where the overturning occurs in the longitudinal plane.

In this case, the front wheels lose contact with the road surface (there are no vertical reactions on the wheels of the front axle) [1–5]. Since the vehicle is free-standing or free-moving on the rise, its stability against overturning is provided only by its own weight, if the effect of the wind on the vehicle is not taken into account. In this case, we consider the stability of the vehicle when moving uphill (longitudinal overturning) based on the design solutions of models, namely, dynamic and mathematical.

Analysis of publications

When analyzing literary sources, it was established that for most modern vehicles with a low center of gravity, rollover in the longitudinal plane is unlikely, rollover around the wheel axes is possible only in vehicles with a very short base and a high center of gravity. In practice, overturning can occur, for example, as a result of a shift of the load on the loading platform back [3 - 7]. However, stability against overturning in the longitudinal plane is one of the properties of stability of movement, because when the stability of the position is lost, the movement of the vehicle or tractor is not possible [8, 9]. In these works, the maximum angle of elevation that vehicles are capable of overcoming is studied.

Thus, in [10], it is proposed to improve the longitudinal and transverse stability of a vehicle with a high center of gravity by installing an air suspension, which, when the transverse position of the center of gravity changes sharply and is close to the critical value of the center of the body, the electrovalve of the receiver is activated, as a result of which automatic filling with compressed with the air of the pneumatic cylinder of the suspension from the side of the uphill.

In work [11], a tractor with a variable base was developed, for example, where increased stability is required, the base is set to the largest, and for work on small areas, where a minimum turning radius is required, its base is set to the smallest.

In [12], the loss of stability of a dump truck when unloaded on a platform with longitudinal and transverse slopes is considered, where the maximum allowable angle of inclination of the unloading platform is determined, the obtained results can be useful for evaluating the stability of dump trucks.

Thus, in works [13, 14], depending on the orientation, the angle of longitudinal and transverse stability was determined using a stand and the coordinate of the center of gravity was determined by the method of axial and on-board weighing on automobile scales, and as a result, methods of increasing stability by lowering the center of gravity were proposed and reduction of road clearance, and optimal placement of cargo.

The article [15] analyzes the stability of the unified dynamic model of the vehicle using Matlab/Simulink under various operating conditions, which allows for a more objective assessment of the stability of various control systems developed for the vehicle chassis.

The works [16-18] consider theoretical methods of determining the longitudinal coordinate of the location of the center of gravity based on statistical reference data of various types of vehicles.

After analyzing the noted works and studies conducted in the field of stability of the considered vehicles, only static methods of determining stability were used or taking into account dynamic loads according to their static equivalent. In this article, the authors do not pretend to be a complete study of sustainability phenomena, and consider sustainability as a technical system. The main methods used in this work in the study of the stability of vehicles, as free-standing or those moving at a constant speed, are analytical methods of analyzing mechanical systems.

Purpose and Tasks

The purpose of the work is to improve the methods of researching the physical foundations of the stability of vehicles when moving uphill on an inclined plane, the formation of mathematical models, which should allow to significantly improve and improve the mathematical support of the methods of calculating the stability of two-axle vehicles, such as a car, truck, tractor, special or specialized vehicle, etc.

To achieve the goal, it is necessary to complete the following tasks:

1) Consider the dynamic model overturning of vehicle;

2) Consider the scheme for determining the angle of elastic inclination of the vehicle;

3) Consider the scheme for determining the critical angle of inclination of the vehicle body;

4) To obtain analytical equations characterizing the stability of the vehicle when driving uphill.

A dynamic model of a vehicle and a method for determining the angle of an elastic inclination of a vehicle

When the vehicle negotiates a steep grade, the rollback can occur around a line passing through the centerline of the rear wheels, point B (see Fig. 1). At the same time, the vehicle changes its position with the movement of the center of gravity in height and up the inclined plane of the road. We believe that the reaction of the road R_A to the front wheels is initially there, and at the moment of separation from the support (point *A)* and longitudinally overturning is zero, and it can be ignored.

The vehicle is in motion on a rise with a road angle α_d and rests on the supports: the rear support, point *B* and the front support, point *A* with reduced stiffness C_n.

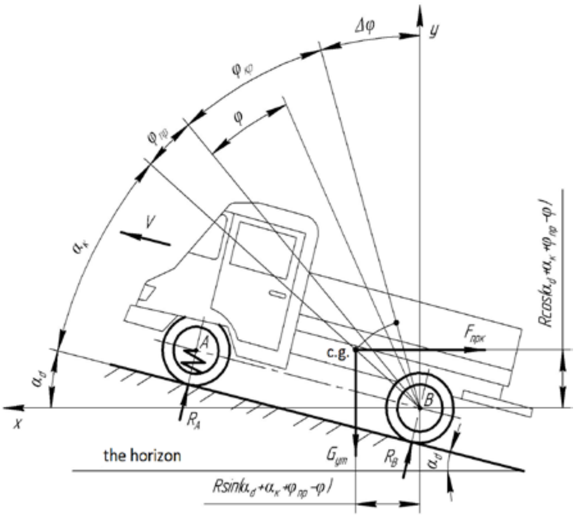

Fig. 1. Dynamic model of vehicle overturning

In Figure 1, the following notations are adopted: *c.g.* – center of gravity of the vehicle; $G_{ут}$ – weight of the vehicle, $F_{прк}$ – brought to the center of gravity overturning force, which is the action of the forces of the elastic support of the uphill (point A) and the air brought to the center of gravity; α_d – angle of inclination of the road to the horizontal; $\alpha_к$ – the angle of the constructive location of the center of gravity; $\varphi_{пр}$ – the angle of elastic inclination of the system to separation from support *A*; φ – the current angle of rotation, which can vary from the elastic angle to the critical angle

$$\varphi_{пр} \le \varphi \le \varphi_{кр} ; \quad (1)$$

where $\varphi_{кр}$ – critical angle, from $\varphi_{пр}$ to the position where the system will be in a state of unstable equilibrium; $\Delta\varphi$ – the angle between the critical angle and the vertical plane passing through the overturning edge (point *B*);

$$R\sin(\alpha_d + \alpha_к + \varphi_{пр} - \varphi) ; \quad (2)$$

$$R\cos(\alpha_d + \alpha_к + \varphi_{пр} - \varphi) ; \quad (3)$$

arms of the acting forces, relative to the axis of rotation; *A* – elastic support of the road surface.

Mathematical model and analysis of its solution

To determine the mathematical model, the principle of possible displacements was used, namely, if a system in equilibrium receives a possible displacement, then the total work of all forces on this displacement is zero [19].

$$\sum_{i=1}^{N} \left(F_i - m_i \ddot{z}_i\right)\delta z_i = 0 . \quad (4)$$

Accordingly, the condition of stability of the vehicle during longitudinal overturning is taken as

$$A_{ут.\varphi кр} \ge A_{прк.\varphi} , \quad (5)$$

where $A_{ут.\varphi кр}$ – work of restraining forces when turning the vehicle to a critical angle (energy reserve), relative to the overturning edge (point *B*).

The work of the restraining forces

$$A_{ут.\varphi кр} = \int_0^{\varphi_{кр}} G_{ут} \cdot R \cdot \sin(\alpha_d + \alpha_к + \varphi_{пр} - \varphi)d\varphi =$$
$$= G_{ут} \cdot R[\cos\left(\alpha_d + \alpha_к + \varphi_{пр} - \varphi_{кр}\right) -$$
$$-\cos\left(\alpha_d + \alpha_к + \varphi_{пр}\right)]. \quad (6)$$

Schematically, the work of the retaining forces depending on the angle of inclination of the system in the form of a graph (see Fig. 2).

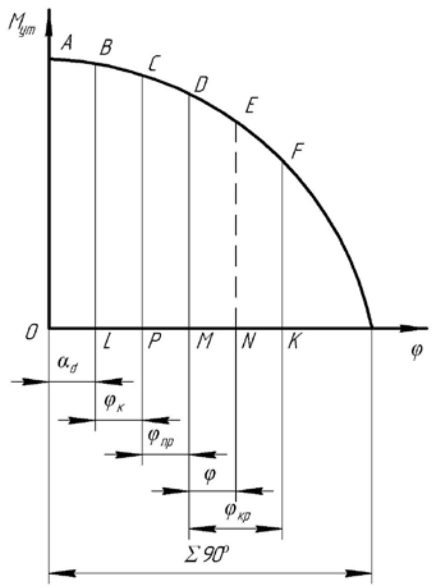

Fig. 2. The graph of changes in the work of forces when the angle changes φ

The graph shows: area *OABL* – conditional work of restraining forces when turning the vehicle to the corner of the road; area *LBCP* – work of restraining forces when turning the vehicle to a constructive angle; *PCDM* area is the work of the restraining forces when turning the vehicle at a corner φ_{np}; *MDFK* area – the work of the restraining forces when turning the vehicle at a corner $\varphi_{\kappa p}$; area *MDEN* is the work of overturning forces when the vehicle turns a corner φ.

At the same time, the work of overturning forces when turning a corner φ is equal to the work of the holding forces when turning at an angle φ

$$A_{np\kappa.}=\int_0^{\varphi} F_{np\kappa}\cdot R\cdot\cos(\alpha_d+\alpha_\kappa+\varphi_{np}-\varphi)d\varphi\,, \quad (7)$$

$$A_{np\kappa.(\alpha_d+\varphi)}=\int_0^{\varphi} G_{ym}\cdot R\cdot\sin\left(\alpha_d+\alpha_\kappa+\varphi_{np}-\varphi\right)d\varphi\,, \quad (8)$$

$$A_{np\kappa.(\alpha_d+\varphi)}=G_{ym}\cdot R\cdot[\cos\left(\alpha_d+\alpha_\kappa+\varphi_{np}-\varphi\right)- -\cos\left(\alpha_d+\alpha_\kappa+\varphi_{np}\right)]\,. \quad (9)$$

Substituting equations (6) and (9) into (5), we get the following equation in the form of an inequality (equality).

$$\cos\left(\alpha_d+\alpha_\kappa+\varphi_{np}-\varphi_{\kappa p}\right)\geq \geq\cos\left(\alpha_d+\alpha_\kappa+\varphi_{np}-\varphi\right). \quad (10)$$

We will analyze the solution of the mathematical model:

1. If $\varphi_{\kappa p}=\varphi_{np}$, $\varphi=\varphi_{np}-\Delta\varphi_1$, then,

$$\cos\left(\alpha_d+\alpha_\kappa+\varphi_{np}-\varphi_{\kappa p}\right)\geq \geq\cos\left(\alpha_d+\alpha_\kappa+\varphi_{np}-\varphi_{np}+\Delta\varphi_1\right); \quad (11)$$

$$\cos\left(\alpha_d+\alpha_\kappa\right)>\cos\left(\alpha_d+\alpha_\kappa+\Delta\varphi_1\right) \quad (12)$$

the system is stable;

2. If $\varphi_{\kappa p}=\varphi_{np}$, $\varphi=\varphi_{np}$, then,

$$\cos\left(\alpha_d+\alpha_\kappa+\varphi_{np}-\varphi_{np}\right)\geq \geq\cos\left(\alpha_d+\alpha_\kappa+\varphi_{np}-\varphi_{np}\right); \quad (13)$$

$$\cos\left(\alpha_d+\alpha_\kappa\right)=\cos\left(\alpha_d+\alpha_\kappa\right) \quad (14)$$

the system is in a state of unstable equilibrium;

3. If $\varphi_{\kappa p}=\varphi_{np}+\Delta\varphi_2$, $\varphi=\varphi_{\kappa p}-\Delta\varphi_3$, then,

$$\cos\left(\alpha_d+\alpha_\kappa+\varphi_{np}-\varphi_{np}-\Delta\varphi_2\right)> >\cos\left(\alpha_d+\alpha_\kappa+\varphi_{np}-\varphi_{np}-\Delta\varphi_2+\Delta\varphi_3\right). \quad (15)$$

$$\cos\left(\alpha_d+\alpha_\kappa-\Delta\varphi_2\right)> >\cos\left(\alpha_d+\alpha_\kappa-\Delta\varphi_2+\Delta\varphi_3\right) \quad (16)$$

the system is stable, but there is a gap between the support and the base (point *A*);

4. If $\varphi_{\kappa p}=\alpha_d+\alpha_\kappa+\varphi_{np}$, $\varphi=\alpha_d+\alpha_\kappa+\varphi_{np}$, then,

$$\cos\left(\alpha_d+\alpha_\kappa+\varphi_{np}-\varphi_{np}\right)= =\cos\left(\alpha_d+\alpha_\kappa+\varphi_{np}-\varphi_{np}\right); \quad (17)$$

the system is in a state of unstable equilibrium at support *B* with a gap in support *A* (movement of the vehicle at the limit of possibilities);

Thus, we were convinced that the mathematical model is legitimate, and its solutions provide clear and rational solutions depending on the component angles: elasticity, critical, actual, taking into account the position of the center of gravity of the vehicle relative to the surface of the road surface with which it is in contact.

As an example, we determine the values of the angles included in the solution of the mathematical model.

Since the deformations of the supports depend on the loads depending on the inclination of the vehicle, the angle of elasticity is determined with the known wheel base *K*, the weight of the vehicle G_{ym}, and measured variable deformations δ_1, supports A and δ_2 supports *B* (see Fig. 3), as the sum of deformations related to the wheel base *K*

$$\frac{\delta_1+\delta_2}{K}=\tan\left(\varphi_{np}\right), \quad (18)$$

or

$$\varphi_{np}=\mathrm{arctg}\left(\frac{\delta_1+\delta_2}{K}\right). \quad (19)$$

The maximum angle of elastic inclination

$$\varphi_{np}=\left(\frac{0,05+0,05}{2}\right)=\mathrm{arctg}\left(0,05\right),$$

where the deformations of the supports are equal to 0.05 m, and the track K = 2 m.

$$\varphi_{пр} = 2°59'.$$

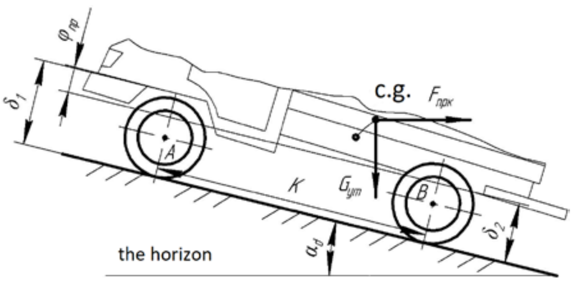

Fig. 3. The scheme for determining the angle of elastic inclination

The angle α_d is determined by direct measurement, relative to the horizon, of the weight of the vehicle (see Fig. 4).

The angle of inclination of the road (see Fig. 4) can be different at the same time in different places. Based on the data [3], where the value of the slope is given from 0° to 15° on a dry road, then, depending on the engaged gear, up to 40°, we tentatively accept α_d = 20°.

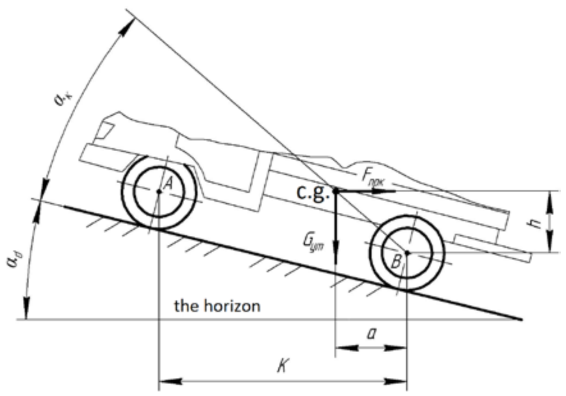

Fig. 4. The scheme for determining the angle $\alpha_к$

The angle $\alpha_к$ is determined on a special stand, or roughly according to the following method. We take the height of the center of mass from the center of the rear wheels h = 0.5 m, and the distance for a horizontal vehicle a = 1.5 m.
Then the angle $\alpha_к$ can be determined from the ratio

$$\tan(\alpha_к) = \frac{h}{a}. \tag{20}$$

where a – the distance from support B to the center of gravity of the vehicle; h – the height of the center of gravity of the vehicle.

$$\tan(\alpha_к) = \frac{0,5}{1,5} = 0,333.$$

or

$$\alpha_к = 18°26'.$$

To determine the critical angle $\alpha_{кр}$,, we consider the scheme of the vehicle with forces brought to the center of gravity of the vehicle, when their net effect passes through the overturning edge (see Fig. 5).

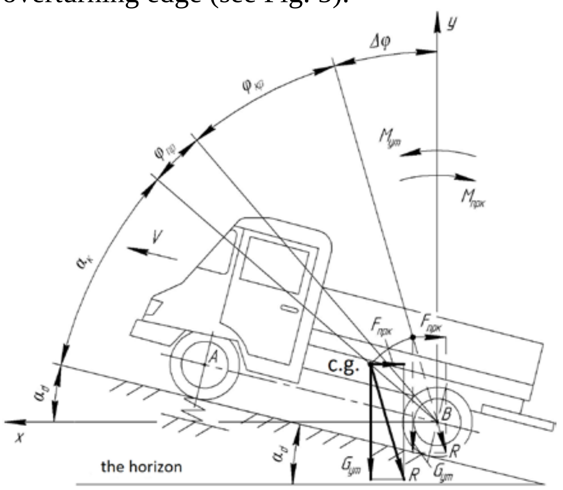

Fig. 5. The scheme of determining the critical angle

The equation for determining the critical angle will look like this

$$\varphi_{кр} = \alpha_d + \alpha_к + \varphi_{пр} - \operatorname{arctg}\left(\frac{F_{прк}}{G_{ут}}\right). \tag{21}$$

where $G_{ут}$ = 30 kN – holding power (vehicle weight); $F_{прк}$ = 10 kN – longitudinally overturning force.

$$\varphi_{кр} = 20° + 18°26' + 2°59' - 18°26' = 22°59'.$$

To determine the current angle of inclination, we will use the calculation scheme (see Fig. 5), on which moments act: clockwise - the moment of overturning forces and the moment of inertia $M_{ут}$, and anti-clockwise moment of restraint and elasticity $M_{прк}$.

Then, on the basis of the principle of possible movements, we obtained the equation of oscillations in the form

$$I_{пр}\frac{d^2\varphi}{dt^2} + C_n\varphi = M_{прк}. \tag{22}$$

where I_{np} – moment of inertia of the vehicle according to fig. 5; φ – the angle of inclination to separation from the left support; $M_{npк}$ – total overturning moment; C_n – the angular stiffness of the system is given;

The solution of equation (22) with the replacement of the overturning moment by the holding moment is obtained in the form

$$\varphi = \frac{M_{ym}}{C_n}\left(1-\cos\left(pt\right)\right), \qquad (23)$$

where p – system oscillation frequency.

Because

$$0 \le \cos\left(pt\right) \le 1, \qquad (24)$$

to separation from the left support A, the overturning moment is variable

$$0 \le \varphi \le \frac{2M_{ym}}{C_n}, \qquad (25)$$

or the current angle φ (see Fig. 5)

$$\varphi = \frac{F_{npк}\cdot R}{C_n}, \qquad (26)$$

where $F_{npк}$ = 10 kN – overturning force, R = 2 m, C_n =123,5 kNm/deg;

At the same time, the radius of rotation of the center of gravity will be determined from the dependence

$$R = \sqrt{a^2 + h^2}, \qquad (27)$$

$$R = \sqrt{1,5^2 + 0,5^2} = 1,58 \text{ м.}$$

Elasticity of support A

$$C_n = \frac{M_{ym}}{\varphi_{np}}, \qquad (28)$$

$$C_n = \frac{10\cdot 1,58}{2°52'} = 5,45 \text{ kNm/deg.}$$

After the separation of the wheels of the vehicle in support A from the surface of the road surface, the vehicle remains resistant to cornering $\varphi_{кр}$, or

$$\varphi_{np} \le \varphi \le \varphi_{кр}, \qquad (29)$$

and if there is $\varphi > \varphi_{кр}$, then the system loses stability: $2°52' \le \varphi \le 22°52'$.

Analysis of the impact of variations in the structural, power and other parameters of vehicles and proposals for their rationality and optimization were not the purpose of this work and will be considered in another work.

Conclusion

Based on the conducted research, the following conclusions can be drawn:

1. Analytical dependences and values of parameters characterizing, in accordance with the physical foundations, the stability of vehicles when they move uphill and possible unwanted longitudinal overturning were obtained.
2. It was established and confirmed that the longitudinal stability of vehicles moving uphill depends on the magnitude of the values of the angles: the longitudinal road, the design of the vehicle, the elasticity of the base, the critical angle, the current angle and the weight of the vehicle.
3. The proposed mathematical model of longitudinal stability and its solution are dependent on the component values of the angles and the structural positions of the vehicle's center of gravity relative to the road surface.
4. Longitudinal stability of the vehicle is lost when moving uphill when the angle of inclination of the vehicle exceeds the value of the critical angle.

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Фізичні основи стійкості транспортного засобу при русі на підйом та при повздовжньому перекиданні назад

Анотація. Проблема. *Розглянуто визначення етапів стійкості транспортних засобів, як технічних систем, при русі на підйом на похилій ділянці дороги та можливому повздовжньому перекиданні, що* *забезпечує безпечну їх експлуатацію в різних умовах та комбінаціях величини параметрів, які входять та дотичні, запропоновані залежності, що характеризують стан їх стійкості.* ***Мета.*** *Мета роботи полягає у вдосконаленні методів дослідження фізичних основ стійкості транспортних засобів при русі на підйом на похилій площині, формування математичних моделей, що дозволяє суттєво покращити і вдосконалити математичне забезпечення методів розрахунків стійкості таких транспортних засобів, як двоосний автомобіль, трактор, автокран, автовишка і т.п.* ***Методологія.*** *Прийняті в роботі підходи досягнення поставленої мети базуються на класичних законах механіки вільностоячих або рухаючих об'єктів, а саме: принципу можливих переміщень, де розрахункові динамічні та математичні моделі із їх рішеннями, здатні зберігати задані умови та параметри, та їх основні характеризувати стан стійкості при повздовжньому перекиданні назад.* ***Результати.*** *Встановлено та підтверджено на чисельному прикладі, що повздовжня стійкість транспортних засобів, які рухаються на підйом, залежить від величини значень кутів: повздовжнього дороги, конструкції засобу, пружності основи, критичного, поточного та величини ваги транспортного засобу.* ***Оригінальність.*** *Запропонована математична модель повздовжньої стійкості транспортних засобів при русі на підйом та її рішення є залежними від складових значень кутів та положень центра мас відносно їх опор.* ***Практичне значення****. Отримані математичні залежності для визначення величин параметрів стійкості транспортних засобів при русі вгору на похилій ділянці дороги, а саме: кутів нахилу дороги, розміщення центру мас засобу, пружного, поточного та критичного, що забезпечують та покращують логістику транспортних засобів в умовах експлуатації.*

Ключові слова: *стійкість, поздовжня стійкість, рух в гору, критичний кут перекидання, поздовжнє перекидання, центр ваги.*

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