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Determination of the possibility of an automobile drive wheel slip

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Abstract. Problem. *The disadvantage of the existing dependencies for determining the possibility of the automobile drive wheel slip with the maximum use of engine power and indicators of the interaction of the drive wheels with the supporting surface is that they are correct only if the motion of the car is steady. This downside is due to the fact that when deriving the equation of the car motion, the assumption is made that the total tractive force is on the drive wheels, that is, without taking into account the energy losses due to the increase in the kinetic energy of the rotating masses of the engine, transmission, and drive wheels. At the same time, the specified energy losses are compensated by a relative increase in the inertia (mass) of the gradually moving car. Such an assumption is quite correct for determining the traction and speed characteristics of a car not only during steady motion but also during acceleration. Determining the possibility of the automobile drive wheel slip during automobile acceleration, this assumption is incorrect because the drive wheels are not affected by the total tractive force, but by a force that is reduced in accordance with energy losses due to an increase in the kinetic energy of the rotating masses of the engine, transmission, and driving wheels. It is necessary to take into account the reduction of the tractive force on the drive wheels during the acceleration of the car in order to eliminate the mentioned shortcoming.* ***Goal.*** *The aim is the further development of the theory of the car by improving the dependencies that allow determining the conditions for the possibility of the automobile drive wheel slip during its acceleration and the modes of its motion.* ***Methodology.*** *The considered approaches for achieving this aim are based on the laws of physics, theoretical mechanics and provisions of the theory of the car.* ***Results.*** *Analytical dependencies have been improved to determine the conditions for the possibility of the automobile drive wheel slip during its acceleration and the modes of its motion. Dependencies for determining the range of the automobile drive wheel slip on the speed of the vehicle during acceleration are obtained. During the theoretical study of the car motion process, it was established that the developed dependencies allow determining the car driving mode and the possibility of the automobile drive wheel slip both during acceleration and steady motion.* ***Originality.*** *The obtained dependences for determining the possibility of the automobile drive wheel slip during acceleration of the car made it possible to clarify the idea of the driving mode of the car and the possibility of the automobile drive wheel slip both during acceleration and during steady motion.* ***Practical value.*** *The obtained dependencies can be used in the design of new and improved sports cars such as dragsters, and for analyzing the dynamics of the car motion during acceleration with total fuel supply and determining the nature of the interaction of the drive wheels with the supporting surface depending on the speed of motion.*

Key words: *automobile (car), equations of motion, dynamic factor, dynamic factor by adhesion, acceleration, total tractive force, coefficient of adhesion, rotating masses.*

Introduction

Tractive and velocity characteristics of the car determine the ranges of changes in driving speeds and maximum acceleration during acceleration concerning traction control. In traction control, the power and torque necessary for motion are supplied from the engine to the drive wheels using the transmission. When the car is moving, the traction force on the drive wheels depends on the engine torque, transmission parameters, wheels and driving mode.

The indicators of car traction and speed characteristics characterize the perfection of its design and make it possible to determine the possibility and mode of motion due to the road conditions. Drive wheel slip significantly affects the indicators of its traction and speed characteristics.

Analysis of publications on determining the possibility of an automobile drive wheel slip

Automobile drive wheel slip has a significant effect on the traction and speed characteristics of the automobile. One often uses the differential equation of its motion, directed to its longitudinal axis so as to analyze the traction and speed characteristics of an automobile [1, 3, 4, 6, 10, 14].

$$m_\text{а}\frac{d^2x}{dt^2}=\frac{G_\text{а}}{g}\cdot j_\text{а}=P_j^{'}=P_\text{т}-P_\text{в}-P_\psi\,, \qquad (1)$$

where $m_\text{а}$ – automobile weight; $\frac{d^2x}{dt^2}=j_\text{а}$ – automobile acceleration; $G_\text{а}$ – gravitational force of an automobile; g – gravitational acceleration; $P_j^{'}$ – inertial force of gradually moving automobile weight; $P_\text{т}$ – tractive force on drive wheels; $P_\text{в}$ – the force of air resistance; P_ψ – rolling resistance.

The solution of the differential equation (1) is possible by a numerical method. We should note that the determination of the tractive force $P_\text{т}$ requires a significant amount of calculations of parameters of the dynamic processes of the engine and transmission.

This fact is because the tractive force $P_\text{т}$ on the driving wheels depends on the driving mode of the car. When an automobile is accelerating, the engine operates in transient modes, which complicates calculations for determining its initial parameters [3, 9]. In addition, in this case, engine energy is spent not only on increasing the energy of the gradually moving automobile mass, but also on rotating parts of the engine, transmission and drive wheels. As a result, the tractive force $P_\text{т}$ of the automobile during its acceleration is less by the amount determined by the energy losses due to the increase in the angular velocity of the rotating masses.

$$P_\text{т}=\frac{M_e\cdot k_p\cdot u_\text{тр}\cdot\eta_\text{тр}}{r_\text{д}}-\frac{J_e\cdot u_\text{тр}^2\cdot\eta_\text{тр}+J_\text{к}}{r_\text{д}\cdot r_\text{к}}\cdot \qquad (2)$$

where M_e – engine torque in steady mode; k_p – a coefficient considering the design features of the engine and operating conditions; J_e, $J_\text{к}$ – the moment of engine inertia and the total moment of inertia of the transmission and drive wheels; $u_\text{тр}$, $\eta_\text{тр}$ – gear ratio and energy conversion efficiency of transmission respectively; $r_\text{д}$,. $r_\text{к}$ – the dynamic radius and the rolling radius of the drive wheels, respectively.

Analyzing the traction and speed characteristics of an automobile with acceptable accuracy, equations of the form are most often used [1 – 4, 8 – 10]

$$P_\text{к}-P_\psi-P_\text{в}-P_j=0\,, \qquad (3)$$

where $P_\text{к}$ – total tractive force on the drive wheels; P_j – the force of resistance to acceleration of the automobile.

When converting equation (1) to type (3), the tractive force $P_\text{т}$ is replaced by relative “total tractive force” $P_\text{к}$, determined for the steady state of the automobile motion [1, 4, 8, 14].

$$P_\text{к}=\frac{M_\text{к}}{r_\text{д}}=\frac{M_e\cdot u_\text{тр}\cdot\eta_\text{тр}}{r_\text{д}}\,, \qquad (4)$$

where $M_\text{к}$ – torque on the drive wheels (in steady state).

In addition, the inertial force of gradually moving automobile weight $P_j^{'}$ is replaced by the “relative” acceleration resistance force [1, 4, 8, 14]

$$P_j=\frac{G_\text{а}}{g}\cdot j_\text{а}\cdot\delta_\text{вр}\,, \qquad (5)$$

where $\delta_\text{вр}=1+\frac{J_e\cdot u_\text{тр}^2\cdot\eta_\text{тр}+\Sigma J_\text{к}}{G_\text{а}\cdot r_\text{д}\cdot r_\text{к}}\cdot g$ – the coefficient of consideration of the rotating automobile masses.

The introduction of the coefficient of consideration of rotating masses $\delta_\text{вр}$ relatively increases the inertia (mass) of the car. The increase in inertia of the car is proportional to the decrease in tractive force compared to the relative “total tractive force”. In this case, the results of calculations according to equation (3) differ from the results of calculations according to equation insignificantly (1).

It is necessary to point out that the well-known term “acceleration resistance force” has no physical significance, because the car, like any other body, cannot resist. The physical meaning of this term is the inertia force, which is the sum of all the forces acting on the car, that is, it is the result of their action, and not the resistance. The force must be additionally applied to the car so that it starts moving with a given acceleration [13].

The equation of the car motion (3) reflects the relationship between driving forces and forces of resistance to motion. It allows determining the conditions of the possibility of motion and the mode of the car motion due to the considered road conditions. At the same time, the traction and speed characteristics of the car are determined not only in the steady state, but also in the general mode of its motion.

From equation (3), the condition of continuous motion of the car is obtained [1 – 4]

$$P_{\text{к}} \ge P_{\psi} + P_{\text{в}}\,. \tag{6}$$

Inequality (6) connects the design factors of the car with the factors that cause resistance to motion. Fulfillment of condition (6) is necessary, but not sufficient for the continuous motion of the car, since the latter is possible only in the absence of drive wheel slip. Considering this fact, continuous motion is possible if the condition [1 – 4] is fulfilled

$$P_{\varphi} \ge P_{\text{к}} \ge P_{\psi} + P_{\text{в}}\,. \tag{7}$$

where P_{φ} – the clutch force of the drive wheels of the car with the supporting surface.

Taking into consideration uniform motion, there is no inertial force (i.e., acceleration resistance force), so the equation of motion of the car will take the following form [1 – 3]

$$P_{\text{к}} - P_{\psi} - P_{\text{в}} = 0\,. \tag{8}$$

The condition of uniform motion of the car without drive wheel slip [1 – 4]

$$P_{\varphi} \ge P_{\text{к}} = P_{\psi} + P_{\text{в}}\,. \tag{9}$$

To determine the possibility and mode of motion of the car, equation (3) is presented in the form of a balance of forces [1, 4]

$$P_{\text{к}} = P_{\psi} + P_{\text{в}} + P_j \tag{10}$$

The solution of the force balance equation (10) is usually performed using a graphic and analytical method [1-4]. At the same time, graphs of the dependences of the total tractive force $P_{\text{к}}$ and the sum of the air resistance $P_{\text{в}}$ and rolling resistance P_{ψ} on the vehicle speed va are constructed (Fig. 1). And the result of their action – the force of resistance to acceleration of the car is defined as the ordinate between the dependences $P_{\text{к}}=f(v_{\text{a}})$ and $P_{\text{в}}+P_{\psi}=f(v_{\text{a}})$.) [3, 4].

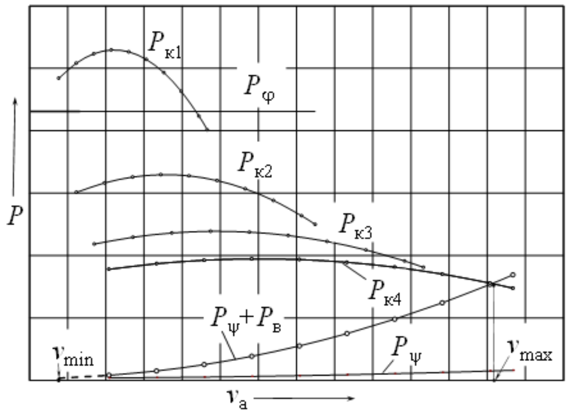

Fig. 1. The graph of the balance of forces on the car

The authors [2, 4, 8] claim that if the condition $P_{\text{к}}>P_{\varphi}$ is met, the drive wheels of the car slip. According to Figure 1, this is the motion of the car in first gear with full fuel supply up to speed v_b. It should be noted that conditions (7), (9) and equation (10) are determined relative to the total tractive force $P_{\text{к}}$, which characterizes the tractive force without taking into account the energy losses due to the increase in the energy of the rotating masses during the accelerated motion of the car. Therefore, determining the possibility of the drive wheel slip is correct only under condition (9) during uniform (steady) motion of the car.

The authors of works [1, 2, 4, 8, 9] propose to determine the possibility of the automobile drive wheel slip based on its dynamic characteristics. At the same time, they claim (11), if the dynamic factor of the car for the traction D exceeds the dynamic factor for the clutch D_{φ}, then the drive wheels slip.

$$D_{\varphi} < D\,. \tag{11}$$

The dynamic factor D is determined in accordance with the total tractive force $P_{\text{к}}$, that is, without taking into account energy losses due to the angular acceleration of the rotating masses of the car. Therefore, the determination of the possibility of the drive wheel slip during the car acceleration under condition (11) is not correct.

In the scientific work [7], the author provides a balance of forces acting on the car during acceleration, considering the energy losses due to the angular acceleration of the rotating masses, but does not determine the possibility of the drive wheel slip. The dependence of the acceleration of the car on the speed of motion with slip of the driving wheels was obtained in works [5, 12].

Therefore, the issue of determining the possibility of the automobile drive wheel slip during its acceleration requires additional research.

Purpose and Tasks

The aim of the article is to improve the dependencies that allow determining the conditions for the possibility of the automobile drive wheel slip during its acceleration.

It is necessary to perform an analysis of the balance of forces on the car and an analysis of its dynamic characteristics, taking into account energy losses due to the angular acceleration of rotating masses, and to determine the conditions for the possibility of the car moving with acceleration without and with wheel slip.

Analysis of the possibility and mode of automobile motion according to the balance of forces

According to condition (9), if the force $P_{\text{к}}$ is increased comparing the sum of $P_{\psi}+P_{\text{в}}$, then it will lead to the car acceleration in accordance with condition (8). The car acceleration will continue until the tractive force is equal to the clutch force P_{φ}. A further increase in $P_{\text{к}}$ will cause only accelerated wheel slip, without affecting the change in car motion parameters. At the same time, car motion parameters are determined by the level of the clutch force and the sum of the resistance forces.

If the condition is met

$$P_{\text{к}} \ge P_{\varphi} > P_{\psi} + P_{\text{в}} . \qquad (12)$$

the car will move with acceleration with the drive wheel slip. As the speed of the car increases, the resistance to its motion rises. Therefore, the car acceleration with the drive wheel slip occurs at the speed at which condition (8) will occur and the car motion will become steady.

$$P_{\text{к}} \ge P_{\varphi} = P_{\psi} + P_{\text{в}} . \qquad (13)$$

As one can contemplate in Figure 2, considering provided rolling conditions, the drive wheel clutch force $P_{\varphi 1}$ and the resistance force $P_{\psi 1}+P_{\text{в}}$, the car moves at an accelerated speed of v_1. At the same time, the level of acceleration determines the ordinate between the dependencies $P_{\varphi 1}$ and $P_{\psi 1}+P_{\text{в}}$.

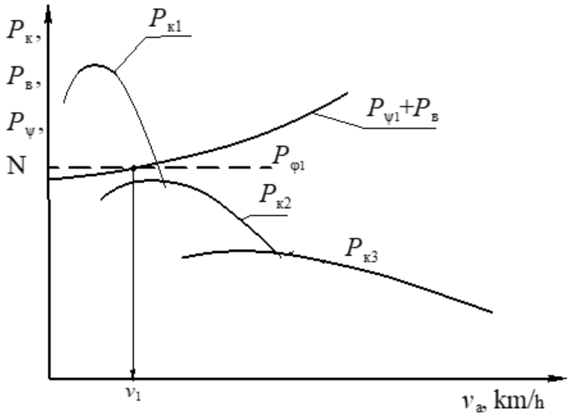

Fig. 2. Assessment of the opportunity to drive and the car driving mode

We should state that the relativity adopted when deriving the equation of motion – the force on the drive wheels $P_{\text{к}}$ refers to the steady motion of the car. Therefore, analyzing the car acceleration, condition (4) is not correct, because it does not take into account the loss due to the increase in the energy of the rotating masses. Considering Figure 2, it is possible to determine the condition of the drive wheel slip only for steady motion at a speed of v_1.

Taking into account the power consumption for the change in the energy of the rotating parts during the torque transmission to the drive wheels, the value of $P_{\text{к}}$ must be reduced according to the coefficient of the rotating masses $\delta_{\text{вр}}$. Equation (10) will be rewritten considering (5) in the following form:

$$P_{\text{к}} = P_{\psi} + P_{\text{в}} + P_j = P_{\psi} + P_{\text{в}} + P_j^{'} \cdot \delta_{\text{вр}} . \qquad (14)$$

Thus, we will get the balance equation taking into account the losses due to the change in the energy of the rotating masses during the torque transmission to the drive wheels

$$\frac{P_{\text{к}}}{\delta_{\text{вр}}} = \frac{P_{\psi} + P_{\text{в}}}{\delta_{\text{вр}}} + P_j^{'} . \qquad (15)$$

Considering the above-mentioned information, the condition of the possibility of motion without wheel slip during the car acceleration will take the form (16), since the clutch force does not depend on the inertia of the rotating masses

$$P_{\varphi} \ge \frac{P_{\text{к}}}{\delta_{\text{вр}}} \ge \frac{P_{\psi} + P_{\text{в}}}{\delta_{\text{вр}}} . \qquad (16)$$

The condition of the possibility of the car acceleration with driving wheel slip is indicated by inequality

$$\frac{P_{\text{к}}}{\delta_{\text{вр}}} \geq P_{\varphi} \geq \frac{P_{\psi}+P_{\text{в}}}{\delta_{\text{вр}}}. \qquad (17)$$

Uniform motion of the car with skidding of the driving wheels is possible if the condition is met

$$\frac{P_{\text{к}}}{\delta_{\text{вр}}} \geq P_{\varphi} = \frac{P_{\psi}+P_{\text{в}}}{\delta_{\text{вр}}}. \qquad (18)$$

In Figure 3, the steady motion of the car occurs without the drive wheel slip at a speed less than v_1. In the section from v_1 to v_2, the force $P_{\text{к}}$ exceeds the force $P_{\varphi 1}$, therefore, when the car is moving at a constant speed with a speed from this range, the drive wheels slip. In this case, it is rational to reduce the fuel supply so as to reduce the tractive force $P_{\text{к}}$ to the level of the clutch force $P_{\varphi 1}$.

In Figure 3 the total tractive force $P_{\text{к}}$ and the clutch force $P_{\varphi 1}$ exceed the total force of motion resistance $P_{\psi} + P_{\text{в}}$, so the car will move with acceleration. In such a case, the determination of the possibility of the drive wheel slip according to the condition (7) as well as the graph $P_{\text{к}}=f(v_{\text{a}})$ is not correct in the entire speed range of the car. The correct conclusion about the possibility of the drive wheel slip is only for a steady mode, that is, due to the condition (9) at v_{amax}.

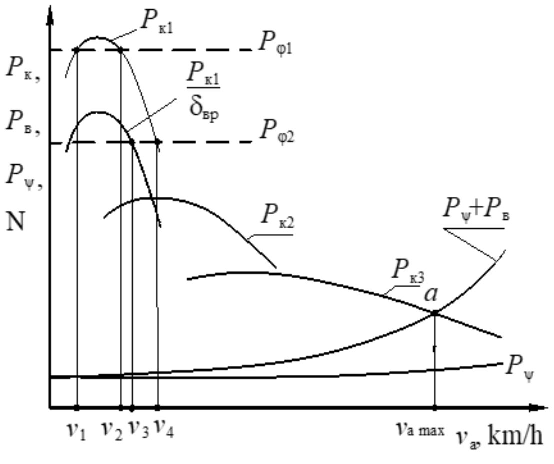

Fig. 3. The determination of the drive wheel slip according to force balance graph

During the acceleration of the car, the tractive force on the drive wheels will decrease in accordance with the energy expenditure for the acceleration of the rotating masses. Therefore, a dependency is added to the traction characteristic $\frac{P_{\text{к}}}{\delta_{\text{вр}}} = f(v_{\text{a}})$ (Fig. 3).

As we can see in Figure 3, during acceleration of the car with the same value of the clutch force $P_{\varphi 1}$ of the drive wheels with the supporting surface, in contrast to steady motion, wheel slip does not occur in the entire speed range.

When the clutch force is reduced to the value $P_{\varphi 2}$, acceleration of the car in the first gear to speed v3 occurs with the drive wheel slip. Further acceleration takes place without the drive wheel slip.

To compare the results of the analysis of the possibility and nature of the car motion and traction and speed characteristics, two equations (10) and (15) were solved on one graph (Fig. 4). A steady mode of motion is obtained, respectively, at points *b* i *b*', which corresponds to the maximum speed of the car. The maximum speed of the car due to the conditions in both cases is equal to v_b. However, the use of equation (10) to determine the possibility of the drive wheel slip at a speed lower than v_b is not correct as the car motion is accelerated.

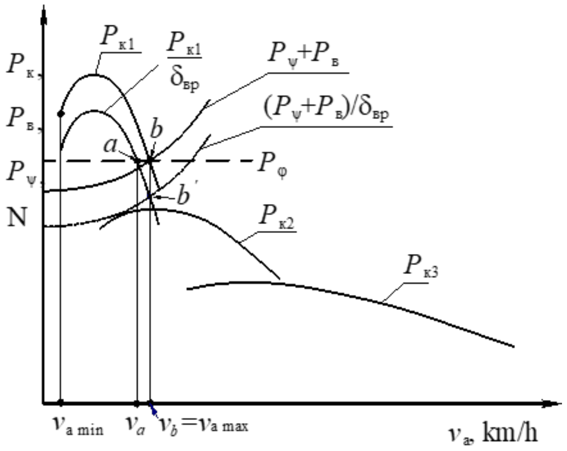

Fig. 4. Analysis of the possibility and type of car motion according to dependencies (10) and (15)

Analyzing the possibility and mode of motion of the car according to equation (15), it is obvious that the car will move with acceleration to speed v_a with slippage of the driving wheels. The motion of the car from speed v_a to speed v_b is also accelerated, but without the drive wheel slip, because of the condition

$$P_{\varphi} > \frac{P_{\text{к}}}{\delta_{\text{вр}}}. \qquad (19)$$

When the car reaches the speed $v_b = v_{a\max}$, its motion becomes constant. In this case, the full tractive force $P_к$ acts on the driving wheels, and the condition $P_к = P_\varphi$ is fulfilled

Analysis of the possibility and mode of motion of the car according to the dynamic characteristics

The graph of the dynamic characteristics of a car is built according to its dynamic balance

$$D = \psi + \frac{j_\text{а}}{g} \cdot \delta_\text{вр}. \qquad (20)$$

At the same time, the graph (Fig. 5) shows the dependences of the dynamic factor D and the total road resistance coefficient ψ on the speed of motion v_a. The second component in the right-hand part of equation (20) reflects the reserve of the dynamic factor D at a certain speed of the car v_a. This component on the graph is defined as the ordinate between dependencies $D=f(v_\text{a})$ і $\psi=f(v_\text{a})$.

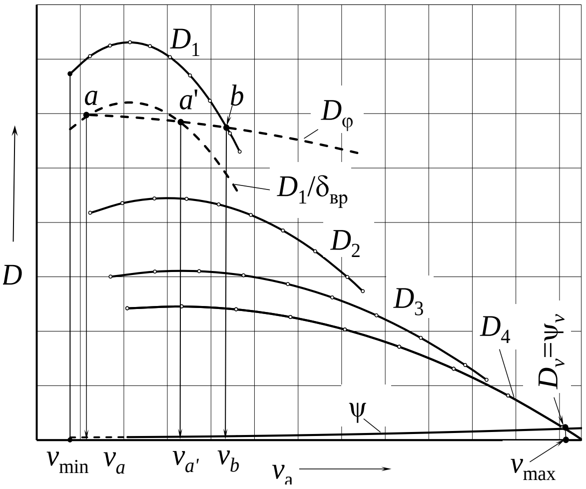

Fig. 5. The graph of the dynamic characteristics of the car

We note that the total tractive force determines the dynamic factor of the car D without taking into account the energy losses due to the increase in the frequency of rotation of the transmission elements and drive wheels. To take into account these losses during acceleration of the car, we transform the dynamic balance equation (20) into the form

$$\frac{D}{\delta_\text{вр}} = \frac{\psi}{\delta_\text{вр}} + \frac{j_\text{а}}{g}. \qquad (21)$$

Equations (20) and (21) characterize the motion of the car if the drive wheels do not slip. From equation (20) we understand that the necessary and sufficient condition for uniform motion of the car without the drive wheel slip

$$D_\varphi \ge D = \psi. \qquad (22)$$

From equation (21), we obtain a necessary and sufficient condition for the accelerated motion of the car without the drive wheel slip

$$D_\varphi \ge \frac{D}{\delta_\text{вр}} \ge \frac{\psi}{\delta_\text{вр}}. \qquad (23)$$

Concerning equation (21), we also write down the condition of accelerated motion of the car with the drive wheel slip

$$\frac{D}{\delta_\text{вр}} > D_\varphi > \frac{\psi}{\delta_\text{вр}}. \qquad (24)$$

In order to analyze the possibility and mode of motion of the car according to the dynamic characteristic, dependencies are added in Figure 5 $D_\varphi = f(v_\text{a})$ та $\frac{D_1}{\delta_\text{вр}} = f(v_\text{a})$.

The dynamic factor of the car by the clutch is defined as a ratio [1, 4, 8, 9]

$$D_\varphi = \frac{P_\varphi - P_w}{G_\text{a}}. \qquad (25)$$

Accordingly [1, 4, 8, 9] the drive wheel slip is possible when moving in the first gear with a speed the value of which corresponds to the interval $v_{\min}$ - v_b. But this is not correct, because with a given coefficient of motion resistance ψ, the motion of the car is accelerated. Condition (25) should be used to determine the possibility of the drive wheel slip during acceleration of the car. Accordingly, in Figure 5, the acceleration of the car from the speed $v_{\min}$ to v_a occurs without the wheel slip. When the car is moving at a speed in the interval $v_{a`}$ - $v_b$, the drive wheel slip is possible, if the fuel supply is not reduced. Accelerated motion of the car at a speed greater than $v_{a`}$` occurs without the drive wheel slip.

Theoretical studies of the driving mode of the car

Theoretical studies of car driving modes have been conducted to compare the results of determining the speeds at which the drive wheels can slip with the results provided in works [5, 12].

For the study, the record-breaking racing car KHADI-37 was chosen with the following parameters: total weight m_a = 732 kg; maximum engine power N_{emax} =220 kW at crankshaft rotation frequency n_N =5800 rpm; base L=2.545 m; the height of the center of gravity h_g =0.668 m, its front longitudinal coordinate a=1.742 m. The possibility and nature of the car motion on a horizontal surface with the clutch coefficient of the drive wheels was studied φ_x=1,2.

According to the results, the car can move with acceleration up to a maximum speed of 242 km/h (Fig. 6). It is obvious that according to condition (16) when accelerating the car to speed v_a, the drive wheels cannot slip. At the speed v_a to the speed v_a ` the motion of the car due to the condition (17) can occur with the drive wheel slip. Further increase in the speed of the car occurs without slip. The obtained data coincide with the results provided in works [5, 12]. In Figure 6, the points of intersection of the dependences of the acceleration of the car j_a and the maximum possible acceleration under the conditions of the clutch j_φ on the speed of motion correspond to the values of the interval determined by condition (17).

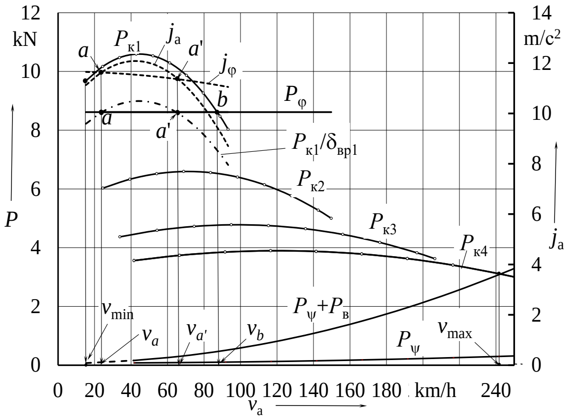

Fig. 6. Analysis of the possibility and type of the car motion according to the balance of forces

The possibility of wheel skidding according to the graph of the dynamic characteristics of the car (Fig. 7) was determined according to conditions (23) – (25). The results of determining the range of speeds during car acceleration according to the balance of forces and dynamic characteristics are the same.

During the operation of the car, the clutch ratio changes in a significant range so that the total tractive force on the drive wheels and in higher gears can exceed the clutch force. In this case, in order to determine the traction and speed characteristics of the car and the possibility of the drive wheel slip during the acceleration and a steady mode of motion, it is necessary to solve equation (15) on the force balance graph.

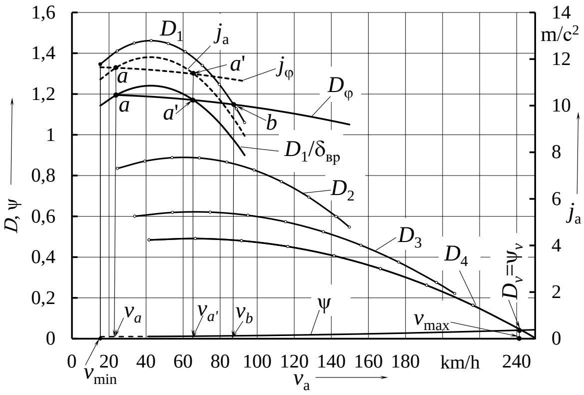

Fig. 7. Analysis of the possibility and type of the car motion according to the dynamic characteristic

Figure 8 shows the balance of forces of the KHADI-37 car according to equations (14) and (15). Since the clutch force is equal to $P_{к1}>P_{\varphi1}$, when accelerating the car, the drive wheels of the car cannot slip (condition (16). Determination of the possibility of the drive wheels slip according to condition (7) [1 – 4], i.e., when $P_{к1}>P_{\varphi1}$ is not correct because the motion is accelerated. Considering the value of $P_{\varphi1}$, due to the condition (7), it is possible to determine the limit of slip (point d), but only with uniform motion, that is, when $P_к=P_\varphi=P_\psi+P_w$.

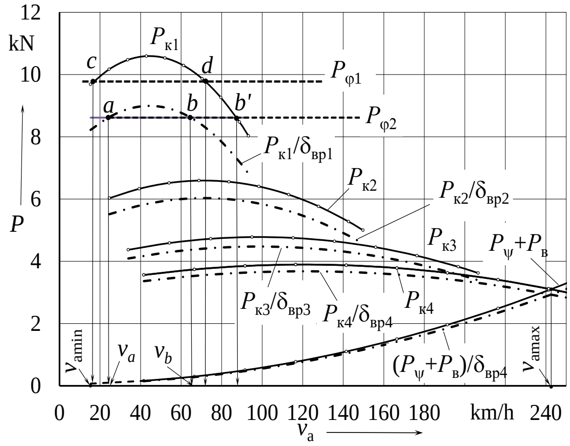

Fig. 8. The schedule of the balance of forces of the KHADI-37 car

The acceleration of the car during acceleration will be the highest if the drive wheels do not slip. Therefore, the correct determination of the possibility of the drive wheel slip is of practical importance. Points *a* and *b* in Figure 8 correspond to the initial and final speed of motion with the possibility of the drive wheel slip on the road with the coefficient of clutch φ_2. To avoid the drive wheel slip and to ensure full use of their traction characteristics, it is necessary to reduce the fuel supply so that equality is achieved

$$\frac{P_к}{\delta_{вр}} = P_\varphi . \qquad (26)$$

При цьому крутний момент двигуна виразимо через його потужність при певної частоті обертання we will find the left side of equation in order to an analytical determination of the range of the automobile speed, where the drive wheel slip is possible, (27). At the same time, the torque of the engine is expressed in terms of its power at a certain rotation frequency n_e.

$$\frac{P_к}{\delta_{вр}} = \frac{M_e \cdot u_{тр} \cdot \eta_{тр}}{\delta_{вр} \cdot r_д} = \frac{9550 N_{e\max} \cdot u_{тр} \cdot \eta_{тр}}{\delta_{вр} \cdot r_д \cdot n_e} \times$$
$$\times \left[A_1 \cdot \frac{n_e}{n_N} + A_2 \cdot \left(\frac{n_e}{n_N}\right)^2 - \left(\frac{n_e}{n_N}\right)^3 \right] , \qquad (27)$$

where $N_{e\max}$ – maximum engine power, kW; n_e, n_N – crankshaft rotation frequency is the answer to the torque M_e, rpm; A_1, A_2 – coefficients of the polynomial describing the dependence of the engine power on the rotation frequency [3, 8, 10, 14].

From equation (27) taking into account dependence (28), we get

$$\frac{A_1}{n_N} + \frac{A_2}{n_N^2} \cdot n_e - \frac{1}{n_N^3} \cdot n_e^2 = \frac{P_\varphi \cdot r_д \cdot \delta_{вр}}{9550 N_{e\max} \cdot u_{тр} \cdot \eta_{тр}} . \qquad (28)$$

We transform equation (29) into

$$n_e^2 + p \cdot n_e + q = 0 , \qquad (29)$$

where $p = -A_2 \cdot n_N$;

$$q = \frac{P_\varphi \cdot r_д \cdot \delta_{вр} \cdot n_N^3}{9550 N_{e\max} \cdot u_{тр} \cdot \eta_{тр}} - A_1 \cdot n_N^2 .$$

The solution of equation (30) has two roots

$$n_{e(1,2)} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q} . \qquad (30)$$

The shaft rotation frequency n_{e1} corresponds to the initial speed v_a (Fig. 8), at which the drive wheels can start to slip, and n_{e2} corresponds to the final speed of the vehicle v_b on the road with the clutch force $P_{\varphi 2}$. A further increase in the rotation frequency n_e will lead to an increase in the car speed without the possibility of the drive wheel slip. Analytical determination of the values of v_a and v_b is performed according to the well-known formula [9].

$$v_{a,b} = 0{,}377 \frac{n_{e(1,2)} \cdot r_д}{u_к \cdot u_0} . \qquad (31)$$

When the KHADI-37 car accelerates on the road with a clutch force of $P_{\varphi 2}$=8616 N, the drive wheel slip can start at the engine shaft rotation frequency n_{e1}=1570 rpm. In order to avoid the drive wheel slip, it is necessary to reduce fuel supply so that equation (27) is fulfilled. After increasing the rotation frequency above n_{e2}=4232 rpm, the drive wheels of the car will not be able to slip, so it is rational to increase the fuel supply to the maximum. Concerning this engine control, one can ensure better acceleration dynamics of the car (Fig. 9).

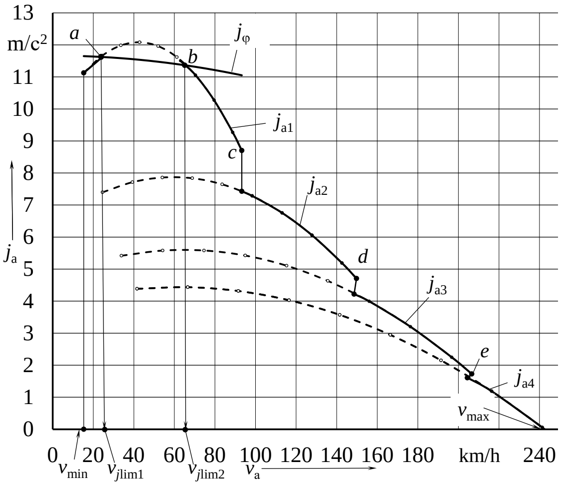

Fig. 9. The acceleration schedule of the car without the drive wheel slip

The motion of the car in the range of speeds $v_{jlim1} – v_{jlim2}$ takes place with the maximum – limit acceleration due to the clutch j_{lim}. The values of the speeds v_a and v_b are determined by formula (32) are 24.1 km/h and 64.7 km/h, respectively, and correspond to the values obtained graphically in Figures 8 and 9.

Conclusions

In the article, the obtained dependencies that allow determining the conditions and modes of the car, the conditions of the possibility of the automobile drive wheel slip, both during steady motion and during acceleration. These dependencies are based on the equation of the car motion and represent the balance of forces acting on the car and the dynamic balance. Graph and analytical methods of solving the balance of forces and dynamic balance allow determining the traction and speed characteristics of the car in the general case of motion due to normal and extreme conditions of the clutch of the drive wheels. Analytical dependences were obtained, which allow determining the value of the engine parameters and the value of the speed of the car at the beginning and at the end of the section with the boundary conditions of the clutch.

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Conflict of interests

The authors declare that there is no conflict of interest concerning the article publication.

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Визначення можливості буксування ведучих коліс автомобіля

Анотація. Проблема. *Недоліком існуючих залежностей визначення можливості буксування ведучих коліс автомобіля при максимальному використанні потужності двигуна і показників взаємодії ведучих коліс з опорною поверхнею є те, що вони є коректними тільки якщо рух автомобіля є сталим. Цей недолік обумовлений тим що при виводі рівняння руху автомобіля прийнята умовність, що на ведучих колесах діє повна сила тяги, тобто без врахування втрат енергії на збільшення кінетичної енергії обертових мас двигуна, трансмісії, ведучих коліс. При цьому вказані втрати енергії компенсуються умовним збільшенням інертності маси автомобіля, що рухається поступово. Таке припущення цілком коректно для визначення тягово-швидкісних властивостей автомобіля не тільки при сталому русі, але і при прискореному. Але при визначенні можливості буксування ведучих коліс при розгоні автомобіля прийняте припущення не коректно так, як на ведучих колесах діє не повна сила тяги, а сила, яка зменшена відповідно втратам енергії на збільшення кінетичної енергії обертових мас двигуна, трансмісії, ведучих коліс. Щоб усунути зазначений недолік необхідно враховувати зменшення сили тяги на ведучих колесах при прискоренні автомобіля .* ***Мета.*** *Мета роботи - подальший розвиток теорії автомобіля шляхом удосконалення залежностей, які дозволяють визначати умови можливості буксування ведучих коліс автомобіля при його прискоренні та режими його руху.* ***Методологія****. Підходи, прийняті в роботі для досягнення цієї мети базуються на законах фізики, теоретичної механіки і положеннях теорії автомобіля.* ***Результати.*** *Удосконалено аналітичні залежності для визначення умов можливості буксування ведучих коліс автомобіля при його прискоренні та режими його руху. Отримано залежності для визначення діапазону буксування ведучих коліс від швидкості руху при розгоні автомобіля. При теоретичному дослідженні процесу руху автомобіля встановлено, що розроблені залежності дозволяють визначити характер руху автомобіля і можливість буксування ведучих коліс як при розгоні, так і при сталому русі.* ***Оригінальність.*** *Отримані залежності для визначення можливості буксування ведучих коліс при розгоні автомобіля дозволили уточнити уявлення про характер руху автомобіля і можливість буксування ведучих коліс як при розгоні, так і при сталому русі.*

***Практичне значення**. Отримані залежності можуть бути використані при проектуванні нових і при удосконаленні спортивних автомобілів типу дрегстер, та для аналізу динаміки руху автомобіля при його розгоні з повної подачею палива та визначення характеру взаємодії ведучих коліс з опорної поверхнею в залежності від швидкості руху.*

***Ключові слова:** автомобіль, рівняння руху, динамічний фактор, динамічний фактор за зчепленням, прискорення, повна сила тяги, коефіцієнт зчеплення, обертові маси.*

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